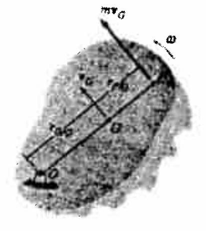
19-1. The rigid body (slab) has a mass m and is rotating with an angular velocity $\boldsymbol{\omega}$ about an axis passing through the fixed point O. Show that the momenta of all the particles composing the body can be represented by a single vector having a magnitude mv_G and acting through point P, called the *center of percussion*, which lies at a distance $r_{P/G} = k_G^2/r_{G/O}$ from the mass center G. Here k_G is the radius of gyration of the body, computed about an axis perpendicular to the plane of motion and passing through G.

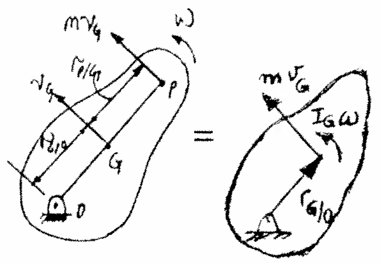


$$H_O = (r_{G/O} + r_{P/G})\, mv_G = r_{G/O}(mv_G) + I_G\omega, \qquad \text{where } I_G = mk_G^2$$

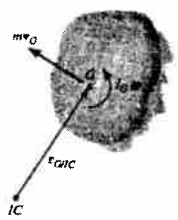
$$r_{G/O}(mv_G) + r_{P/G}(mv_G) = r_{G/O}(mv_G) + \left(mk_G^2\right)\omega$$

$$r_{P/G} = \frac{k_G^2}{v_G/\omega} \qquad \text{However, } v_G = \omega r_{G/O} \text{ or } r_{G/O} = \frac{v_G}{\omega}$$

$$r_{P/G} = \frac{k_G^2}{r_{G/O}} \qquad \textbf{(QED)}$$



19-2. At a given instant, the body has a linear momentum $\mathbf{L} = m\mathbf{v}_G$ and an angular momentum $\mathbf{H}_G = I_G\boldsymbol{\omega}$ computed about its mass center. Show that the angular momentum of the body computed about the instantaneous center of zero velocity IC can be expressed as $\mathbf{H}_{IC} = I_{IC}\boldsymbol{\omega}$, where I_{IC} represents the body's moment of inertia computed about the instantaneous axis of zero velocity. As shown, the IC is located at a distance $r_{G/IC}$ away from the mass center G.



$$H_{IC} = r_{G/IC}(mv_G) + I_G\omega, \qquad \text{where } v_G = \omega\, r_{G/IC}$$

$$= r_{G/IC}(m\omega\, r_{G/IC}) + I_G\omega$$

$$= \left(I_G + mr_{G/IC}^2\right)\omega$$

$$= I_{IC}\,\omega \qquad \textbf{(QED)}$$

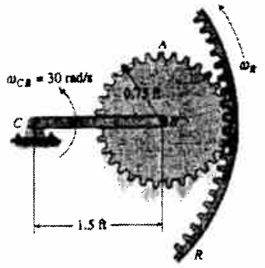
19-3. Show that if a slab is rotating about a fixed axis perpendicular to the slab and passing through its mass center G, the angular momentum is the same when computed about any other point P on the slab.

Since $v_G = 0$, the linear momentum $L = mv_G = 0$. Hence the angular momentum about any point P is

$$H_P = I_G\,\omega$$

Here ω is a free vector so is $\mathbf{H}_P$. **(QED)**

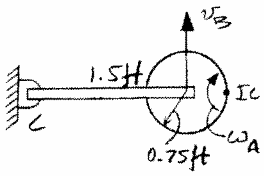
***19-4.** Gear A rotates along the inside of the circular gear rack R. If A has a weight of 4 lb and a radius of gyration of $k_B = 0.5$ ft, determine its angular momentum about point C when $\omega_{CB} = 30$ rad/s and (a) $\omega_R = 0$, (b) $\omega_R = 20$ rad/s.



a)

$v_B = (1.5)(30) = 45$ ft/s

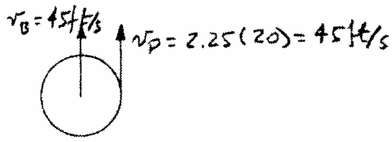
$\omega_A = \dfrac{45}{0.75} = 60$ rad/s



$(\circlearrowleft+)\quad H_C = \left(\dfrac{4}{32.2}\right)(45)(1.5) - \left[\left(\dfrac{4}{32.2}\right)(0.5)^2\right](60)$

$= 6.52$ slug · ft²/s **Ans**

b)



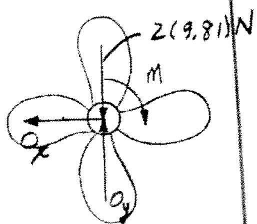
$v_B = 1.5(30) = 45$ ft/s

$\omega_A = 0$

$(\circlearrowleft+)\quad H_C = \left(\dfrac{4}{32.2}\right)(45)(1.5)$

$= 8.39$ slug · ft²/s **Ans**

19-5. Solve Prob. 17-55 using the principle of impulse and momentum.



$(\circlearrowleft+)\qquad (H_O)_1 + \Sigma\int M_O\,dt = (H_O)_2$

$0 + \int_0^4 3\left(1 - e^{-0.2t}\right)dt = (0.18)\,\omega$

$3\left(t + 5e^{-0.2t}\right)\Big|_0^4 = 0.18\omega$

$\omega = 20.8$ rad/s **Ans**

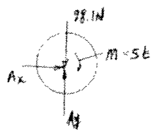
19–6. Solve Prob. 17–54 using the principle of impulse and momentum.

$$(H_A)_1 + \Sigma\int M_A\,dt = (H_A)_2$$

$$0 + \int_0^3 5\,t\,dt = [10(0.2)^2]\omega$$

$$\frac{5}{2}(3)^2 = 0.4\omega$$

$$\omega = 56.2 \text{ rad/s}$$ **Ans**



$$m(v_G)_1 + \Sigma\int F\,dt = m(v_G)_2$$

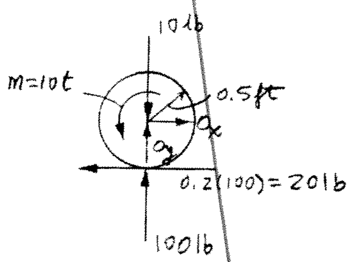
$(\xrightarrow{+})$ $\quad 0 + A_x(3) = 0$

$$A_x = 0$$ **Ans**

$(+\uparrow)$ $\quad 0 + A_y(3) - 98.1(3) = 0$

$$A_y = 98.1 \text{ N}$$ **Ans**

19-7. Solve Prob. 17-69 using the principle of impulse and momentum.



Time to start motion :

$$F = 100(0.3) = 30 \text{ lb}$$

$$+\Sigma M_O = 0; \qquad 10t - 30(0.5) = 0$$

$$t = 1.5 \text{ s}$$

$(\circlearrowleft +)$ $\qquad (H_O)_1 + \Sigma\int M_O\,dt = (H_O)_2$

$$0 + \int_{1.5}^{2} 10t\,dt - 20(2 - 1.5)(0.5) = \left[\frac{1}{2}\left(\frac{10}{32.2}\right)(0.5)^2\right]\omega$$

$$5t^2\Big|_{1.5}^{2} - 5 = 0.038820\omega$$

$$\omega = 96.6 \text{ rad/s}$$ **Ans**

***19-8.** Solve Prob. 17-80 using the principle of impulse and momentum.

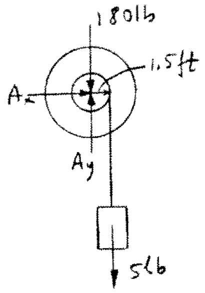
System :

$v_B = \omega(1.5)$

$(\circlearrowleft +) \qquad (H_A)_1 + \Sigma \int M_A\, dt = (H_A)_2$

$$0 + 5(1.5)(3) = \left[\left(\frac{180}{32.2}\right)(1.25)^2\right]\omega + \left(\frac{5}{32.2}\right)[\omega(1.5)](1.5)$$

$\omega = 2.48$ rad/s **Ans**

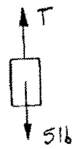


Block :

$v_B = \omega(1.5)$

$(+\downarrow) \qquad m(v_G)_1 + \Sigma \int F_y\, dt = m(v_G)_2$

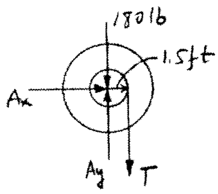
$$0 + 5(3) - T(3) = \frac{5}{32.2}[\omega(1.5)] \qquad (1)$$



Spool :

$(\circlearrowleft +) \qquad (H_A)_1 + \Sigma \int M_A\, dt = (H_A)_2$

$$0 + T(1.5)(3) = \left[\left(\frac{180}{32.2}\right)(1.25)^2\right]\omega \qquad (2)$$



Solving Eqs. (1) and (2) :

$T = 4.81$ lb

$\omega = 2.48$ rad/s **Ans**

19-9. Solve Prob. 17-73 using the principle of impulse and momentum.

$(\overset{+}{\rightarrow}) \qquad m(v_{Gx})_1 + \Sigma \int F_x\, dt = m(v_{Gx})_2$

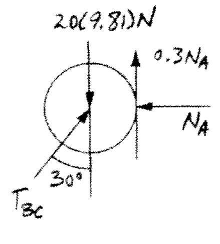
$$0 + T_{BC}\sin 30^\circ (t) - N_A t = 0$$

$$0.5T_{BC} = N_A$$

$(+\uparrow) \qquad m(v_{Gy})_1 + \Sigma \int F_y\, dt = m(v_{Gy})_2$

$$0 + T_{BC}\cos 30^\circ t - 20(9.81)t + 0.3N_A t = 0$$

$$0.86603T_{BC} + 0.3N_A = 196.2$$



Solving :

$T_{BC} = 193$ N **Ans**

$N_A = 96.553$ N

$(\circlearrowleft +) \qquad (H_B)_1 + \Sigma \int M_B\, dt = (H_B)_2$

$$\left[\frac{1}{2}(20)(0.15)^2\right](60) - 0.3(96.553)t(0.15) = 0$$

$t = 3.11$ s **Ans**

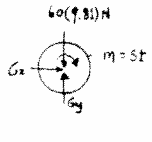
4

19-10. A flywheel has a mass of 60 kg and a radius of gyration of $k_G = 150$ mm about an axis of rotation passing through its mass center. If a motor supplies a clockwise torque having a magnitude of $M = (5t)$ N·m, where t is in seconds, determine the flywheel's angular velocity in $t = 3$ s. Initially the flywheel is rotating clockwise at $\omega_1 = 2$ rad/s.

$(\curvearrowright +) \qquad (H_G)_1 + \Sigma\int M\,dt = (H_G)_2$

$$60(0.15)^2(2) + \int_0^3 5t\,dt = 60(0.15)^2\,\omega$$

$\omega = 18.7$ rad/s **Ans**



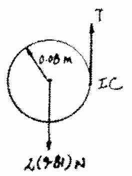
5

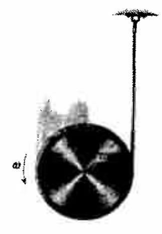
19-11. A wire of negligible mass is wrapped around the outer surface of the 2-kg disk. If the disk is released from rest, determine its angular velocity in 3 s.

$(\curvearrowright +) \qquad I_{IC}\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_{IC}\,dt = I_{IC}\,\omega_2$

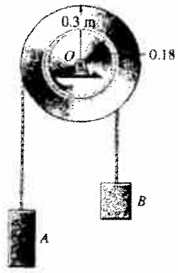
$$0 + 2(9.81)(0.08)(3) = \left[\frac{1}{2}(2)(0.08)^2 + 2(0.08)^2\right]\omega_2$$

$\omega_2 = 245$ rad/s **Ans**





***19-12.** The spool has a mass of 30 kg and a radius of gyration $k_O = 0.25$ m. Block A has a mass of 25 kg, and block B has a mass of 10 kg. If they are released from rest, determine the time required for block A to attain a speed of 2 m/s. Neglect the mass of the ropes.



$v_A = 2$ m/s

$$\omega = \frac{2}{0.3} = 6.667 \text{ rad/s}$$

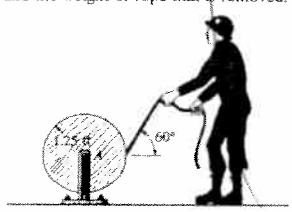
$v_B = 6.667(0.18) = 1.20$ m/s

$(\curvearrowleft +) \qquad (H_O)_1 + \Sigma\int M\,dt = (H_O)_2$

$$0 + 25(9.81)(0.3)\,t - 10(9.81)(0.18)(t) = 25(2)(0.3) + 30(0.25)^2(6.667) + 10(1.20)(0.8)$$

$t = 0.530$ s **Ans**

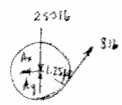
19-13. The man pulls the rope off the reel with a constant force of 8 lb in the direction shown. If the reel has a weight of 250 lb and radius of gyration $k_G = 0.8$ ft about the trunnion (pin) at A, determine the angular velocity of the reel in 3 s starting from rest. Neglect friction and the weight of rope that is removed.



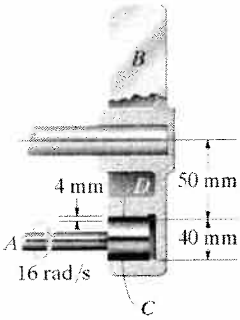
$(\curvearrowleft +) \qquad (H_A)_1 + \Sigma\int M_A\,dt = (H_A)_2$

$$0 + 8(1.25)(3) = \left[\frac{250}{32.2}(0.8)^2\right]\omega$$

$\omega = 6.04$ rad/s **Ans**



19–14. Angular motion is transmitted from a driver wheel A to the driven wheel B by friction between the wheels at C. If A always rotates at a constant rate of 16 rad/s, and the coefficient of kinetic friction between the wheels is $\mu_k = 0.2$, determine the time required for B to reach a constant angular velocity once the wheels make contact with a normal force of 50 N. What is the final angular velocity of wheel B? Wheel B has a mass of 90 kg and a radius of gyration about its axis of rotation of $k_G = 120$ mm.



$F = 0.2(50) = 10$ N

$\circlearrowleft + \quad (H_O)_1 + \Sigma\int M_O dt = (H_O)_2$

$0 + (10)(0.09)\, t = [90(0.120)^2]\,\omega$

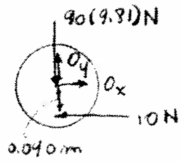
$\omega_A r_A = \omega_B r_B$

$16(20) = \omega_B(90)$

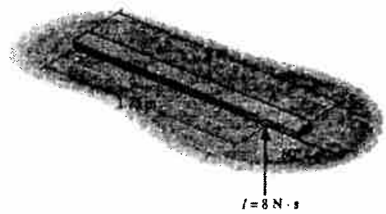
$\omega_B = 3.556$ rad/s $= 3.56$ rad/s **Ans**

Thus,

$t = 5.12$ s **Ans**



19-15. The 4-kg slender rod rests on a smooth floor. If it is kicked so as to receive a horizontal impulse $I = 8$ N·s at point A as shown, determine its angular velocity and the speed of its mass center.



$(\overset{+}{\leftarrow}) \quad m(v_{Gx})_1 + \Sigma\int F_x dt = m(v_{Gx})_2$

$0 + 8\cos 60° = 4(v_G)_x$

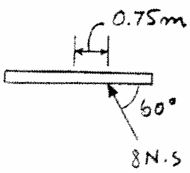
$(v_G)_x = 1$ m/s

$(+\uparrow) \quad m(v_{Gy})_1 + \Sigma\int F_y dt = m(v_{Gy})_2$

$0 + 8\sin 60° = 4(v_G)_y$

$(v_G)_y = 1.732$ m/s

$v_G = \sqrt{(1.732)^2 + (1)^2} = 2$ m/s **Ans**



$(\circlearrowleft +) \quad (H_G)_1 + \Sigma\int M_G\, dt = (H_G)_2$

$0 + 8\sin 60°(0.75) = \left[\frac{1}{12}(4)(2)^2\right]\omega$

$\omega = 3.90$ rad/s **Ans**

*19–~~16~~ 8. A cord of negligible mass is wrapped around the outer surface of the ~~50-lb cylinder and its end is subjected~~ 250N to a constant horizontal force of P = ~~2 lb~~ 10N. If the cylinder rolls without slipping at A, determine its angular velocity in 4 s starting from rest. Neglect the thickness of the cord.

$v_G = 0.6\omega$

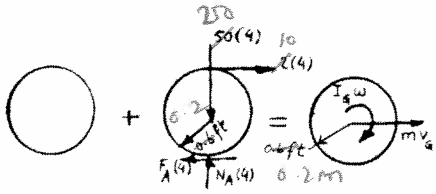
$(\circlearrowleft +) \quad (H_A)_1 + \Sigma\int M_A\, dt = (H_A)_2$

$$0 + \cancel{2}^{10}(4)\cancel{(1.2)}^{0.4} = \left[\frac{1}{2}\left(\frac{\overset{250}{50}}{\underset{9.81}{32.2}}\right)(0.\cancel{6})^{2}\right]\omega + \left(\frac{\overset{250}{50}}{\underset{9.81}{32.2}}\right)(0.\cancel{6}\omega)(0.\cancel{6})$$

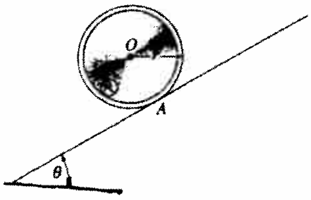
0.2 0.2

$\omega = 11.4$ rad/s **Ans**

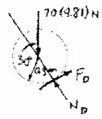
10.46 rad/s



19-~~17~~ 9. The drum has a mass of 70 kg, a radius of 300 mm, and radius of gyration k_O = 125 mm. If the coefficients of static and kinetic friction at A are μ_s = 0.4 and μ_k = 0.3, respectively, determine the drum's angular velocity 2 s after it is released from rest. Take θ = 30°.



Assume no slipping



$\circlearrowright + \quad (H_A)_1 + \Sigma\int M_A\, dt = (H_A)_2$

$0 + 70(9.81)(\sin 30°)(0.3)(2) = (70(0.125)^2 + 70(0.3)^2)\omega$

$\omega = 27.863$ rad/s $= 27.9$ rad/s **Ans**

$m(v_x)_1 + \Sigma\int F_x\, dt = m(v_x)_2$

$+\swarrow \quad 0 + 70(9.81)\sin 30°(2) - F(2) = 70(27.863)(0.3)$

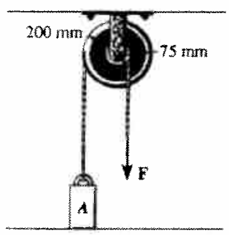
$F = 50.79$ N

$+\nwarrow \quad 0 + N_D(2) - 70(9.81)\cos 30°(2) = 0$

$N_D = 594.7$ N

$F_{max} = 0.4[(594.7] = 237.87$ N > 50.79 N **OK**

19-18. The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass of 15 kg and a radius of gyration k_O = 110 mm. If the block at A has a mass of 40 kg, determine the speed of the block in 3 s after a constant force F = 2 kN is applied to the rope wrapped around the inner hub of the pulley. The block is originally at rest. Neglect the mass of the rope.

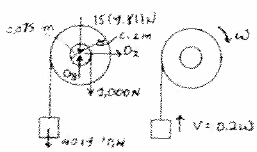


$(\circlearrowleft +) \quad (H_O)_1 + \Sigma\int M_O dt = (H_O)_2$

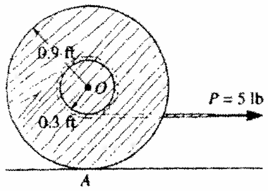
$0 + 2000(0.075)(3) - 40(9.81)(0.2)(3) = 15(0.110)^2\omega + 40(0.2\omega)(0.2)$

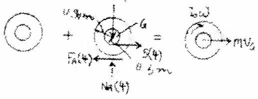
$\omega = 120.4$ rad/s

$v_A = 0.2(120.4) = 24.1$ m/s **Ans**



19-19. The spool has a weight of 30 lb and a radius of gyration $k_O = 0.45$ ft. A cord is wrapped around its inner hub and the end subjected to a horizontal force $P = 5$ lb. Determine the spool's angular velocity in 4 s starting from rest. Assume the spool rolls without slipping.





$(+\uparrow)\quad m(v_y)_1 + \Sigma\int F_y\,dt = m(v_y)_2$

$0 + N_A(4) - 30(4) = 0$

$N_A = 30$ lb

$(\overset{+}{\rightarrow})\quad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$

$0 + 5(4) - F_A(4) = \frac{30}{32.2}v_G$

$(\circlearrowleft +)\quad (H_G)_1 + \Sigma\int M_G\,dt = (H_G)_2$

$0 + F_A(4)(0.9) - 5(4)(0.3) = \frac{30}{32.2}(0.45)^2\,\omega$

Since no slipping occurs

Set $v_G = 0.9\omega$

$F_A = 2.33$ lb

$\omega = 12.7$ rad/s **Ans**

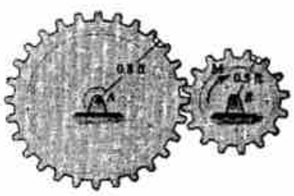
Also,

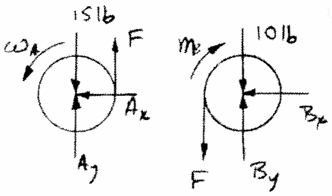
$(\circlearrowright +)\quad (H_A)_1 + \Sigma M_A\,dt = (H_A)_2$

$0 + 5(4)(0.6) = \left[\frac{30}{32.2}(0.45)^2 + \frac{30}{32.2}(0.9)^2\right]\omega$

$\omega = 12.7$ rad/s **Ans**

***19-20.** The two gears A and B have weights and radii of gyration of $W_A = 15$ lb, $k_A = 0.5$ ft and $W_B = 10$ lb, $k_B = 0.35$ ft, respectively. If a motor transmits a couple moment to gear B of $M = 2(1 - e^{-0.5t})$ lb·ft, where t is in seconds, determine the angular velocity of gear A in $t = 5$ s, starting from rest.





$\omega_A(0.8) = \omega_B(0.5)$

$\omega_B = 1.6\omega_A$

Gear B :

$(\circlearrowleft +)\qquad (H_B)_1 + \Sigma\int M_B\,dt = (H_B)_2$

$$0 + \int_0^5 2\left(1 - e^{-0.5t}\right)dt - \int 0.5F\,dt = \left[\left(\frac{10}{32.2}\right)(0.35)^2\right](1.6\omega_A)$$

$$6.328 = 0.5\int F\,dt + 0.06087\omega_A \qquad (1)$$

Gear A :

$(\circlearrowright +)\qquad (H_A)_1 + \Sigma\int M_A\,dt = (H_A)_2$

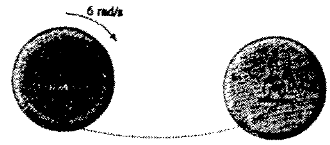
$$0 + \int 0.8F\,dt = \left[\left(\frac{15}{32.2}\right)(0.5)^2\right]\omega_A$$

$$0 = 0.8\int F\,dt - 0.1165\omega_A \qquad (2)$$

Eliminate $\int F\,dt$ between Eqs. (1) and (2), and solving for ω_A,

$\omega_A = 47.3$ rad/s **Ans**

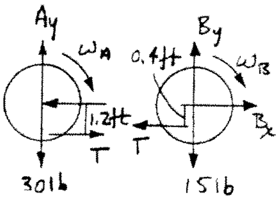
19-21. Spool B is at rest and spool A is rotating at 6 rad/s when the slack in the cord connecting them is taken up. Determine the angular velocity of each spool immediately after the cord is jerked tight by the spinning of spool A. The weights and radii of gyration of A and B are $W_A = 30$ lb, $k_A = 0.8$ ft and $W_B = 15$ lb, $k_B = 0.6$ ft, respectively.



Spool A :

$$(\circlearrowleft +) \qquad (H_A)_1 + \Sigma\int M_A\, dt = (H_A)_2$$

$$\left[\left(\frac{30}{32.2}\right)(0.8)^2\right](6) - \int T\, dt(1.2) = \left[\left(\frac{30}{32.2}\right)(0.8)^2\right]\omega_A \qquad (1)$$



Spool B :

$$(\circlearrowleft +) \qquad (H_B)_1 + \Sigma\int M_B\, dt = (H_B)_2$$

$$0 + \int T\, dt(0.4) = \left[\left(\frac{15}{32.2}\right)(0.6)^2\right]\omega_B \qquad (2)$$

Since $\quad 1.2\omega_A = 0.4\omega_B$

$$\omega_B = 3\omega_A \qquad (3)$$

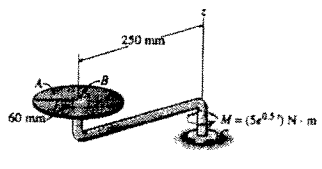
Solving Eqs. (1) – (3),

$\omega_A = 1.70$ rad/s **Ans**

$\omega_B = 5.10$ rad/s **Ans**

10

19-22. A 4-kg disk A is mounted on arm BC, which has a negligible mass. If a torque of $M = (5e^{0.5t})$ N·m, where t is in seconds, is applied to the arm at C, determine the angular velocity of BC in 2 s starting from rest. Solve the problem assuming that (a) the disk is set in a smooth bearing at B so that it rotates with curvilinear translation, (b) the disk is fixed to the shaft BC, and (c) the disk is given an initial freely spinning angular velocity of $\boldsymbol{\omega}_D = \{-80\mathbf{k}\}$ rad/s prior to application of the torque.



a)

$$(H_z)_1 + \Sigma\int M_z\, dt = (H_z)_2$$

$$0 + \int_0^2 5e^{0.5t} dt = 4(v_B)(0.25)$$

$$\frac{5}{0.5}e^{0.5t}\Big|_0^2 = v_B$$

$$v_B = 17.18 \text{ m/s}$$

Thus,

$$\omega_{BC} = \frac{17.18}{0.25} = 68.7 \text{ rad/s} \qquad \textbf{Ans}$$

b)

$$(H_z)_1 + \Sigma\int M_z\, dt = (H_z)_2$$

$$0 + \int_0^2 5e^{0.5t} dt = 4(v_B)(0.25) + \left[\frac{1}{2}(4)(0.06)^2\right]\omega_{BC}$$

Since $v_B = 0.25\omega_{BC}$, then

$\omega_{BC} = 66.8$ rad/s **Ans**

c)

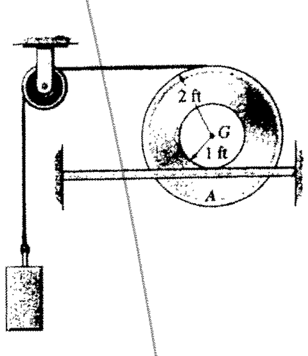
$$(H_z)_1 + \Sigma\int M_z\, dt = (H_z)_2$$

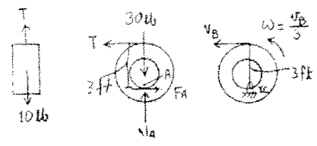
$$-\left[\frac{1}{2}(4)(0.06)^2\right](80) + \int_0^2 5e^{0.5t} dt = 4(v_B)(0.25) - \left[\frac{1}{2}(4)(0.06)^2\right](80)$$

Since $v_B = 0.25\omega_{BC}$,

$\omega_{BC} = 68.7$ rad/s **Ans**

19-23. The inner hub of the wheel rests on the horizontal track. If it does not slip at A, determine the speed of the 10-lb block in 2 s after the block is released from rest. The wheel has a weight of 30 lb and a radius of gyration $k_G = 1.30$ ft. Neglect the mass of the pulley and cord.





Spool,

$(\circlearrowleft +) \quad (H_A)_1 + \Sigma \int M_A \, dt = (H_A)_2$

$$0 + T(3)(2) = [\frac{30}{32.2}(1.3)^2 + \frac{30}{32.2}(1)^2](\frac{v_B}{3})$$

Block,

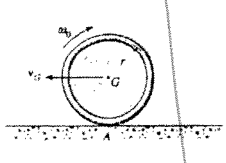
$(+\downarrow) \quad m(v_y)_1 + \Sigma \int F_y \, dt = m(v_y)_2$

$$0 + 10(2) - T(2) = \frac{10}{32.2} v_B$$

$v_B = 34.0$ ft/s **Ans**

$T = 4.73$ lb

***19-24.** If the hoop has a weight W and radius r and is thrown onto a *rough surface* with a velocity $\mathbf{v}_G$ parallel to the surface, determine the amount of backspin, ω_0, it must be given so that it stops spinning at the same instant that its forward velocity is zero. It is not necessary to know the coefficient of kinetic friction at A for the calculation.



$(\overset{+}{\leftarrow}) \quad m(v_{Gx})_1 + \Sigma \int F_x \, dt = m(v_{Gx})_2$

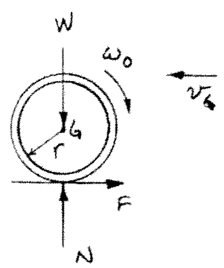
$$\frac{W}{g} v_G - Ft = 0 \qquad (1)$$

$(\circlearrowleft +) \quad (H_G)_1 + \Sigma \int M_G \, dt = (H_G)_2$

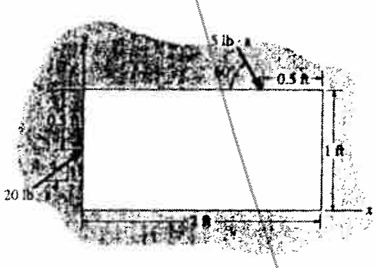
$$-\left(\frac{W}{g} r^2\right)\omega_0 + Ft(r) = 0 \qquad (2)$$

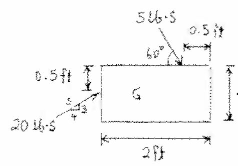
Eliminate Ft between Eqs. (1) and (2),

$\omega_0 = \frac{v_G}{r}$ **Ans**



19-25. The 10-lb rectangular plate is at rest on a smooth *horizontal* floor. If it is given the horizontal impulses shown, determine its angular velocity and the velocity of the mass center.





$(\overset{+}{\rightarrow})\qquad m(v_{Gx})_1 + \Sigma\int F_x\,dt = m(v_{Gx})_2$

$$0 + 20(\frac{4}{5}) + 5\cos 60° = \frac{10}{32.2}(v_G)_x$$

$(+\uparrow)\qquad m(v_{Gy})_1 + \Sigma\int F_y\,dt = m(v_{Gy})_2$

$$0 + 20(\frac{3}{5}) - 5\sin 60° = \frac{10}{32.2}(v_G)_y$$

$(\circlearrowleft +)\qquad I_G\omega_1 + \Sigma\int M\,dt = I_G\omega_2$

$$0 + 20(\frac{3}{5})(1) + 5\sin 60°(0.5) + 5\cos 60°(0.5) = \frac{1}{12}(\frac{10}{32.2})[(1)^2 + (2)^2]\omega$$

Solving,

$\omega = 119$ rad/s **Ans**

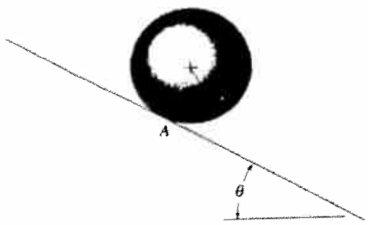
$(v_G)_x = 59.6$ ft/s

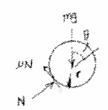
$(v_G)_y = 24.7$ ft/s

$v_G = \sqrt{(59.6)^2 + (24.7)^2} = 64.5$ ft/s **Ans**

$\theta = \tan^{-1}\frac{24.7}{59.6} = 22.5°$ $\measuredangle\theta$

19-26. The ball of mass m and radius r rolls along an inclined plane for which the coefficient of static friction is μ. If the ball is released from rest, determine the maximum angle θ for the incline so that it rolls without slipping at A.





$+\nearrow\qquad m(v_y)_1 + \Sigma\int F_y\,dt = m(v_y)_2$

$$0 + N(t) - (mg\cos\theta)t = 0$$

$\searrow +\qquad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$

$$0 + (mg\sin\theta)t - \mu Nt = m(r\omega)$$

$\circlearrowleft +\qquad (H_G)_1 + \Sigma\int M_G dt = (H_G)_2$

$$0 + \mu Nrt = (\frac{2}{5}mr^2)\omega$$

$$N = mg\cos\theta$$

$$mg\sin\theta\, t - \mu mg\cos\theta t = mr\omega$$

$$t = \frac{r\omega}{g(\sin\theta - \mu\cos\theta)}$$

$$\mu(mg\cos\theta)r(\frac{r\omega}{g(\sin\theta - \mu\cos\theta)}) = \frac{2}{5}mr^2\omega$$

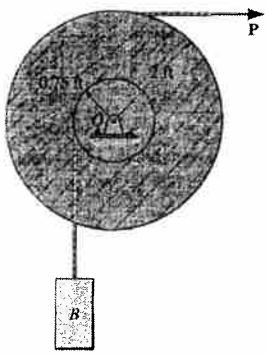
$$\mu\cos\theta = \frac{2}{5}(\sin\theta - \mu\cos\theta)$$

$$3.5\mu\cos\theta = \sin\theta$$

$$\tan\theta = 3.5\mu$$

$\theta = \tan^{-1}(3.5\mu)$ **Ans**

19-27. The spool has a weight of 75 lb and a radius of gyration $k_O = 1.20$ ft. If the block B weighs 60 lb, and a force $P = 25$ lb is applied to the cord, determine the speed of the block in 5 s starting from rest. Neglect the mass of the cord.

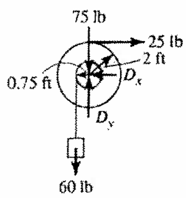


$$(H_O)_1 + \sum \int M_O dt = (H_O)_2$$

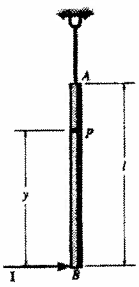
$$0 - 60(0.75)(5) + 25(2)(5) = \frac{75}{32.2}(1.20)^2\omega + \left[\frac{60}{32.2}(0.75\omega)\right](0.75)$$

$\omega = 5.679$ rad/s

$v_B = \omega r = (5.679)(0.75) = 4.26$ ft/s **Ans**



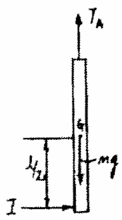
***19-28.** The slender rod has a mass m and is suspended at its end A by a cord. If the rod receives a horizontal blow giving it an impulse **I** at its bottom B, determine the location y of the point P about which the rod appears to rotate during the impact.



Principle of impulse and momentum :

$$I_G\omega_1 + \Sigma\int_{t_1}^{t_2} M_G dt = I_G\omega_2$$

$$0 + I\left(\frac{l}{2}\right) = \left[\frac{1}{12}ml^2\right]\omega \qquad I = \frac{1}{6}ml\omega$$

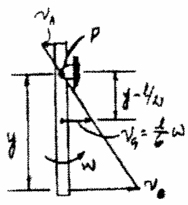


$$m(v_{Ax})_1 + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_{Ax})_2$$

$$0 + \frac{1}{6}ml\omega = mv_G \qquad v_G = \frac{l}{6}\omega$$

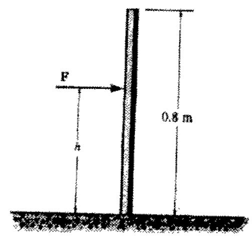
Kinematics : Point P is the IC.

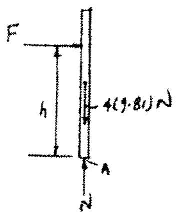
$$v_B = \omega y$$



Using similar triangles $\quad \frac{\omega y}{y} = \frac{\frac{l}{6}\omega}{y - \frac{l}{2}} \qquad y = \frac{2}{3}l$ **Ans**

19-29. A thin rod having a mass of 4 kg is balanced vertically as shown. Determine the height h at which it can be struck with a horizontal force **F** and not slip on the floor. This requires that the frictional force at A be essentially zero.





$(\xrightarrow{+})$ $\quad m(v_{Gx})_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_{Gx})_2$

$0 + F(t) = 4v_G \qquad (1)$

$(\curvearrowleft +)$ $\quad I_A\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = I_A\,\omega_2$

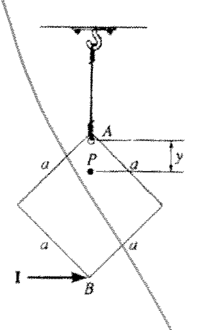
$0 + Fh(t) = \left[\frac{1}{3}(4)(0.8)^2\right]\omega \qquad (2)$

However, $\quad v_G = \omega(0.4) \qquad (3)$

Susitute Eq.(3) into Eq. (1) $\qquad Ft = 1.6\omega \qquad (4)$

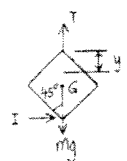
Divide Eq.(2) by Eq.(4) $\qquad h = \dfrac{\frac{1}{3}(4)(0.8)^2}{1.6} = 0.533\text{ m}$ $\qquad$ **Ans**

19-30. The square plate has a mass m and is suspended at its corner A by a cord. If it receives a horizontal impulse **I** at corner B, determine the location y of the point P about which the plate appears to rotate during the impact.



$(\curvearrowleft +)$ $\quad (H_G)_1 + \Sigma\int M_G\,dt = (H_G)_2$

$$0 + I\left(\frac{a}{\sqrt{2}}\right) = \frac{m}{12}(a^2 + a^2)\omega$$



$(\xrightarrow{+})$ $\quad m(v_{Gx})_1 + \Sigma\int F_x\,dt = m(v_{Gx})_2$

$$0 + I = mv_G$$

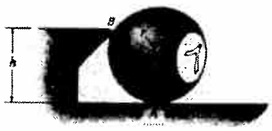
$$\omega = \frac{6I}{\sqrt{2}am}$$

$$v_G = \frac{I}{m}$$

$$y' = \frac{v_G}{\omega} = \frac{\frac{I}{m}}{\frac{6I}{\sqrt{2}\,am}} = \frac{\sqrt{2}a}{6}$$

$$y = \frac{3\sqrt{2}}{6}a - \frac{\sqrt{2}}{6}a = \frac{\sqrt{2}}{3}a$$ **Ans**

19-31. Determine the height h of the bumper of the pool table, so that when the pool ball of mass m strikes it, no frictional force will be developed between the ball and the table at A. Assume the bumper exerts only a horizontal force on the ball.

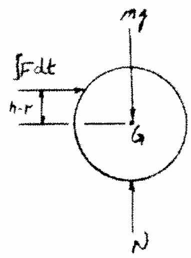


$(\overset{+}{\rightarrow})$ $\quad \Sigma \int_{t_1}^{t_2} F_x\,dt = m\Delta(v_{Ax})$

$$\int F\,dt = m\Delta v_G \qquad (1)$$

$(\curvearrowleft +)$ $\quad \Sigma \int_{t_1}^{t_2} M_G dt = I_G \Delta\omega_2$

$$(h-r)\int F\,dt = \left[\frac{2}{5}mr^2\right]\Delta\omega \qquad (2)$$

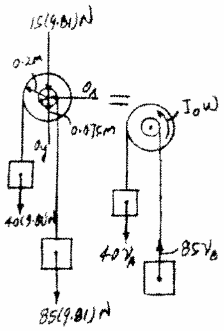


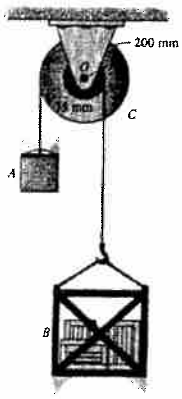
However, $\quad \Delta v_G = \Delta\omega r \qquad (3)$

Substitute Eq.(3) into Eq. (1) yields $\quad \int F\,dt = m\Delta\omega r \qquad (4)$

Divide Eq.(2) by Eq.(4) yields $\quad h - r = \frac{2}{5}r \qquad h = \frac{7}{5}r \qquad$ **Ans**

***19-32.** The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass of 15 kg and a radius of gyration of k_O = 110 mm. If the block at A has a mass of 40 kg and the container at B has a mass of 85 kg, including its contents, determine the speed of the container when t = 3 s after it is released from rest.





The angular velocity of the pulley can be related to the speed of container B by $\omega = \frac{v_B}{0.075} = 13.333 v_B$. Also the speed of block A $v_A = \omega(0.2) = 13.33 v_B(0.2) = 2.667 v_B$.

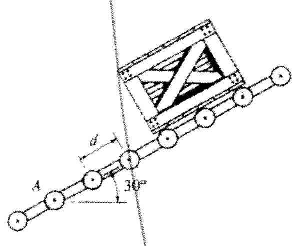
$(\curvearrowleft +)$ $\quad \left(\Sigma \text{Syst. Ang. Mom.}\right)_{O1} + (\Sigma\text{Syst. Ang. Imp.})_{O(1-2)} = \left(\Sigma\text{Syst. Ang. Mom.}\right)_{O2}$

$$0 + 40(9.81)(0.2)(3) - 85(9.81)(0.075)(3)$$

$$= \left[15(0.110)^2\right](13.333 v_B) + 85 v_B(0.075) + 40(2.667 v_B)(0.2)$$

$$v_B = 1.59 \text{ m/s} \qquad \textbf{Ans}$$

19-33. The crate has a mass m_c. Determine the constant speed v_0 it acquires as it moves down the conveyor. The rollers each have a radius of r, mass m, and are spaced d apart. Note that friction causes each roller to rotate when the crate comes in contact with it.

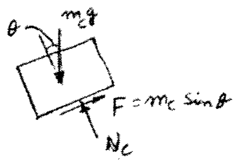


The number of rollers per unit length is $1/d$.

Thus in one second, $\frac{v_0}{d}$ rollers are contacted.

If a roller is brought to full angular speed of $\omega = \frac{v_0}{r}$ in t_0 seconds,

then the moment of inertia that is effected is



$$I' = I(\frac{v_0}{d})(t_0) = (\frac{1}{2}m\,r^2)(\frac{v_0}{d})t_0$$

Since the frictional impluse is

$F = m_c \sin\theta$ then

$$\circlearrowleft + \quad (H_G)_1 + \Sigma\int M_G dt = (H_G)_2$$

$$0 + (m_c \sin\theta)\, r\, t_0 = \left[\left(\frac{1}{2}m\,r^2\right)\left(\frac{v_0}{d}\right)t_0\right]\left(\frac{v_0}{r}\right)$$

$$v_0 = \sqrt{(2g\,\sin\theta\; d)(\frac{m_c}{m})} \qquad \textbf{Ans}$$

14

19-34. Two wheels A and B have masses m_A and m_B, and radii of gyration about their central vertical axes of k_A and k_B, respectively. If they are freely rotating in the same direction at ω_A and ω_B about the same vertical axis, determine their common angular velocity after they are brought into contact and slipping between them stops.

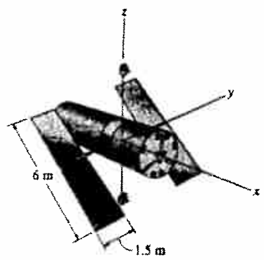
$$(\Sigma\text{Syst. Angular Momentum})_1 = (\Sigma\text{Syst. Angular Momentum})_2$$

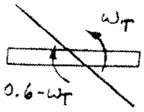
$$(m_A k_A^2)\,\omega_A + (m_B k_B^2)\,\omega_B = (m_A k_A^2)\,\omega'_A + (m_B k_B^2)\,\omega'_B$$

Set $\omega'_A = \omega'_B = \omega$, then

$$\omega = \frac{m_A k_A^2 \omega_A + m_B k_B^2 \omega_B}{m_A k_A^2 + m_B k_B^2} \qquad \textbf{Ans}$$

19-35. The Hubble Space Telescope is powered by two solar panels as shown. The body of the telescope has a mass of 11 Mg and radii of gyration $k_x = 1.64$ m and $k_y = 3.85$ m, whereas the solar panels can be considered as thin plates, each having a mass of 54 kg. Due to an internal drive, the panels are given an angular velocity of $\{0.6\mathbf{j}\}$ rad/s, measured relative to the telescope. Determine the angular velocity of the telescope due to the rotation of the panels. Prior to rotating the panels, the telescope was originally traveling at $\mathbf{v}_G = \{-400\mathbf{i} + 250\mathbf{j} + 175\mathbf{k}\}$ m/s. Neglect its orbital rotation.





$(H_y)_1 = (H_y)_2$

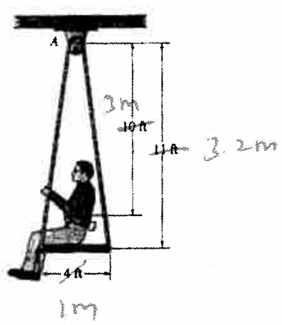
$$0 = 2\left[\frac{1}{12}(54)(6)^2\right](0.6-\omega_T) - \left[(11\,000)(3.85)^2\right]\omega_T$$

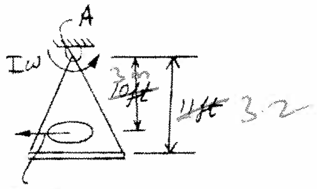
$\omega_T = 1.19\,(10^{-3})$ rad/s **Ans**

18 100kg

***19-36.** The platform swing consists of a ~~200-lb~~ flat plate suspended by four rods of negligible weight. When the swing is at rest, the 150-lb man jumps off the platform when his center of gravity G is 10 ft from the pin at A. This is done with a horizontal velocity of 5 ft/s, measured relative to the swing at the level of G. Determine the angular velocity he imparts to the swing just after jumping off.

75
3m
2 m/s





$75\ \frac{150}{32.2}(5-10\omega)$

$(\circlearrowleft+)\quad (H_A)_1 = (H_A)_2$

$$0+0 = \left[\frac{1}{12}\left(\frac{\overset{100}{\cancel{200}}}{\cancel{32.2}}\right)(\overset{1}{\cancel{4}})^2 + \frac{\overset{100}{\cancel{200}}}{\cancel{32.2}}(\overset{3.2}{\cancel{11}})^2\right]\omega - \left[\left(\frac{\overset{75}{\cancel{150}}}{\cancel{32.2}}\right)(\overset{(2-3\omega)}{\cancel{5-10\omega}})\right](\overset{3}{\cancel{10}})$$

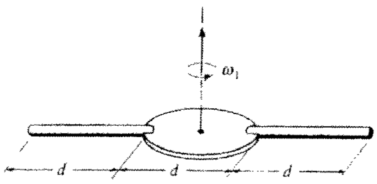
$= ~~833~~\ 1032.3\,\omega - 450 + 675\,\omega$

$\omega = 0.$~~190 rad/s~~ **Ans**

$= 0.264$ rad/s

16

19-37. Each of the two slender rods and the disk have the same mass m. Also, the length of each rod is equal to the diameter d of the disk. If the assembly is rotating with an angular velocity ω_1 when the rods are directed outward, determine the angular velocity of the assembly if by internal means the rods are brought to an upright vertical position.



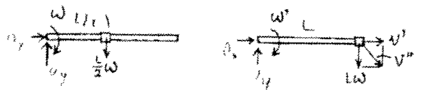
$H_1 = H_2$

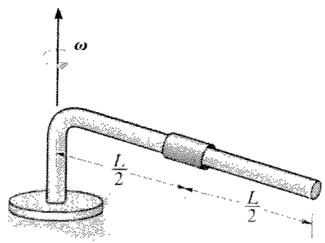
$$\left[\frac{1}{2}m\left(\frac{d}{2}\right)^2\right]\omega_1 + 2\left(\frac{1}{12}md^2\right)\omega_1 + 2(md^2\omega_1) = \left[\frac{1}{2}m\left(\frac{d}{2}\right)^2\right]\omega' + 2m\left(\frac{d}{2}\right)^2\omega'$$

$\omega' = \frac{11}{3}\omega_1$ **Ans**

17

19-38. The rod has a length L and mass m. A smooth collar having a negligible size and one-fourth the mass of the rod is placed on the rod at its midpoint. If the rod is freely rotating at ω about its end and the collar is released, determine the rod's angular velocity just before the collar flies off the rod. Also, what is the speed of the collar as it leaves the rod?





$H_1 = H_2$

$$\left(\frac{1}{3}mL^2\right)\omega + \frac{m}{4}\left(\frac{L}{2}\right)\omega\left(\frac{L}{2}\right) = \frac{1}{3}mL^2\omega' + \frac{m}{4}L^2\omega'$$

$\omega' = \frac{19}{28}\omega$ **Ans**

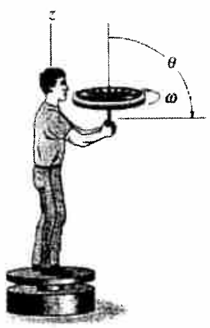
$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 + \frac{1}{2}\left(\frac{m}{4}\right)\left(\frac{L}{2}\omega\right)^2 = \frac{1}{2}\left(\frac{m}{4}\right)v'^2 + \frac{1}{2}\left(\frac{m}{4}\right)(L\omega')^2 + \frac{1}{2}\left(\frac{1}{3}mL^2\right)(\omega')^2$$

$v'^2 = 0.71342L^2\omega^2$

$$v'' = \sqrt{(0.71342L^2\omega^2) + \left[L\left(\frac{19}{28}\omega\right)\right]^2} = 0.985\omega L$$ **Ans**

19-39. A man has a moment of inertia I_z about the z axis. He is originally at rest and standing on a small platform which can turn freely. If he is handed a wheel when it is at *rest* and he starts it spinning with an angular velocity ω, determine his angular velocity if (a) he holds the wheel upright as shown, (b) turns the wheel out, $\theta = 90°$, and (c) turns the wheel downward, $\theta = 180°$.



(a)

$$\sum(H_z)_1 = \sum(H_z)_2; \quad 0 + I\omega = I_z\omega_M + I\omega \quad \omega_M = 0 \qquad \textbf{Ans}$$

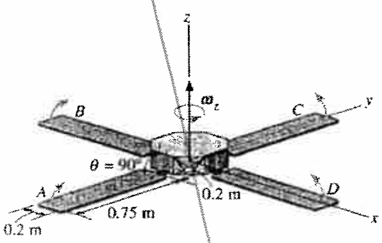
(b)

$$\sum(H_z)_1 = \sum(H_z)_2; \quad 0 + I\omega = I_z\omega_M + 0 \quad \omega_M = \frac{I}{I_z}\omega \qquad \textbf{Ans}$$

(c)

$$\sum(H_t)_1 = \sum(H_z)_2; \quad 0 + I\omega = I_z\omega_M - I\omega \quad \omega_M = \frac{2I}{I_z}\omega \qquad \textbf{Ans}$$

•19-40. The space satellite has a mass of 125 kg and a moment of inertia $I_z = 0.940\ \text{kg}\cdot\text{m}^2$, excluding the four solar panels A, B, C, and D. Each solar panel has a mass of 20 kg and can be approximated as a thin plate. If the satellite is originally spinning about the z axis at a constant rate $\omega_z = 0.5$ rad/s when $\theta = 90°$, determine the rate of spin if all the panels are raised and reach the upward position, $\theta = 0°$, at the same instant.

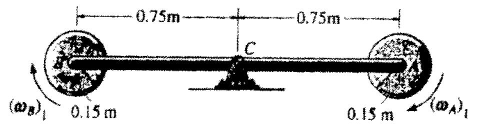


$$+\ H_1 = H_2$$

$$2\left[\frac{1}{2}(4)(0.15)^2\right](5) = 2\left[\frac{1}{2}(4)(0.15)^2\right]\omega + 2[4(0.75\omega)(0.75)] + \left[\frac{1}{12}(2)(1.50)^2\right]\omega$$

$$\omega = 0.0906\ \text{rad/s} \qquad \textbf{Ans}$$

19–41. The 2-kg rod ACB supports the two 4-kg disks at its ends. If both disks are given a clockwise angular velocity $(\omega_A)_1 = (\omega_B)_1 = 5$ rad/s while the rod is held stationary and then released, determine the angular velocity of the rod after both disks have stopped spinning relative to the rod due to frictional resistance at the pins A and B. Motion is in the *horizontal plane*. Neglect friction at pin C.

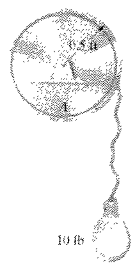


$$\curvearrowleft + \quad H_1 = H_2$$

$$2[\frac{1}{2}(4)(0.15)^2](5) = 2[\frac{1}{2}(4)(0.15)^2]\omega + 2[4(0.75\omega)(0.75)] + [\frac{1}{12}(2)(1.50)^2]\omega$$

$\omega = 0.0906$ rad/s **Ans**

19–42. Disk A has a weight of 20 lb. An inextensible cable is attached to the 10-lb weight and wrapped around the disk. The weight is dropped 2 ft before the slack is taken up. If the impact is perfectly elastic, i.e., $e = 1$, determine the angular velocity of the disk just after impact.



For the weight

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 10(2) = \frac{1}{2}\left(\frac{10}{32.2}\right)v_2^2$$

$$v_2 = 11.35 \text{ ft/s}$$

$$(H_A)_2 = (H_A)_3$$

$$mv_2(0.5) + 0 = mv_3(0.5) + I_A\,\omega$$

$$\left(\frac{10}{32.2}\right)(11.35)(0.5) + 0 = \left(\frac{10}{32.2}\right)v_3(0.5) + \left[\frac{1}{2}\left(\frac{20}{32.2}\right)(0.5)^2\right]\omega \qquad [1]$$

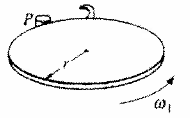
$$(+\downarrow) \qquad e = \frac{0.5\omega - v_3}{v_2 - 0} \qquad 1 = \frac{0.5\omega - v_3}{11.35 - 0} \qquad 11.35 = 0.5\omega - v_3 \qquad [2]$$

Solving Eqs.[1] and [2] yields :

$\omega = 22.7$ rad/s **Ans**

$v_3 = 0$

19-43. A thin disk of mass m has an angular velocity ω_1 while rotating on a smooth surface. Determine its new angular velocity just after the hook at its edge strikes the peg P and the disk starts to rotate about P without rebounding.

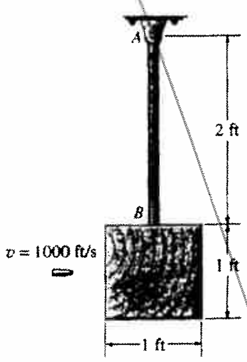


$$H_1 = H_2$$

$$\left(\frac{1}{2}mr^2\right)\omega_1 = \left[\frac{1}{2}mr^2 + mr^2\right]\omega_2$$

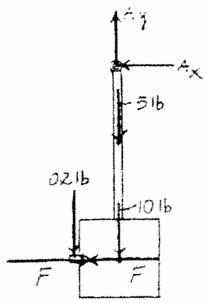
$$\omega_2 = \frac{1}{3}\omega_1 \qquad \textbf{Ans}$$

***19-44.** The pendulum consists of a 5-lb slender rod AB and a 10-lb wooden block. A projectile weighing 0.2 lb is fired into the center of the block with a velocity of 1000 ft/s. If the pendulum is initially at rest, and the projectile embeds itself into the block, determine the angular velocity of the pendulum just after the impact.



Mass Moment of Inertia : The mass moment inertia of the pendulum and the embeded bullet about point A is

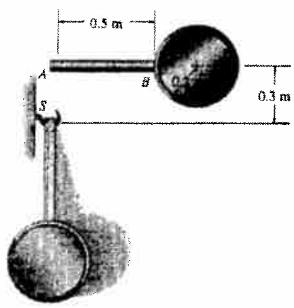
$$(I_A)_2 = \frac{1}{12}\left(\frac{5}{32.2}\right)(2^2) + \frac{5}{32.2}(1^2)$$
$$+ \frac{1}{12}\left(\frac{10}{32.2}\right)(1^2 + 1^2) + \frac{10}{32.2}(2.5^2) + \frac{0.2}{32.2}(2.5^2)$$
$$= 2.239 \text{ slug}\cdot\text{ft}^2$$



Conservation of Angular Momentum : Since force F due to the impact is *internal* to the system consisting of the pendulum and the bullet, it will cancel out. Thus, angular momentum is conserved about point A. Applying Eq. 19 – 17, we have

$$(H_A)_1 = (H_A)_2$$
$$(m_b v_b)(r_b) = (I_A)_2\,\omega_2$$
$$\left(\frac{0.2}{32.2}\right)(1000)(2.5) = 2.239\,\omega_2$$
$$\omega_2 = 6.94 \text{ rad/s} \qquad \textbf{Ans}$$

19-45. The pendulum consists of a slender 2-kg rod AB and 5-kg disk. It is released from rest without rotating. When it falls 0.3 m, the end A strikes the hook S, which provides a permanent connection. Determine the angular velocity of the pendulum after it has rotated 90°. Treat the pendulum's weight during impact as a nonimpulsive force.



$$T_0 + V_0 = T_1 + V_1$$

$$0 + 2(9.81)(0.3) + 5(9.81)(0.3) = \frac{1}{2}(2)(v_G)_1^2 + \frac{1}{2}(5)(v_G)_1^2$$

$(v_G)_1 = 2.4261$ m/s

$$\Sigma(H_S)_1 = \Sigma(H_S)_2$$

$$2(2.4261)(0.25) + 5(2.4261)(0.7) = \left[\frac{1}{12}(2)(0.5)^2 + 2(0.25)^2 + \frac{1}{2}(5)(0.2)^2 + 5(0.7)^2\right]\omega$$

$\omega = 3.572$ rad/s

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}\left[\frac{1}{12}(2)(0.5)^2 + 2(0.25)^2 + \frac{1}{2}(5)(0.2)^2 + 5(0.7)^2\right](3.572)^2 + 0$$

$$= \frac{1}{2}\left[\frac{1}{12}(2)(0.5)^2 + 2(0.25)^2 + \frac{1}{2}(5)(0.2)^2 + 5(0.7)^2\right]\omega^2$$

$$+ 2(9.81)(-0.25) + 5(9.81)(-0.7)$$

$\omega = 6.45$ rad/s **Ans**

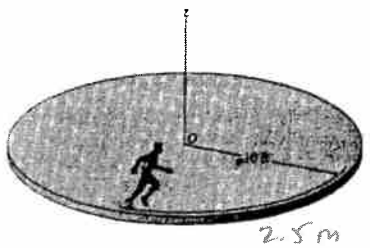
21

19-46. A horizontal circular platform has a weight of ~~300 lb~~ and a radius of gyration k_z = ~~8 ft~~ about the z axis passing through its center O. The platform is free to rotate about the z axis and is initially at rest. A man having a weight of 150 lb begins to run along the edge in a circular path of radius 10 ft. If he has a speed of 4 ft/s and maintains this speed relative to the platform, determine the angular velocity of the platform. Neglect friction.

1500 N 2 m

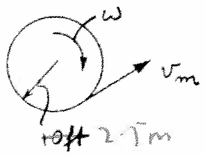
750 N 1 m/s

2.5 m



$v_m = v_p + v_{m/p}$

$(\overset{+}{\rightarrow})$ $v_m = -10\omega + 4$ $\quad -2.5\omega + 1$



$(\circlearrowleft +)$ $(H_z)_1 = (H_z)_2$

$$0 = -\left(\frac{\overset{1500}{\cancel{300}}}{\underset{9.81}{\cancel{32.2}}}\right)(\cancel{8})^2\,\omega + \left(\frac{\overset{750}{\cancel{150}}}{\underset{9.81}{\cancel{32.2}}}\right)(-\cancel{10}\omega + \cancel{4})(\cancel{10})$$

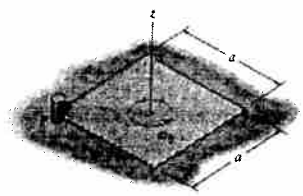
2 2.5 1 2.5

~~$\omega = 0.175$ rad/s **Ans**~~

$= -611.6\omega - 477.8\omega + 191.1$

$= 0.175$ rad/s

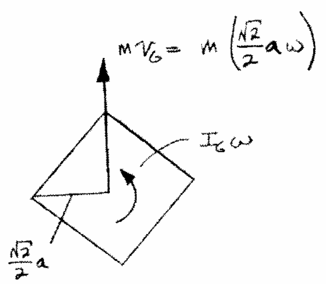
19-47. The square plate has a weight W and is rotating on the smooth surface with a constant angular velocity ω_0. Determine the new angular velocity of the plate just after its corner strikes the peg P and the plate starts to rotate about P without rebounding.



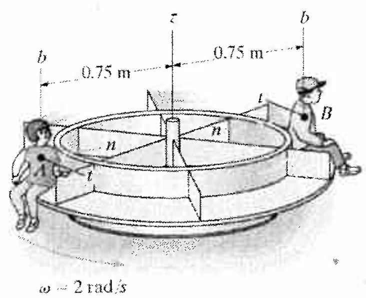
$$\Sigma(H_P)_0 = \Sigma(H_P)_1$$

$$\left[\frac{1}{12}\left(\frac{W}{g}\right)(a^2+a^2)\right]\omega_0 = \left[\frac{1}{12}\left(\frac{W}{g}\right)(a^2+a^2) + \left(\frac{W}{g}\right)\left(\frac{\sqrt{2}}{2}a\right)^2\right]\omega$$

$$\omega = \frac{1}{4}\omega_0 \qquad \textbf{Ans}$$



***19-48.** Two children A and B, each having a mass of 30 kg, sit at the edge of the merry-go-round which is rotating at $\omega = 2$ rad/s. Excluding the children, the merry-go-round has a mass of 180 kg and a radius of gyration $k_z = 0.6$ m. Determine the angular velocity of the merry-go-round if A jumps off horizontally in the $-n$ direction with a speed of 2 m/s, measured with respect to the merry-go-round. What is the merry-go-round's angular velocity if B then jumps off horizontally in the $+t$ direction with a speed of 2 m/s, measured with respect to the merry-go-round? Neglect friction and the size of each child.



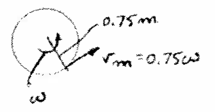
A jumps off,

$$(H_z)_0 = (H_z)_1$$

$$[180(0.6)^2](2) + 2[30(0.75)(2)](0.75) = [180(0.6)^2]\omega_1 + [30(0.75)\omega_1](0.75)$$

$$\omega_1 = 2.41 \text{ rad/s} \qquad \textbf{Ans}$$

B jumps off,



$$\mathbf{v}_B = \mathbf{v}_M + \mathbf{v}_{B/M}$$

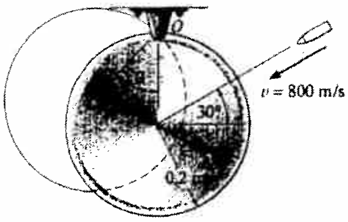
$$v_B = 0.75\omega_2 + 2$$

$$(H_z)_1 = (H_z)_2$$

$$[180(0.6)^2](2.41) + [30(0.75)(2.41)](0.75) = [180(0.6)^2]\omega_2 + [30(0.75\omega_2 + 2)]0.75$$

$$\omega_2 = 1.86 \text{ rad/s} \qquad \textbf{Ans}$$

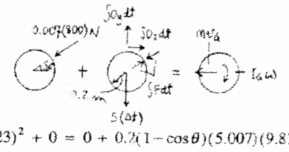
19-49. A 7-g bullet having a velocity of 800 m/s is fired into the edge of the 5-kg disk as shown. Determine the angular velocity of the disk just after the bullet becomes embedded in it. Also, calculate how far θ the disk will swing until it stops. The disk is originally at rest.



$\circlearrowleft + \quad (H_O)_1 + \Sigma\int M_O\,dt = (H_O)_2$

$0.007(800)\cos 30°(0.2) + 0 = \frac{1}{2}(5.007)(0.2)^2\omega + 5.007(0.2\omega)(0.2)$

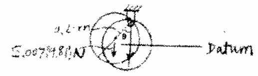
$\omega = 3.23$ rad/s **Ans**



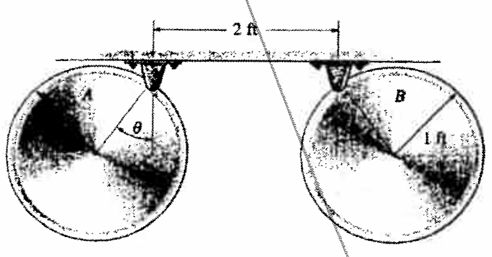
$T_1 + V_1 = T_2 + V_2$

$\frac{1}{2}(5.007)[3.23(0.2)]^2 + \frac{1}{2}[\frac{1}{2}(5.007)(0.2)^2](3.23)^2 + 0 = 0 + 0.2(1 - \cos\theta)(5.007)(9.81)$

$\theta = 32.8°$ **Ans**



19-50. The two disks each weigh 10 lb. If they are released from rest when $\theta = 30°$, determine θ after they collide and rebound from each other. The coefficient of restitution is $e = 0.75$. When $\theta = 0°$, the disks hang so that they just touch one another.



$I_C = \frac{1}{2}(\frac{10}{32.2})(1)^2 + \frac{10}{32.2}(1)^2 = 0.46584\ \text{slug}\cdot\text{ft}^2$

$T_1 + V_1 = T_2 + V_2$

$0 + 10(1 - \cos 30°) = \frac{1}{2}(0.46584)\omega_1^2 + 0$

$\omega_1 = 2.398$ rad/s

Coefficient of restitution:

$e = \frac{(v_D)_{B_2} - (v_D)_{A_2}}{(v_D)_{A_1} - (v_D)_{B_1}} = 0.75 = \frac{\omega_2 - (-\omega_2)}{2.398 - (-2.398)} \qquad (1)$

Where, v_D is the speed of point D on disk A or B. Note that $(v_D)_B = -(v_D)_A$ and $(v_D)_A = r\omega = (v_D)_B$.

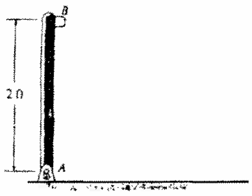
Solving Eq.(1); $\omega_2 = 1.799$ rad/s

$T_1 + V_1 = T_2 + V_2$

$\frac{1}{2}(0.46584)(1.799)^2 + 0 = 0 + 10(1 - \cos\theta)$

$\theta = 22.4°$ **Ans**

19-51. The 15-lb rod AB is released from rest in the vertical position. If the coefficient of restitution between the floor and the cushion at B is $e = 0.7$, determine how high the end of the rod rebounds after impact with the floor.



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 15(1) = \frac{1}{2}\left[\frac{1}{3}\left(\frac{15}{32.2}\right)(2)^2\right]\omega_2^2$$

$\omega_2 = 6.950$ rad/s Hence $(v_B)_2 = 6.950(2) = 13.90$ rad/s

$$(+\downarrow) \qquad e = \frac{0 - (v_B)_3}{(v_B)_2 - 0}; \qquad 0.7 = \frac{0 - (v_B)_3}{13.90}$$

$$(v_B)_3 = -9.730 \text{ ft/s} = 9.730 \text{ ft/s} \uparrow$$

$$\omega_3 = \frac{(v_B)_3}{2} = \frac{9.730}{2} = 4.865 \text{ rad/s}$$

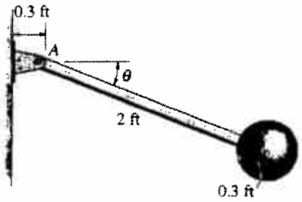
$$T_3 + V_3 = T_4 + V_4$$

$$\frac{1}{2}\left[\frac{1}{3}\left(\frac{15}{32.2}\right)(2)^2\right](4.865)^2 = 0 + 15(h_G)$$

$h_G = 0.490$ ft

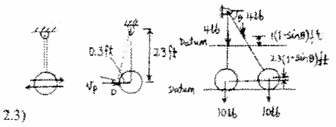
$h_B = 2h_G = 0.980$ ft **Ans**

***19-52.** The pendulum consists of a 10-lb solid ball and 4-lb rod. If it is released from rest when $\theta_1 = 0°$, determine the angle θ_2 after the ball strikes the wall, rebounds, and the pendulum swings up to the point of momentary rest. Take $e = 0.6$.



$$I_A = \frac{1}{3}\left(\frac{4}{32.2}\right)(2)^2 + \frac{2}{5}\left(\frac{10}{32.2}\right)(0.3)^2 + \frac{10}{32.2}(2.3)^2 = 1.8197 \text{ slug}\cdot\text{ft}^2$$

Just before impact:



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}[1.8197]\omega^2 - 4(1) - 10(2.3)$$

$\omega = 5.4475$ rad/s

$v_P = 2.3(5.4475) = 12.53$ ft/s

Since wall does not move

$$e = 0.6 = \frac{v_P'}{12.529}$$

$v_P' = 7.518$ ft/s

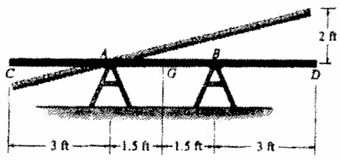
$$\omega' = \frac{7.518}{2.3} = 3.2685 \text{ rad/s}$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(1.8197)(3.2685)^2 = 4(1)(1 - \sin\theta) + 10(2.3)(1 - \sin\theta)$$

$\theta = 39.8°$ **Ans**

19-53. The plank has a weight of 30 lb, center of gravity at G, and it rests on the two sawhorses at A and B. If the end D is raised 2 ft above the top of the sawhorses and is released from rest, determine how high end C will rise from the top of the sawhorses after the plank falls so that it rotates clockwise about A, strikes and pivots on the sawhorse at B, and rotates clockwise off the sawhorse at A.



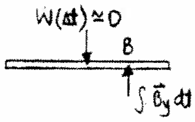
Establishing a datum through AB, the angular velocity of the plank just before striking B is

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 30\left[\frac{2}{6}(1.5)\right] = \frac{1}{2}\left[\frac{1}{12}\left(\frac{30}{32.2}\right)(9)^2 + \frac{30}{32.2}(1.5)^2\right](\omega_{CD})_2^2 + 0$$

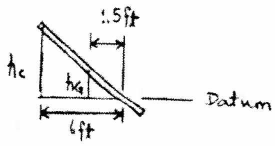
$$(\omega_{CD})_2 = 1.8915 \text{ rad/s}$$

$$(v_G)_2 = 1.8915(1.5) = 2.837 \text{ m/s}$$



$$(\circlearrowleft +) \qquad (H_B)_2 = (H_B)_3$$

$$\left[\frac{1}{12}\left(\frac{30}{32.2}\right)(9)^2\right](1.8915) - \frac{30}{32.2}(2.837)(1.5) = \left[\frac{1}{12}\left(\frac{30}{32.2}\right)(9)^2\right](\omega_{AB})_3 + \frac{30}{32.2}(v_G)_3(1.5)$$



Since $(v_G)_3 = 1.5(\omega_{AB})_3$

$$(\omega_{AB})_3 = 0.9458 \text{ rad/s}$$

$$(v_G)_3 = 1.4186 \text{ m/s}$$

$$T_3 + V_3 = T_4 + V_4$$

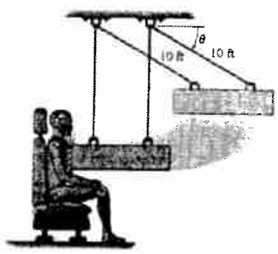
$$\frac{1}{2}\left[\frac{1}{12}\left(\frac{30}{32.2}\right)(9)^2\right](0.9458)^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)(1.4186)^2 + 0 = 0 + 30\,h_G$$

$$h_G = 0.125$$

Thus,

$$h_C = \frac{6}{1.5}(0.125) = 0.500 \text{ ft} \qquad \textbf{Ans}$$

19-54. Tests of impact on the fixed crash dummy are conducted using the 300-lb ram that is released from rest at $\theta = 30°$, and allowed to fall and strike the dummy at $\theta = 90°$. If the coefficient of restitution between the dummy and the ram is $e = 0.4$, determine the angle θ to which the ram will rebound before momentarily coming to rest.



Datum through pin support at ceiling.

$$T_1 + V_1 = T_2 + V_2$$

$$0 - 300(10\sin 30°) = \frac{1}{2}\left(\frac{300}{32.2}\right)(v)^2 - 300(10)$$

$$v = 17.944 \text{ ft/s}$$

$$(\xrightarrow{+}) \qquad e = 0.4 = \frac{v' - 0}{0 - (-17.944)}$$

$$v' = 7.178 \text{ ft/s}$$

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}\left(\frac{300}{32.2}\right)(7.178)^2 - 300(10) = 0 - 300(10\sin\theta)$$

$$\theta = 66.9° \qquad \textbf{Ans}$$

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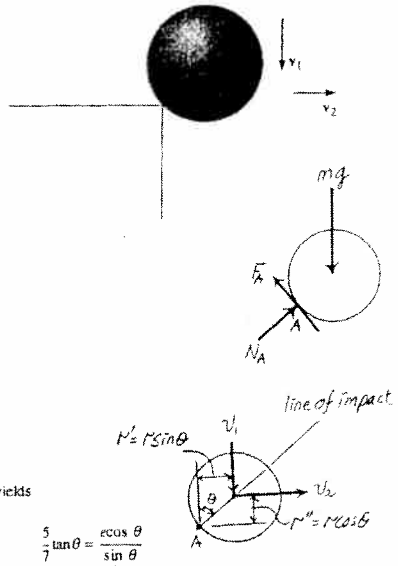
19-55. The solid ball of mass m is dropped with a velocity $\mathbf{v}_1$ onto the edge of the rough step. If it rebounds horizontally off the step with a velocity $\mathbf{v}_2$, determine the angle θ at which contact occurs. Assume no slipping when the ball strikes the step. The coefficient of restitution is e.



Conservation of Angular Momentum : Since the weight of the solid ball is a *nonimpulsive force*, then angular momentum is conserved about point A. The mass moment of inertia of the solid ball about its mass center is $I_G = \frac{2}{5}mr^2$. Here, $\omega_2 = \frac{v_2\cos\theta}{r}$. Applying Eq. 19 – 17, we have

$$(H_A)_1 = (H_A)_2$$
$$[m_b(v_b)_1](r') = I_G\omega_2 + [m_b(v_b)_2](r'')$$
$$(mv_1)(r\sin\theta) = \left(\frac{2}{5}mr^2\right)\left(\frac{v_2\cos\theta}{r}\right) + (mv_2)(r\cos\theta)$$
$$\frac{v_2}{v_1} = \frac{5}{7}\tan\theta \qquad [1]$$

Coefficient of Restitution : Applying Eq. 19 – 20, we have

$$e = \frac{0-(v_b)_2}{(v_b)_1 - 0}$$
$$e = \frac{-(v_2\sin\theta)}{-v_1\cos\theta}$$
$$\frac{v_2}{v_1} = \frac{e\cos\theta}{\sin\theta} \qquad [2]$$

Equating Eqs. [1] and [2] yields

$$\frac{5}{7}\tan\theta = \frac{e\cos\theta}{\sin\theta}$$
$$\tan^2\theta = \frac{7}{5}e$$
$$\theta = \tan^{-1}\left(\sqrt{\frac{7}{5}e}\right) \qquad \textbf{Ans}$$

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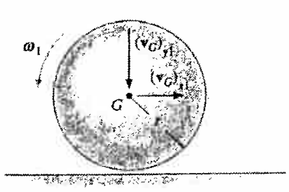
***19-56.** A solid ball with a mass m is thrown on the ground such that at the instant of contact it has an angular velocity $\boldsymbol{\omega}_1$ and velocity components $(\mathbf{v}_G)_{x1}$ and $(\mathbf{v}_G)_{y1}$ as shown. If the ground is rough so no slipping occurs, determine the components of the velocity of its mass center just after impact. The coefficient of restitution is e.



Coefficient of Restitution (y direction) :

$$(+\downarrow) \qquad e = \frac{0-(v_G)_{y2}}{(v_G)_{y1} - 0} \qquad (v_G)_{y2} = -e(v_G)_{y1} = e(v_G)_{y1}\uparrow \qquad \textbf{Ans}$$

Conservation of angular momentum about point on the ground :

$$(\circlearrowleft +) \qquad (H_A)_1 = (H_A)_2$$

$$-\frac{2}{5}mr^2\omega_1 + m(v_G)_{x1}r = \frac{2}{5}mr^2\omega_2 + m(v_G)_{x2}r$$

Since no slipping, $(v_G)_{x2} = \omega_2 r$ then,

$$\omega_2 = \frac{5\left((v_G)_{x1} - \frac{2}{5}\omega_1 r\right)}{7r}$$

Therefore

$$(v_G)_{x2} = \frac{5}{7}\left((v_G)_{x1} - \frac{2}{5}\omega_1 r\right) \qquad \textbf{Ans}$$