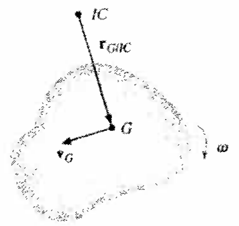
18-1. At a given instant the body of mass m has an angular velocity ω and its mass center has a velocity $\mathbf{v}_G$. Show that its kinetic energy can be represented as $T = \frac{1}{2} I_{IC}\omega^2$, where I_{IC} is the moment of inertia of the body computed about the instantaneous axis of zero velocity, located a distance $r_{G/IC}$ from the mass center as shown.



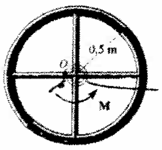
$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$ where $v_G = \omega r_{G/IC}$

$= \frac{1}{2} m (\omega r_{G/IC})^2 + \frac{1}{2} I_G \omega^2$

$= \frac{1}{2}\left(m r_{G/IC}^2 + I_G\right)\omega^2$ However $m r_{G/IC}^2 + I_G = I_{IC}$

$= \frac{1}{2} I_{IC}\omega^2$ **QED**

18-2. The wheel is made from a 5-kg thin ring and two 2-kg slender rods. If the torsional spring attached to the wheel's center has a stiffness $k = 2\ \text{N}\cdot\text{m/rad}$, so that the torque on the center of the wheel is $M = (2\theta)\ \text{N}\cdot\text{m}$, where θ is in radians, determine the maximum angular velocity of the wheel if it is rotated two revolutions and then released from rest.



$I_O = 2[\frac{1}{12}(2)(1)^2] + 5(0.5)^2 = 1.583$

$T_1 + \Sigma U_{1-2} = T_2$

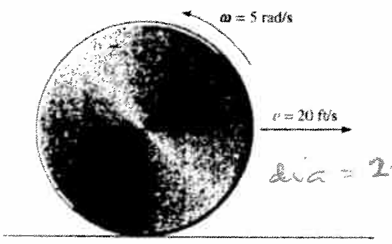
$0 + \int_0^{4\pi} 2\theta\, d\theta = \frac{1}{2}(1.583)\omega^2$

$(4\pi)^2 = 0.7917\omega^2$

$\omega = 14.1$ rad/s **Ans**

15 kg

18-3. At the instant shown, the ~~30-lb~~ disk has a counterclockwise angular velocity of 5 rad/s when its center has a velocity of ~~20 ft/s~~. Determine the kinetic energy of the disk at this instant. 10 m/s

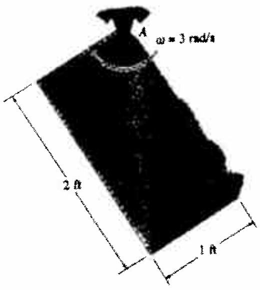


dia = 2m

$T = \frac{1}{2}\left[\frac{1}{2}\left(\frac{30}{32.2}\right)(2)^2\right](5)^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)(20)^2 = 210$ ft·lb **Ans**

$= \frac{1}{2}\left[\frac{1}{2} \times 15 \times (1)^2\right](5)^2 + \frac{1}{2}(15)(10)^2 = 843.8\ \text{J}$

***18-4.** The uniform rectangular plate weighs 30 ~~lb~~ kg. If the plate is pinned at A and has an angular velocity of 3 rad/s, determine the kinetic energy of the plate.

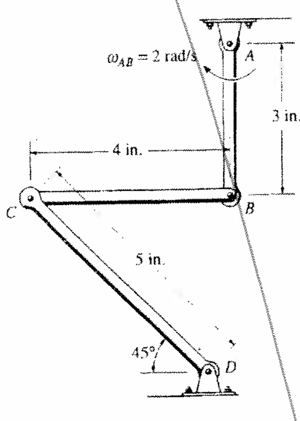


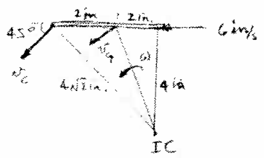
$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

$$T = \frac{1}{2}\left(\frac{30}{\cancel{32.2}}\right)\left((3)\sqrt{1^2 + (0.5)^2}\right)^2 + \frac{1}{2}\left[\frac{1}{12}\left(\frac{30}{\cancel{32.2}}\right)\left(1^2 + 2^2\right)\right](3)^2 = \cancel{6.99\ \text{ft}\cdot\text{lb}} \qquad \textbf{Ans}$$

$= 225$ J

18-5. At the instant shown, link AB has an angular velocity $\omega_{AB} = 2$ rad/s. If each link is considered as a uniform slender bar with a weight of 0.5 lb/in., determine the total kinetic energy of the system.





$$\omega_{BC} = \frac{6}{4} = 1.5 \text{ rad/s}$$

$$v_C = 1.5(4\sqrt{2}) = 8.4853 \text{ in./s}$$

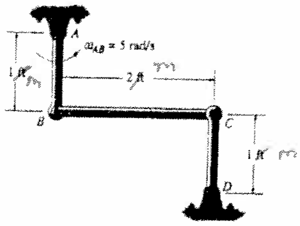
$$r_{IC-G} = \sqrt{(2)^2 + (4)^2} = 4.472$$

$$v_G = 1.5(4.472) = 6.7082 \text{ in./s}$$

$$\omega_{DC} = \frac{8.4853}{5} = 1.697 \text{ rad/s}$$

$$T = \frac{1}{2}\left[\frac{1}{3}\left(\frac{3(0.5)}{32.2}\right)\left(\frac{3}{12}\right)^2\right](2)^2 + \frac{1}{2}\left[\frac{4(0.5)}{32.2}\right]\left(\frac{6.7082}{12}\right)^2 + \frac{1}{2}\left[\frac{1}{12}\left(\frac{4(0.5)}{32.2}\right)\left(\frac{4}{12}\right)^2\right](1.5)^2$$
$$+ \frac{1}{2}\left[\frac{1}{3}\left(\frac{5(0.5)}{32.2}\right)\left(\frac{5}{12}\right)^2\right](1.697)^2 = 0.0188 \text{ ft lb} \qquad \textbf{Ans}$$

18-~~6~~ 5 Determine the kinetic energy of the system of three links. Links AB and CD each weigh ~~10 lb~~ 100N, and link BC weighs ~~20 lb~~ 200N.



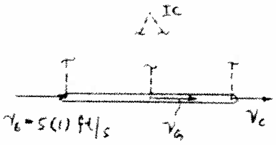
Link BC is subjected to general plane motion. Using the IC

$$r_{B/IC} = r_{C/IC} = r_{G/IC} = \infty$$

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{5(1)}{\infty} = 0$$

$v_C = v_G = v_B = 5(1) = 5$ ~~ft/s~~ 5 m/s

$$\omega_{CD} = \frac{v_C}{r_{CD}} = \frac{5}{1} = 5 \text{ rad/s}$$



Kinetic energy :

$$T_{AB} = \frac{1}{2}I_A\,\omega_{AB}^2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{\cancel{10}\ 100}{\cancel{32.2}\ 9.81}\right)(1)^2\right](5)^2 = \cancel{1.2940 \text{ ft}\cdot\text{lb}}\quad 42.47\text{ J}$$

$$T_{BC} = \frac{1}{2}m v_G^2 + \frac{1}{2}I_G\,\omega_{BC}^2 = \frac{1}{2}\left[\frac{\cancel{20}}{\cancel{32.2}}\ 200/9.81\right](5)^2 + 0 = \cancel{7.7640 \text{ ft}}\cdot\text{lb}\quad 254.8\text{ J}$$

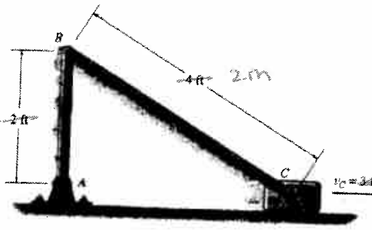
$$T_{CD} = \frac{1}{2}I_D\,\omega_{CD}^2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{\cancel{10}\ 100}{\cancel{32.2}\ 9.81}\right)(1)^2\right](5)^2 = \cancel{1.2940 \text{ ft}\cdot\text{lb}}\quad 42.47\text{ J}$$

T_T = ~~1.2940 + 7.7640 + 1.2940 = 10.4 ft~~ · lb **Ans**

42.47 + 254.8 + 42.47 = 339.5 J

18-~~7~~ 6 The mechanism consists of two rods, AB and BC, which weigh 10 lb and 20 lb, respectively, and a ~~4-lb block~~ at C. Determine the kinetic energy of the system at the instant shown, when the block is moving at ~~3 ft/s~~. 1 m/s

10 kg and 20 kg
4 kg



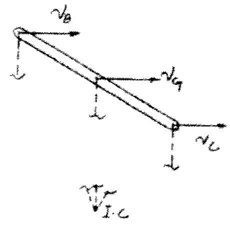
Link BC is subjected to general plane motion. Using the IC

$$r_{B/IC} = r_{C/IC} = r_{G/IC} = \infty$$

$$\omega_{BC} = \frac{v_C}{r_{C/IC}} = \frac{\cancel{3}\ 1}{\infty} = 0$$

$v_B = v_G = v_C =$ ~~3 ft/s~~ 1 m/s

$$\omega_{AB} = \frac{v_B}{r_{AB}} = \frac{\cancel{3}\ 1}{\cancel{2}\ 1} = \cancel{1.5 \text{ rad/s}}\quad 1 \text{ rad/s}$$



Kinetic energy : For the links

$$T_{AB} = \frac{1}{2}I_A\,\omega_{AB}^2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{10}{\cancel{32.2}}\right)(\cancel{2})^2\right](\cancel{1.5})^2 = \cancel{0.4658 \text{ ft}\cdot\text{lb}}\quad 1.67\text{ J}$$

$$T_{BC} = \frac{1}{2}m v_G^2 + \frac{1}{2}I_G\,\omega_{BC}^2 = \frac{1}{2}\left[\frac{20}{\cancel{32.2}}\right](\cancel{3})^2 + 0 = \cancel{2.7950 \text{ ft}}\cdot\text{lb}\quad 10\text{ J}$$

For the block

$$T_C = \frac{1}{2}m_C\,v_C^2 = \frac{1}{2}\left(\frac{\cancel{4}\ 4}{\cancel{32.2}}\right)(\cancel{3})^2 = \cancel{0.5590 \text{ ft}\cdot\text{lb}}\quad 2\text{ J}$$

T_T = ~~0.4658 + 2.7950 + 0.5590 = 3.82 ft · lb~~ **Ans**

1.67 + 10 + 2 = 13.67 J

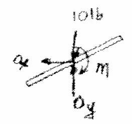
*18-8. Solve Prob. 17–59 using the principle of work and energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

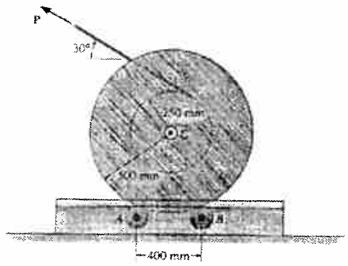
$$0 + \int_0^{\pi/2} 5\theta \, d\theta = \frac{1}{2}\left[\frac{1}{12}\left(\frac{10}{32.2}\right)(2)^2\right]\omega^2$$

$$\frac{5}{2}\left(\frac{\pi}{2}\right)^2 = 0.05176\omega^2$$

$\omega = 10.9$ rad/s **Ans**



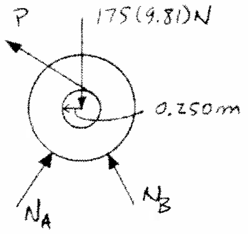
18-9. A force of $P = 20$ N is applied to the cable, which causes the 175-kg reel to turn since it is resting on the two rollers A and B of the dispenser. Determine the angular velocity of the reel after it has made two revolutions starting from rest. Neglect the mass of the rollers and the mass of the cable. The radius of gyration of the reel about its center axis is $k_G = 0.42$ m.



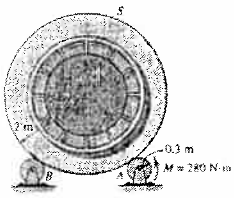
$$T_1 + \Sigma U_{1-2} = T_2$$

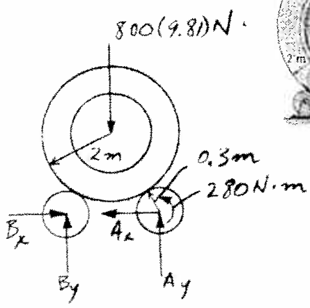
$$0 + 20(2)(2\pi)(0.250) = \frac{1}{2}\left[175(0.42)^2\right]\omega^2$$

$\omega = 2.02$ rad/s **Ans**



18-10. The rotary screen S is used to wash limestone. When empty it has a mass of 800 kg and a radius of gyration of $k_G = 1.75$ m. Rotation is achieved by applying a torque of $M = 280$ N·m about the drive wheel A. If no slipping occurs at A and the supporting wheel at B is free to roll, determine the angular velocity of the screen after it has rotated 5 revolutions. Neglect the mass of A and B.





$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 280(\theta_A) = \frac{1}{2}\left[800(1.75)^2\right]\omega^2$$

$$\theta_S(2) = \theta_A(0.3)$$

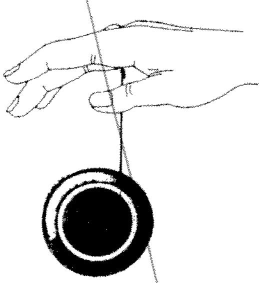
$$5(2\pi)(2) = \theta_A(0.3)$$

$\theta_A = 209.4$ rad

Thus

$\omega = 6.92$ rad/s **Ans**

18-11. A yo-yo has a weight of 0.3 lb and a radius of gyration $k_O = 0.06$ ft. If it is released from rest, determine how far it must descend in order to attain an angular velocity $\omega = 70$ rad/s. Neglect the mass of the string and assume that the string is wound around the central peg such that the mean radius at which it unravels is $r = 0.02$ ft.

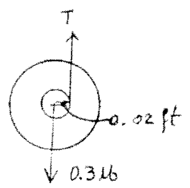


$$v_G = (0.02)70 = 1.40 \text{ ft/s}$$

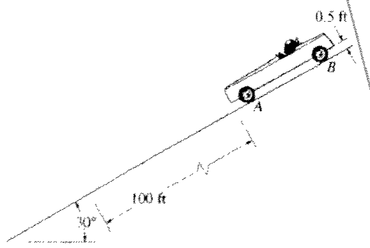
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + (0.3)(s) = \frac{1}{2}(\frac{0.3}{32.2})(1.40)^2 + \frac{1}{2}[(0.06)^2(\frac{0.3}{32.2})](70)^2$$

$s = 0.304$ ft **Ans**



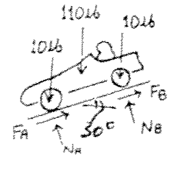
***18-12.** The soap-box car has a weight of 110 lb, including the passenger but *excluding* its four wheels. Each wheel has a weight of 5 lb, radius of 0.5 ft, and a radius of gyration $k = 0.3$ ft, computed about an axis passing through the wheel's axle. Determine the car's speed after it has traveled 100 ft starting from rest. The wheels roll without slipping. Neglect air resistance.



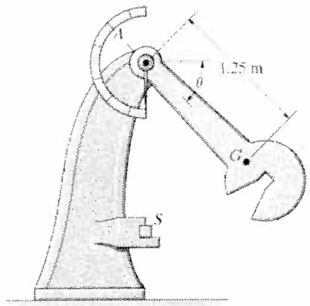
$$T_1 + \Sigma U_{1-2} = T_2$$

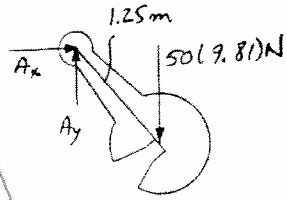
$$0 + 130(100\sin 30°) = 4\{\frac{1}{2}(\frac{5}{32.2})(v_C)^2 + \frac{1}{2}[(\frac{5}{32.2})(0.3)^2](\frac{v_C}{0.5})^2\} + \frac{1}{2}(\frac{110}{32.2})(v_C)^2$$

$v_C = 55.2$ ft/s **Ans**



18-13. The pendulum of the Charpy impact machine has a mass of 50 kg and a radius of gyration of $k_A = 1.75$ m. If it is released from rest when $\theta = 0°$, determine its angular velocity just before it strikes the specimen S, $\theta = 90°$.



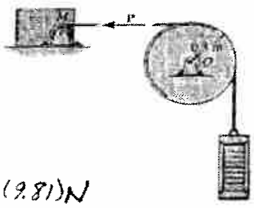


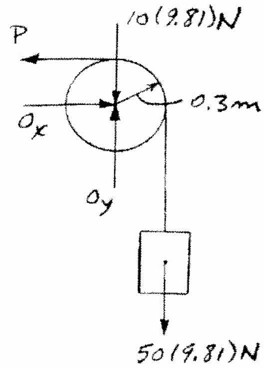
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + (50)(9.81)(1.25) = \frac{1}{2}\left[(50)(1.75)^2\right]\omega_2^2$$

$\omega_2 = 2.83$ rad/s **Ans**

18-14. The 10-kg pulley has a radius of gyration about O of $k_O = 0.21$ m. If a motor M supplies a force to the cable of $P = 800(3 - 2e^{-x})$ N, where x is the amount of cable wound up in meters, determine the speed of the 50-kg crate when it has been hoisted 2 m starting from rest. Neglect the mass of the cable and assume the cable does not slip on the pulley.





$$T_1 + \Sigma U_{1-2} = T_2$$

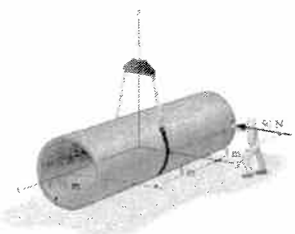
$$0 + \int_0^2 800(3 - 2e^{-x})\,dx - 50(9.81)(2) = \frac{1}{2}\left[(10)(0.21)^2\right]\left(\frac{v}{0.3}\right)^2 + \frac{1}{2}(50)(v)^2$$

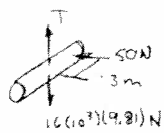
$$800(3x + 2e^{-x})|_0^2 - 981 = 27.45v^2$$

$$2435.54 = 27.45v^2$$

$v = 9.42$ m/s **Ans**

18-15. The uniform pipe has a mass of 16 Mg and radius of gyration about the z axis of $k_G = 2.7$ m. If the worker pushes on it with a horizontal force of 50 N, applied perpendicular to the pipe, determine the pipe's angular velocity when it has rotated 90° about the z axis, starting from rest. Assume the pipe does not swing.





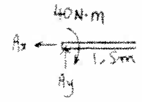
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 50(3)\left(\frac{\pi}{2}\right) = \frac{1}{2}\left[16(10^3)(2.7)^2\right]\omega^2$$

$\omega = 0.0636$ rad/s **Ans**

10

***18-16.** The 4-kg slender rod is subjected to the force and couple moment. When it is in the position shown it has an angular velocity $\omega_1 = 6$ rad/s. Determine its angular velocity at the instant it has rotated downward 90°. The force is always applied perpendicular to the axis of the rod. Motion occurs in the vertical plane.



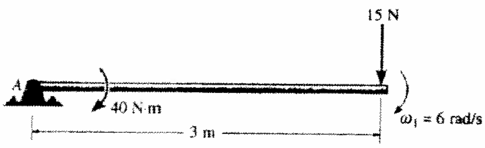
$$T_1 + \Sigma U_{1-2} = T_2$$

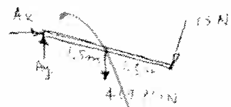
$$\frac{1}{2}[\frac{1}{3}(4)(3)^2](6)^2 + 15(\frac{\pi}{2})(3) + 4(9.81)(1.5) + 40(\frac{\pi}{2}) = \frac{1}{2}[\frac{1}{3}(4)(3)^2]\alpha$$

$\omega = 8.25$ rad/s **Ans**

11

18-17. The 4-kg slender rod is subjected to the force and couple moment. When the rod is in the position shown it has a angular velocity $\omega_1 = 6$ rad/s. Determine its angular velocity at the instant it has rotated 360°. The force is always applied perpendicular to the axis of the rod and motion occurs in the vertical plane.



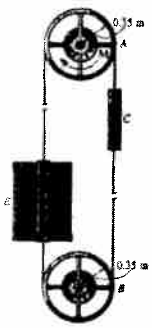


$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}[\frac{1}{3}(4)(3)^2](6)^2 + 15(2\pi)(3) + 40(2\pi) = \frac{1}{2}[\frac{1}{3}(4)(3)^2]\omega^2$$

$\omega = 11.2$ rad/s **Ans**

18-18. The elevator car E has a mass of 1.80 Mg and the counterweight C has a mass of 2.30 Mg. If a motor turns the driving sheave A with a constant torque of $M = 100$ N·m, determine the speed of the elevator when it has ascended 10 m starting from rest. Each sheave A and B has a mass of 150 kg and a radius of gyration of $k = 0.2$ m about its mass center or pinned axis. Neglect the mass of the cable and assume the cable does not slip on the sheaves.



$$\theta = \frac{10}{0.35} = 28.57 \text{ rad}$$

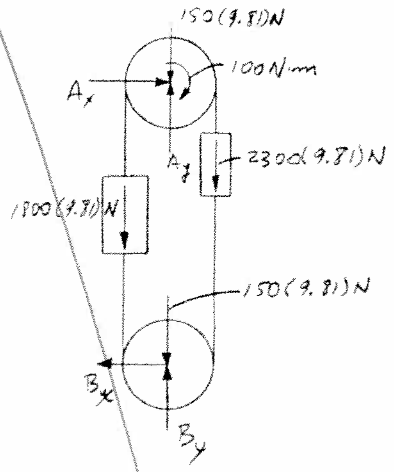
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 2300(9.81)(10) - 1800(9.81)(10) + 100(28.57)$$

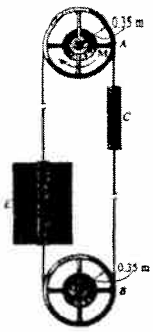
$$= \frac{1}{2}(1800)(v)^2 + \frac{1}{2}(2300)(v)^2 + (2)\frac{1}{2}[150(0.2)^2]\left(\frac{v}{0.35}\right)^2$$

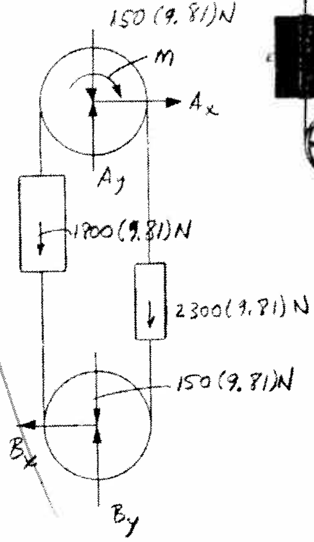
$$51\,907.1 = 2099v^2$$

$v = 4.97$ m/s **Ans**



18-19. The elevator car E has a mass of 1.80 Mg and the counterweight C has a mass of 2.30 Mg. If a motor turns the driving sheave A with a torque of $M = (0.06\theta^2 + 7.5)$ N·m, where θ is in radians, determine the speed of the elevator when it has ascended 12 m starting from rest. Each sheave A and B has a mass of 150 kg and a radius of gyration of $k = 0.2$ m about its mass center or pinned axis. Neglect the mass of the cable and assume the cable does not slip on the sheaves.





$$\theta = \frac{12}{0.35} = 34.29 \text{ rad}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 2300(9.81)(12) - 1800(9.81)(12) + \int_0^{34.29} \left(0.06\theta^2 + 7.5\right) d\theta$$

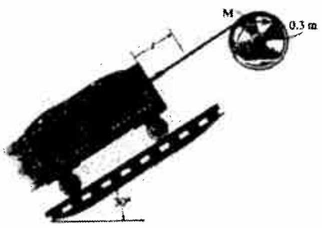
$$= \frac{1}{2}(1800)(v)^2 + \frac{1}{2}(2300)(v)^2 + (2)\frac{1}{2}\left[150(0.2)^2\right]\left(\frac{v}{0.35}\right)^2$$

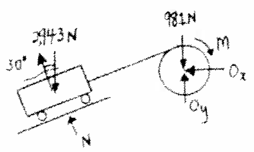
$$58860 + \left(0.02\theta^3 + 7.5\theta\right)\Big|_0^{34.29} = 2098.98v^2$$

$v = 5.34$ m/s **Ans**

12

***18-20.** The wheel has a mass of 100 kg and a radius of gyration $k_O = 0.2$ m. A motor supplies a torque $M = (40\theta + 900)$ N·m, where θ is in radians, about the drive shaft at O. Determine the speed of the loading car, which has a mass of 300 kg, after it travels $s = 4$ m. Initially the car is at rest when $s = 0$ and $\theta = 0°$. Neglect the mass of the attached cable and the mass of the car's wheels.





$$s = 0.3\theta = 4$$

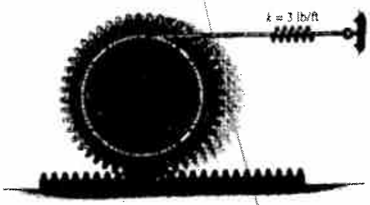
$$\theta = 13.33 \text{ rad}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$[0 + 0] + \int_0^{13.33} (40\theta + 900)\,d\theta - 300(9.81)\sin 30°(4) = \frac{1}{2}(300)v_C^2 + \frac{1}{2}\left[100(0.20)^2\right]\left(\frac{v_C}{0.3}\right)^2$$

$v_C = 7.49$ m/s **Ans**

18-21. The gear has a weight of 15 lb and a radius of gyration $k_G = 0.375$ ft. If the spring is unstretched when the torque $M = 6$ lb·ft is applied, determine the gear's angular velocity after its mass center G has moved to the left $s = 2$ ft.



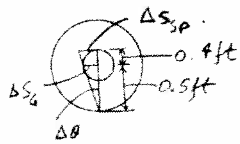
$\Delta s_G = 0.5\Delta\theta$

$\Delta s_{sp} = 0.9\Delta\theta$

$\Delta s_{sp} = 1.8\Delta s_G$

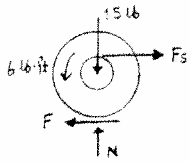
For $\Delta s_G = 2$ ft, then

$\Delta s_{sp} = 3.6$ ft



Also,

$$\theta = \frac{2}{0.5} = 4 \text{ rad}$$

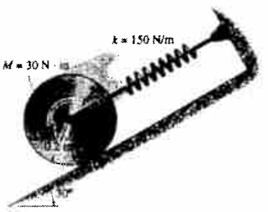


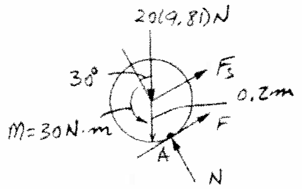
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 6(4) - \frac{1}{2}(3)(3.6)^2 = \frac{1}{2}\left(\frac{15}{32.2}\right)(0.5\omega)^2 + \frac{1}{2}\left[\left(\frac{15}{32.2}\right)(0.375)^2\right]\omega^2$$

$\omega = 7.08$ rad/s **Ans**

18-22. The 20-kg disk is originally at rest, and the spring holds it in equilibrium. A couple moment of $M = 30\ \text{N}\cdot\text{m}$ is then applied to the disk as shown. Determine its angular velocity at the instant its mass center G has moved $s = 0.8$ m down along the inclined plane. The disk rolls without slipping.





Initial tension in spring :

$+\Sigma M_A = 0; \quad -F_s(0.2) + 20(9.81)\sin 30°(0.2) = 0$

$F_s = 98.1$ N

$s_1 = \dfrac{98.1}{150} = 0.654$ m

When $s = 0.8$ m the disk rotates $\theta = \dfrac{0.8}{0.2} = 4$ rad

$T_1 + \Sigma U_{1-2} = T_2$

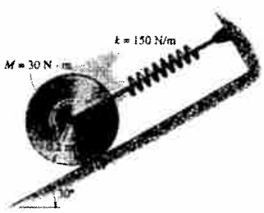
$$0 + 20(9.81)(0.8\sin 30°) + 30(4) - \left[\frac{1}{2}(150)(0.8 + 0.654)^2 - \frac{1}{2}(150)(0.654)^2\right]$$

$$= \frac{1}{2}\left[\frac{1}{2}(20)(0.2)^2\right]\omega^2 + \frac{1}{2}(20)(0.2\omega)^2$$

$72 = 0.6\omega^2$

$\omega = 11.0$ rad/s **Ans**

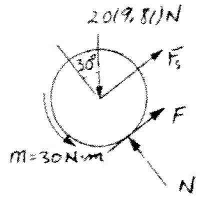
18-23. The 20-kg disk is originally at rest, and the spring holds it in equilibrium. A couple moment of $M = 30\ \text{N}\cdot\text{m}$ is then applied to the disk as shown. Determine how far the center of mass of the disk travels down along the incline, measured from the equilibrium position, before it stops. The disk rolls without slipping.



$+\Sigma M_A = 0; \quad -F_s(0.2) + 20(9.81)\sin 30°(0.2) = 0$

$F_s = 98.1$ N

$s_1 = \dfrac{98.1}{150} = 0.654$ m



When G moves s, the disk rotates $\theta = \dfrac{s}{0.2}$

$T_1 + \Sigma U_{1-2} = T_2$

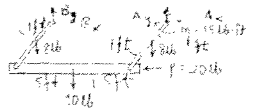
$$0 + 20(9.81)(s)\sin 30° + 30\left(\frac{s}{0.2}\right) - \left[\frac{1}{2}(150)(s + 0.654)^2 - \frac{1}{2}(150)(0.654)^2\right] = 0$$

$248.1s = 75(s^2 + 1.308s + 0.4277) - 32.08$

$248.1 = 75s + 98.1$

$s = 2.00$ m **Ans**

***18-24.** The linkage consists of two 8-lb rods AB and CD and a 10-lb bar AD. When $\theta = 0°$, rod AB is rotating with an angular velocity $\omega_{AB} = 2$ rad/s. If rod CD is subjected to a couple moment $M = 15$ lb · ft and bar AD is subjected to a horizontal force $P = 20$ lb as shown, determine ω_{AB} at the instant $\theta = 90°$.

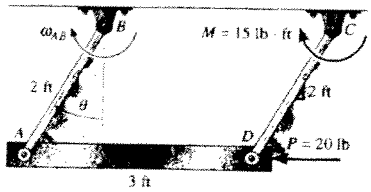


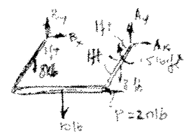
$$T_1 + \Sigma U_{1-2} = T_2$$

$$2[\frac{1}{2}\{\frac{1}{3}(\frac{8}{32.2})(2)^2\}(2)^2] + \frac{1}{2}(\frac{10}{32.2})(4)^2 + [20(2) + 15(\frac{\pi}{2}) - 2(8)(1) - 10(2)] =$$
$$2[\frac{1}{2}\{\frac{1}{3}(\frac{8}{32.2})(2)^2\}\omega^2] + \frac{1}{2}(\frac{10}{32.2})(2\omega)^2$$

$\omega_{AB} = 5.74$ rad/s **Ans**

18-25. The linkage consists of two 8-lb rods AB and CD and a 10-lb bar AD. When $\theta = 0°$, rod AB is rotating with an angular velocity $\omega_{AB} = 2$ rad/s. If rod CD is subjected to a couple moment $M = 15$ lb · ft and bar AD is subjected to a horizontal force $P = 20$ lb as shown, determine ω_{AB} at the instant $\theta = 45°$.



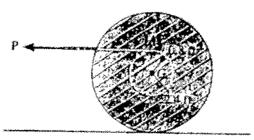


$$T_1 + \Sigma U_{1-2} = T_2$$

$$2[\frac{1}{2}\{\frac{1}{3}(\frac{8}{32.2})(2)^2\}(2)^2] + \frac{1}{2}(\frac{10}{32.2})(4)^2$$
$$+ [20(2\sin 45°) + 15(\frac{\pi}{4}) - 2(8)(1 - \cos 45°) - 10(2 - 2\cos 45°)]$$
$$= 2[\frac{1}{2}\{\frac{1}{3}(\frac{8}{32.2})(2)^2\}\omega^2] + \frac{1}{2}(\frac{10}{32.2})(2\omega)^2$$

$\omega_{AB} = 5.92$ rad/s **Ans**

18-26. The spool has a weight of 500 lb and a radius of gyration of $k_G = 1.75$ ft. A horizontal force of $P = 15$ lb is applied to a cable wrapped around its inner core. If the spool is originally at rest, determine its angular velocity after the mass center G has moved 6 ft to the left. The spool rolls without slipping. Neglect the mass of the cable.

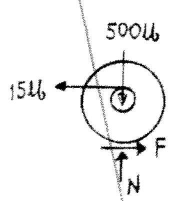


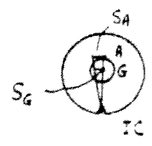
$$\frac{s_G}{2.4} = \frac{s_A}{3.2}$$

For $s_G = 6$ ft, then $s_A = 8$ ft

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 15(8) = \frac{1}{2}\left[\left(\frac{500}{32.2}\right)(1.75)^2\right]\omega^2 + \frac{1}{2}\left(\frac{500}{32.2}\right)(2.4\omega)^2$$





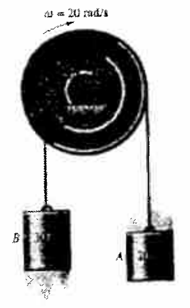
$\omega = 1.32$ rad/s **Ans**

14

18-27. The double pulley consists of two parts that are attached to one another. It has a weight of 50 lb and a centroidal radius of gyration of $k_O = 0.6$ ft and is turning with an angular velocity of 20 rad/s clockwise. Determine the kinetic energy of the system. Assume that neither cable slips on the pulley.

mass 5 50kg

m

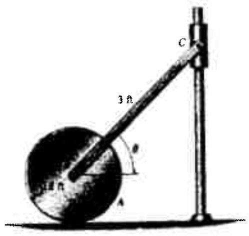


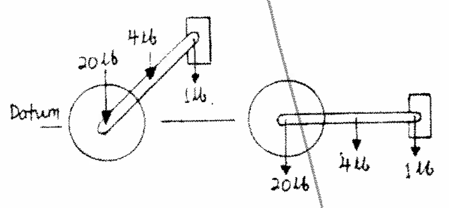
$$T = \frac{1}{2} I_O \omega_O^2 + \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

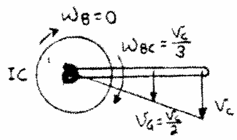
$$T = \frac{1}{2}\left(\frac{50}{32.2}(0.6)^2\right)(20)^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)[(20)(1)]^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)[(20)(0.5)]^2 = \cancel{283\ \text{ft}\cdot\text{lb}}$$ **Ans**

9.11 KJ

***18-28.** The system consists of a 20-lb disk A, 4-lb slender rod BC, and a 1-lb smooth collar C. If the disk rolls without slipping, determine the velocity of the collar at the instant the rod becomes horizontal, i.e., $\theta = 0°$. The system is released from rest when $\theta = 45°$.





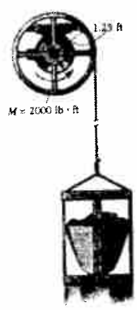


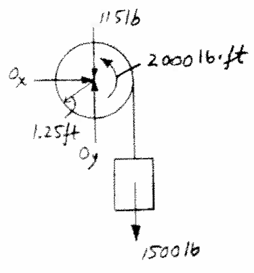
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 4(1.5\sin 45°) + 1(3\sin 45°) = \frac{1}{2}\left[\frac{1}{3}\left(\frac{4}{32.2}\right)(3)^2\right]\left(\frac{v_C}{3}\right)^2 + \frac{1}{2}\left(\frac{1}{32.2}\right)(v_C)^2 + 0$$

$v_C = 13.3$ ft/s **Ans**

18-29 The 1500-lb cement bucket is hoisted using a motor that supplies a torque of $M = 2000$ lb·ft to the axle of the wheel. If the wheel has a weight of 115 lb and a radius of gyration about O of $k_O = 0.95$ ft, determine the speed of the bucket when it has been hoisted 10 ft starting from rest.



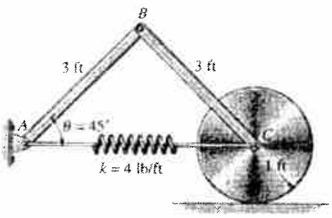


$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 2000\left(\frac{10}{1.25}\right) - 1500(10) = \frac{1}{2}\left(\frac{1500}{32.2}\right)v^2 + \frac{1}{2}\left[\left(\frac{115}{32.2}\right)(0.95)^2\right]\left(\frac{v}{1.25}\right)^2$$

$v = 6.41$ ft/s **Ans**

18-30. The assembly consists of two 15-lb slender rods and a 20-lb disk. If the spring is unstretched when $\theta = 45°$ and the assembly is released from rest at this position, determine the angular velocity of rod AB at the instant $\theta = 0°$. The disk rolls without slipping.

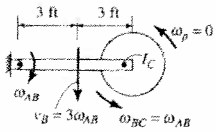


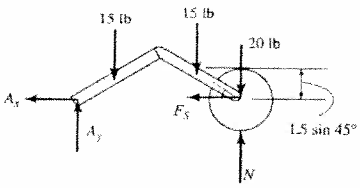
$$T_1 + \sum U_{1-2} = T_2$$

$$[0 + 0] + 2(15)(1.5)\sin 45° - \frac{1}{2}(4)[6 - 2(3)\cos 45°]^2$$

$$= 2\left[\frac{1}{2}\left(\frac{1}{3}\left(\frac{15}{32.2}\right)(3)^2\right)\omega_{AB}^2\right]$$

$\omega_{AB} = 4.28$ rad/s **Ans**





15

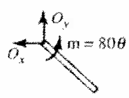
18-31. The uniform door has a mass of 20 kg and can be treated as a thin plate having the dimensions shown. If it is connected to a torsional spring at A, which has a stiffness of $k = 80$ N · m/rad, determine the required initial twist of the spring in radians so that the door has an angular velocity of 12 rad/s when it closes at $\theta = 0°$ after being opened at $\theta = 90°$ and released from rest. *Hint:* For a torsional spring $M = k\theta$, when k is the stiffness and θ is the angle of twist.

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + \int_{\theta_0}^{\theta_0+\frac{\pi}{2}} 80\theta\, d\theta = \frac{1}{2}\left[\frac{1}{3}(20)(0.8)^2\right](12)^2$$

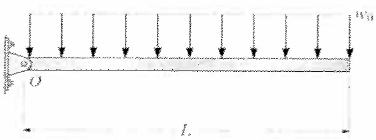
$$40\left[\left(\theta_0 + \frac{\pi}{2}\right)^2 - \theta_0^2\right] = 307.2$$

$\theta_0 = 1.66$ rad **Ans**



16

***18-32.** The uniform slender bar has a mass m and a length L is subjected to a uniform distributed load w_0 which is always erected perpendicular to the axis of the bar. If it is released from rest from the position shown, determine its angular velocity at the instant it has rotated 90°. Solve the problem for rotation in (a) the horizontal plane, and (b) the vertical plane.



a) $T_1 + \Sigma U_{1-2} = T_2$

$$[0] + \int_0^{\frac{\pi}{2}}\int_0^L (w_0\,dx)(x\,d\theta) = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2$$

$$\int_0^{\frac{\pi}{2}} \frac{w_0 L^2}{2}\,d\theta = \frac{1}{6}mL^2\omega^2$$

$$\frac{w_0 L^2}{2}\left(\frac{\pi}{2}\right) = \frac{1}{6}mL^2\omega^2$$

$\omega = \sqrt{\frac{3\pi}{2}\left(\frac{w_0}{m}\right)}$ **Ans**

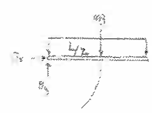
Note : The work of the distributed load can also be determined from its resultant.

$$U_{1-2} = w_0 L\left(\frac{\pi}{2}\right)\left(\frac{L}{2}\right) = \frac{w_0}{4}\pi L^2$$

b) $T_1 + \Sigma U_{1-2} = T_2$

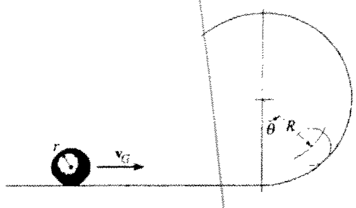
$$[0] + \frac{w_0}{4}\pi L^2 + mg\left(\frac{L}{2}\right) = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2$$

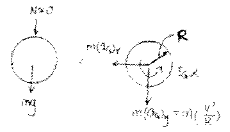
$$\omega^2 = \frac{3}{2}\frac{w_0 \pi L}{mL} + \frac{mg(6)}{2mL}$$



$\omega = \sqrt{\frac{3\pi}{2}\frac{w_0}{m} + \frac{3g}{L}}$ **Ans**

18-33. A ball of mass m and radius r is cast onto the horizontal surface such that it rolls without slipping. Determine the minimum speed v_G of its mass center G so that it rolls completely around the loop of radius $R + r$ without leaving the track.





$$+\downarrow \Sigma F_y = m(a_G)_y; \quad mg = m(\frac{v^2}{R})$$

$$v^2 = gR$$

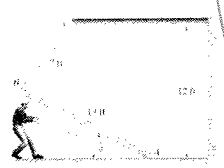
$$T_1 + \Sigma U_{1-2} = T_2$$

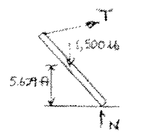
$$\frac{1}{2}(\frac{2}{5}mr^2)(\frac{v_G^2}{r^2}) + \frac{1}{2}mv_G^2 - mg(2R) = \frac{1}{2}(\frac{2}{5}mr^2)(\frac{gR}{r^2}) + \frac{1}{2}m(gR)$$

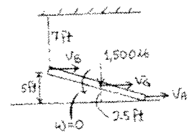
$$\frac{1}{5}v_G^2 + \frac{1}{2}v_G^2 = 2gR + \frac{1}{5}gR + \frac{1}{2}gR$$

$v_G = 3\sqrt{\frac{3}{7}gR}$ **Ans**

18-34. The beam has a weight of 1500 lb and is being raised to a vertical position by pulling very slowly on its bottom end A. If the cord fails when $\theta = 60°$ and the beam is essentially at rest, determine the speed of A at the instant cord BC becomes vertical. Neglect friction and the mass of the cords, and treat the beam as a slender rod.







$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 1500(5.629) - 1500(2.5) = \frac{1}{2}(\frac{1500}{32.2})(v_G)^2$$

$v_G = v_A = 14.2$ ft/s **Ans**

18-35. Solve Prob. 18-13 using the conservation of energy equation.

Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 50(9.81)(1.25) = \frac{1}{2}[50(1.75)^2]\omega^2 + 0$$

$\omega = 2.83$ rad/s **Ans**

***18-36.** Solve Prob. 18-12 using the conservation of energy equation.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 4[\frac{1}{2}(\frac{5}{32.2})v_C^2 + \frac{1}{2}(\frac{5}{32.2})(0.3)^2(\frac{v_C}{0.5})^2] + \frac{1}{2}(\frac{110}{32.2})v_C^2 - 130(100\sin 30°)$$

$v_C = 55.2$ ft/s **Ans**

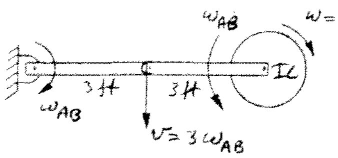
18-37. Solve Prob. 18-30 using the conservation of energy equation.

Datum at lowest point.

$T_1 + V_1 = T_2 + V_2$

$0 + 2\left[15(1.5\sin 45^\circ)\right] = 2\left[\frac{1}{2}\left(\frac{1}{3}\left(\frac{15}{32.2}\right)(3)^2\right)\omega_{AB}^2\right] + \frac{1}{2}(4)\left[6 - 2(3\cos 45^\circ)\right]^2 + 0$

$\omega_{AB} = 4.28$ rad/s **Ans**



18–38. Solve Prob. 18–11 using the conservation of energy equation.

$v_G = (0.02)\omega = 0.02(70) = 1.4$ ft/s

$T_1 + V_1 = T_2 + V_2$

$0 + 0 = \frac{1}{2}(\frac{0.3}{32.2})(1.4)^2 + \frac{1}{2}(0.06)^2(\frac{0.3}{32.2})](70)^2 - (0.3)s$

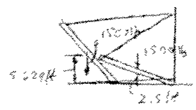
$s = 0.304$ ft **Ans**

18–39. Solve Prob. 18–34 using the conservation of energy equation.

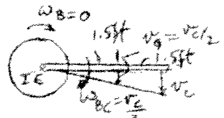
$T_1 + V_1 = T_2 + V_2$

$0 + 1500(5.629) = \frac{1}{2}(\frac{1500}{32.2})(v_G)^2 + (1500)(2.5)$

$v_G = v_A = 14.2$ ft/s **Ans**



***18–40.** Solve Prob. 18–28 using the conservation of energy equation.



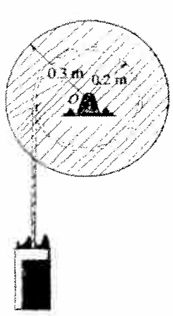
$T_1 + V_1 = T_2 + V_2$

$0 + 4(1.5\sin 45^\circ) + 1(3\sin 45^\circ) = \frac{1}{2}[\frac{1}{3}(\frac{4}{32.2})(3)^2](\frac{v_C}{3})^2 + \frac{1}{2}(\frac{1}{32.2})(v_C)^2 + 0$

$v_C = 13.3$ ft/s **Ans**

17

18-~~41~~. The spool has a mass of 50 kg and a radius of gyration $k_O = 0.280$ m. If the 20-kg block A is released from rest, determine the distance the block must fall in order for the spool to have an angular velocity $\omega = 5$ rad/s. Also, what is the tension in the cord while the block is in motion? Neglect the mass of the cord.



$v_A = 0.2\,\omega = 0.2(5) = 1$ m/s

System :

$T_1 + V_1 = T_2 + V_2$

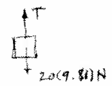
$[0 + 0] + 0 = \frac{1}{2}(20)(1)^2 + \frac{1}{2}[50(0.280)^2](5)^2 - 20(9.81)\,s$

$s = 0.30071$ m $= 0.301$ m **Ans**

Block :

$T_1 + \Sigma U_{1-2} = T_2$

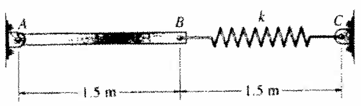
$0 + 20(9.81)(0.30071) - T(0.30071) = \frac{1}{2}(20)(1)^2$



$T = 163$ N **Ans**

18

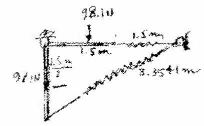
18-~~42~~. When the slender 10-kg bar AB is horizontal it is at rest and the spring is unstretched. Determine the stiffness k of the spring so that the motion of the bar is momentarily stopped when it has rotated downward 90°.



$T_1 + V_1 = T_2 + V_2$

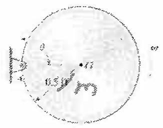
$0 + 0 = 0 + \frac{1}{2}(k)(3.3541 - 1.5)^2 - 98.1(\frac{1.5}{2})$

$k = 42.8$ N/m **Ans**



19

18-~~43~~. The 15-~~lb~~ kg disk is rotating about pin A in the vertical plane with an angular velocity $\omega = 2$ rad/s when $\theta = 0°$. Determine its angular velocity at the instant $\theta = 90°$. Also, compute the horizontal and vertical components of reaction at A at this instant.



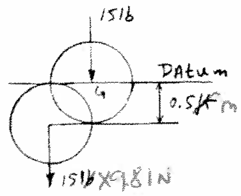
Datum through G at initial point.

$T_1 + V_1 = T_2 + V_2$

$\frac{1}{2}\left[\frac{3}{2}\left(\frac{15}{\cancel{32.2}}\right)(0.5)^2\right](2)^2 + 0 = \frac{1}{2}\left[\frac{3}{2}\left(\frac{15}{\cancel{32.2}}\right)(0.5)^2\right]\omega^2 - 15(0.5)$

2.58

$\omega =$ ~~9.48 rad/s~~ **Ans**



$\xleftarrow{+} \Sigma F_n = m(a_G)_n;\quad A_x = \left(\frac{15}{\cancel{32.2}}\right)(\cancel{9.48})^2(0.5)$

2.58

$A_x =$ ~~20.9 lb~~ ~~Ans~~ 49.9 N

$\circlearrowleft + \Sigma M_A = I_A\alpha;\quad 15(0.5) = \frac{3}{2}\left(\frac{15}{\cancel{32.2}}\right)(0.5)^2\,\alpha$

×9.81

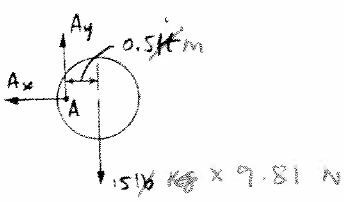
$\alpha =$ ~~42.93 rad/s²~~ 13.08 rad/s²

13.08

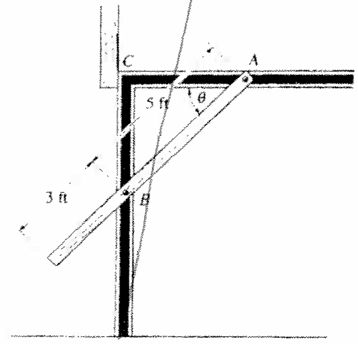
$+\downarrow \Sigma F_t = m(a_G)_t;\quad 9.81 \times 15 - A_y = \left(\frac{15}{\cancel{32.2}}\right)(\cancel{42.93})(0.5)$

$A_y =$ ~~5.00 lb~~ **Ans**

49.1 N



***18-44.** The door is made from one piece, whose ends move along the horizontal and vertical tracks. If the door is in the open position, $\theta = 0°$, and then released, determine the speed at which its end A strikes the stop at C. Assume the door is a 180-lb thin plate having a width of 10 ft.

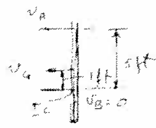


$$T_1 + V_1 = T_2 + V_2$$

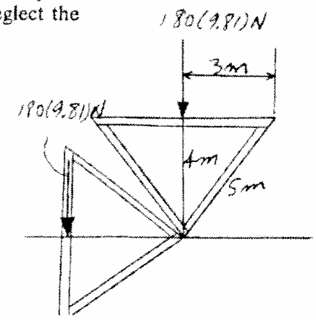
$$0 + 0 = \frac{1}{2}\left[\frac{1}{12}\left(\frac{180}{32.2}\right)(8)^2\right]\omega^2 + \frac{1}{2}\left(\frac{180}{32.2}\right)(1\omega)^2 - 180(4)$$

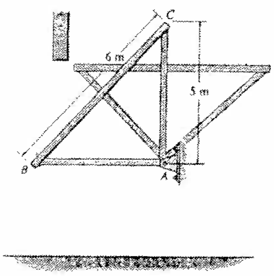
$$\omega = 6.3776 \text{ rad/s}$$

$$v_C = \omega(5) = 6.3776(5) = 31.9 \text{ m/s} \qquad \textbf{Ans}$$



18-~~45~~ 20. The overhead door BC is pushed slightly from its open position and then rotates downward about the pin at A. Determine its angular velocity just before its end B strikes the floor. Assume the door is a thin plate having a mass of 180 kg and length of 6 m. Neglect the mass of the supporting frame AB and AC.





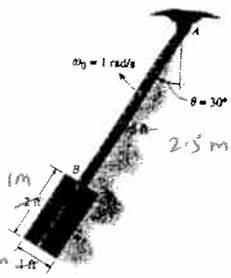
Datum at lowest point

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 180(9.81)4 = \frac{1}{2}\left[\frac{1}{12}(180)(6)^2\right]\omega^2 + \frac{1}{2}(180)(4\omega)^2 + 0$$

$$\omega = 2.03 \text{ rad/s} \qquad \textbf{Ans}$$

18-46. The 80-~~lb~~ kg cylinder is attached to the 10-~~lb~~ kg slender rod which is pinned at point A. At the instant $\theta = 30°$ the rod has an angular velocity of $\omega_0 = 1$ rad/s as shown. Determine the angle θ to which the rod swings before it momentarily stops.



$$I_A = \frac{1}{12}\left(\frac{80}{\cancel{32.2}}\right)\left[3(\cancel{0.5})^2 + (\cancel{2})^2\right] + \left(\frac{80}{\cancel{32.2}}\right)(\cancel{6})^2 + \frac{1}{3}\left(\frac{10}{\cancel{32.2}}\right)(\cancel{6})^2 = \cancel{93.01 \text{ slug}\cdot\text{ft}^2}$$

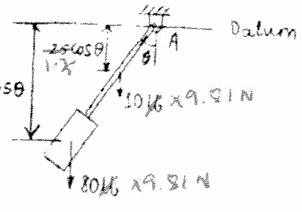
0.25, 1, 3, 2.5 $= 748.8 \text{ kg m}^2$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(\cancel{93.01})(1)^2 - (10)(\cancel{2.5}\cos 30°) - 80(\cancel{6}\cos 30°) = 0 - (10)(\cancel{2.5}\cos\theta) - 80(\cancel{6}\cos\theta)$$

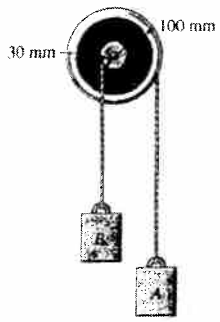
748.8; ×9.81; 1.25; ×9.81; 3; ×9.81; 1.25; ×9.81; 3

$\theta = \cancel{39.3°}$ **Ans** 44.4°



22

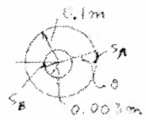
18-47. The compound disk pulley consists of a hub and attached outer rim. If it has a mass of 3 kg and a radius of gyration $k_G = 45$ mm, determine the speed of block A after A descends 0.2 m from rest. Blocks A and B each have a mass of 2 kg. Neglect the mass of the cords.



$T_1 + V_1 = T_2 + V_2$

$$[0 + 0 + 0] + [0 + 0] = \frac{1}{2}[3(0.045)^2]\omega^2 + \frac{1}{2}(2)(0.03\omega)^2 + \frac{1}{2}(2)(0.1\omega)^2 - 2(9.81)s_A + 2(9.81)s_B$$

$\theta = \dfrac{s_B}{0.03} = \dfrac{s_A}{0.1}$



$s_B = 0.3\, s_A$

Set $s_A = 0.2$ m, $\quad s_B = 0.06$ m

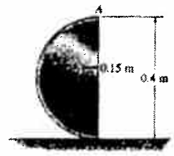
Substituting and solving yields,

$\omega = 14.04$ rad/s

$v_A = 0.1(14.04) = 1.40$ m/s **Ans**

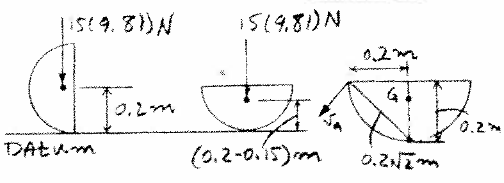
23

***18-48.** The 15-kg semicircular segment is released from rest in the position shown. Determine the velocity of point A when it has rotated counterclockwise 90°. Assume that the segment rolls without slipping on the surface. The moment of inertia about its mass center is $I_G = 0.25$ kg·m^2.



$T_1 + V_1 = T_2 + V_2$

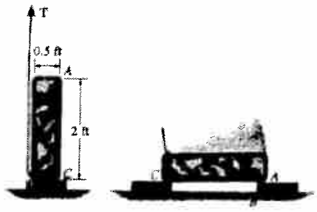
$$0 + 15(9.81)(0.2) = \frac{1}{2}(0.25)\omega^2 + \frac{1}{2}(15)\left[(0.2 - 0.15)\omega\right]^2 + 15(9.81)(0.2 - 0.15)$$

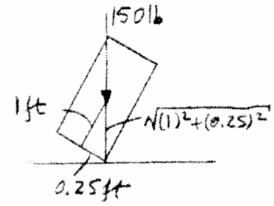


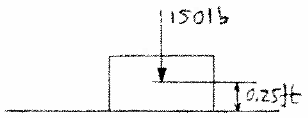
$\omega = 12.39$ rad/s

$v_A = 12.39\left(0.2\sqrt{2}\right) = 3.50$ m/s 45° **Ans**

18-49. The uniform 150-lb stone (rectangular block) is being turned over on its side by pulling the vertical cable *slowly* upward until the stone begins to tip. If it then falls freely ($\mathbf{T} = \mathbf{0}$) from an essentially balanced at-rest position, determine the speed at which the corner A strikes the pad at B. The stone does not slip at its corner C as it falls.







$T_1 + V_1 = T_2 + V_2$

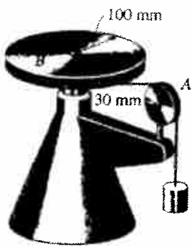
$$0 + 1.0308(150) = \frac{1}{2}\left[\frac{1}{12}\left(\frac{150}{32.2}\right)\left(0.5^2 + 2^2\right)\right]\omega^2 + \frac{1}{2}\left(\frac{150}{32.2}\right)(1.0308\,\omega)^2 + (0.25)(150)$$

$\omega = 5.958$ rad/s

The IC is at C.

$v_A = 2\omega = 2(5.958) = 11.9$ ft/s **Ans**

18-50. The assembly consists of a 3-kg pulley A and 10-kg pulley B. If a 2-kg block is suspended from the cord, determine the block's speed after it descends 0.5 m starting from rest. Neglect the mass of the cord and treat the pulleys as thin disks. No slipping occurs.



$T_1 + V_1 = T_2 + V_2$

$$[0 + 0 + 0] + [0] = \frac{1}{2}[\frac{1}{2}(3)(0.03)^2]\omega_A^2 + \frac{1}{2}[\frac{1}{2}(10)(0.1)^2]\omega_B^2 + \frac{1}{2}(2)(v_C)^2 - 2(9.81)(0.5)$$

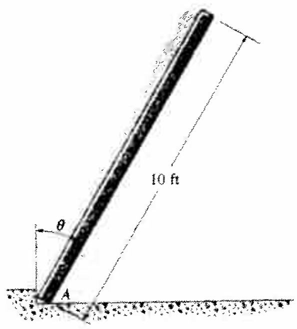
$v_C = \omega_B(0.1) = 0.03\omega_A$

Thus,

$\omega_B = 10\,v_C$

$\omega_A = 33.33\ldots$

18-51. A uniform ladder having a weight of 30 lb is released from rest when it is in the vertical position. If it is allowed to fall freely, determine the angle θ at which the bottom end A starts to lift off the ground. For the calculation, assume the ladder to be a slender rod and neglect friction at A.



Potential Energy : Datum is set at point A. When the ladder is at its initial and final position, its center of gravity is located 5 ft and ($5\cos\theta$) ft *above* the datum. Its initial and final gravitational potential energy are $30(5) = 150$ ft · lb and $30(5\cos\theta) = 150\cos\theta$ ft · lb respectively. Thus, the initial and final potential energy are

$$V_1 = 150 \text{ ft} \cdot \text{lb} \qquad V_2 = 150\cos\theta \text{ ft} \cdot \text{lb}$$

Kinetic Energy : The mass moment inertia of the ladder about point A is $I_A = \frac{1}{12}\left(\frac{30}{32.2}\right)(10^2) + \left(\frac{30}{32.2}\right)(5^2) = 31.06 \text{ slug} \cdot \text{ft}^2$. Since the ladder is initially at rest, the initial kinectic energy is $T_1 = 0$. The final kinetic energy is given by

$$T_2 = \frac{1}{2} I_A \omega^2 = \frac{1}{2}(31.06)\,\omega^2 = 15.53\omega^2$$

Conservation of Energy : Applying Eq. 18 – 18, we have

$$T_1 + V_1 = T_2 + V_2$$
$$0 + 150 = 15.53\omega^2 + 150\cos\theta$$
$$\omega^2 = 9.66(1 - \cos\theta)$$

Equation of Motion : The mass moment inertia of the ladder about its mass center is $I_G = \frac{1}{12}\left(\frac{30}{32.2}\right)(10^2) = 7.764 \text{ slug} \cdot \text{ft}^2$. Applying Eq. 17 – 16, we have

$$+\Sigma M_A = \Sigma(M_k)_A; \qquad -30\sin\theta(5) = -7.764\alpha - \left(\frac{30}{32.2}\right)[\alpha(5)](5)$$
$$\alpha = 4.83\sin\theta$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad A_y - 30 = -\frac{30}{32.2}[9.66(1-\cos\theta)(5)]\cos\theta - \frac{30}{32.2}[4.83\sin\theta(5)]\sin\theta$$

$$\begin{aligned} A_y &= 30 - \frac{30}{32.2}\left(48.3\cos\theta - 48.3\cos^2\theta + 24.15\sin^2\theta\right) \\ &= 30 - 45.0\cos\theta + 45.0\cos^2\theta - 22.5\sin^2\theta \\ &= 30 - 45.0\cos\theta + 45.0\cos^2\theta - 22.5\left(1 - \cos^2\theta\right) \\ &= 7.50\left(9\cos^2\theta - 6\cos\theta + 1\right) \\ &= 7.50(3\cos\theta - 1)^2 \end{aligned}$$

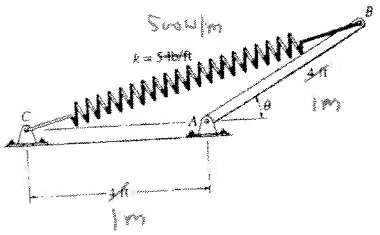
If the ladder lifts off the ground, then $A_y = 0$. Thus,

$$7.50(3\cos\theta - 1)^2 = 0$$
$$\theta = 70.5° \qquad \textbf{Ans}$$

---

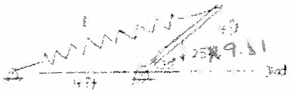
m kg

***18-52.** The 25-~~lb~~ slender rod AB is attached to a spring BC which, has an unstretched length of ~~4 ft~~. If the rod is released from rest when $\theta = 30°$, determine its angular velocity at the instant $\theta = 90°$. 1m



3.73m

$$l = \sqrt{(\cancel{4})^2 + (\cancel{4})^2 - 2(\cancel{4})(\cancel{4})\cos 150°} = \cancel{7.727\text{ ft}}$$
1 1 1 1



$$T_1 + V_1 = T_2 + V_2$$

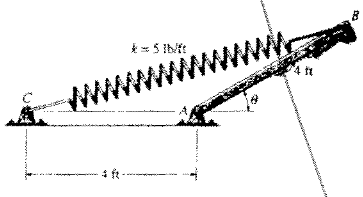
×9.81 500 5W

$$0 + 25(\cancel{2})\sin 30° + \frac{1}{2}(\cancel{5})(\cancel{7.727 - 4})^2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{25}{\cancel{32.2}}\right)(\cancel{4})^2\right]\omega^2 + \cancel{25}(\cancel{2}) + \frac{1}{2}(\cancel{5})(\cancel{4}\sqrt{2} - \cancel{4})^2$$
1 (3.73 − 1) 1 25×9.81 ×1 1 1

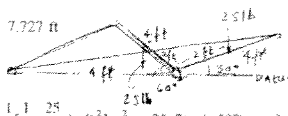
$$\omega = \cancel{1.18\text{ rad/s}} \qquad \textbf{Ans}$$

= 20.18 rad/s

18-53. The 25-lb slender rod AB is attached to a spring BC which has an unstretched length of 4 ft. If the rod is released from rest when $\theta = 30°$, determine the angular velocity of the rod the instant the spring becomes unstretched.



$$l = \sqrt{(4)^2 + (4)^2 - 2(4)(4)\cos 150°} = 7.727 \text{ ft}$$

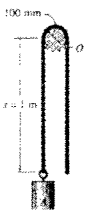


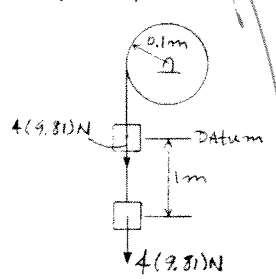
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 25(2)\sin 30° + \frac{1}{2}(5)(7.727 - 4)^2 = \frac{1}{2}[\frac{1}{3}(\frac{25}{32.2})(4)^2]\omega^2 + 25(2)(\sin 60°) + 0$$

$\omega = 2.82$ rad/s **Ans**

18-54. A chain that has a negligible mass is draped over a sprocket which has a mass of 2 kg and a radius of gyration of $k_O = 50$ mm. If the 4-kg block A is released from rest in the position $s = 1$ m, determine the angular velocity of the sprocket at the instant $s = 2$ m.



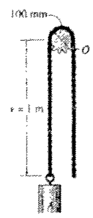


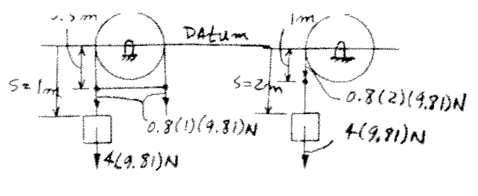
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 + 0 = \frac{1}{2}(4)(0.1\omega)^2 + \frac{1}{2}\left[2(0.05)^2\right]\omega^2 - 4(9.81)(1)$$

$\omega = 41.8$ rad/s **Ans**

18-55. Solve Prob. 18-54 if the chain has a mass of 0.8 kg/m. For the calculation neglect the portion of the chain that wraps over the sprocket.





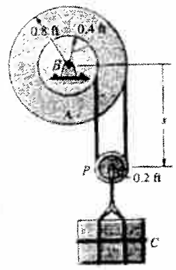
$$T_1 + V_1 = T_2 + V_2$$

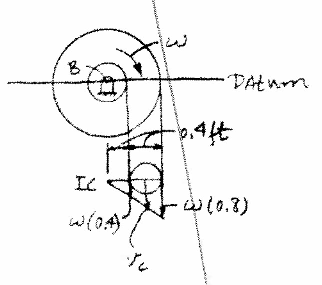
$$0 - 4(9.81)(1) - 2\left[0.8(1)(9.81)(0.5)\right] = \frac{1}{2}(4)(0.1\omega)^2 + \frac{1}{2}\left[2(0.05)^2\right]\omega^2 + \frac{1}{2}(0.8)(2)(0.1\omega)^2$$

$$-4(9.81)(2) - 0.8(2)(9.81)(1)$$

$\omega = 39.3$ rad/s **Ans**

***18-56.** Pulley A has a weight of 30 lb and a centroidal radius of gyration $k_B = 0.6$ ft. Determine the speed of the 20-lb crate C at the instant $s = 10$ ft. Initially, the crate is released from rest when $s = 5$ ft. The pulley at P "rolls" downward on the cord without slipping. For the calculation, neglect the mass of this pulley and the cord as it unwinds from the inner and outer hubs of pulley A.





$$\frac{v_C}{0.6} = \frac{\omega(0.4)}{0.4}$$

$\omega = 1.667 v_C$

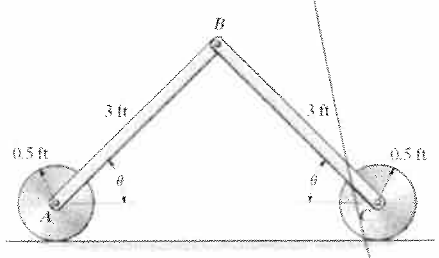
Datum through B.

$T_1 + V_1 = T_2 + V_2$

$$0 - 5(20) = \frac{1}{2}\left(\frac{20}{32.2}\right)v_C^2 + \frac{1}{2}\left[\left(\frac{30}{32.2}\right)(0.6)^2\right](1.667 v_C)^2 - 10(20)$$

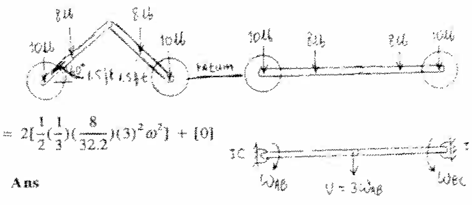
$v_C = 11.3$ ft/s **Ans**

18–57. The assembly consists of two 8-lb bars which are pinconnected to the two 10-lb disks. If the bars are released from rest when $\theta = 60°$, determine their angular velocities at the instant $\theta = 0°$. Assume the disks roll without slipping.



$\omega_{AB} = \omega_{BC}$

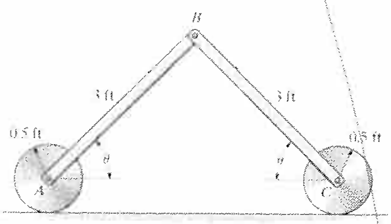
$T_1 + V_1 = T_2 + V_2$



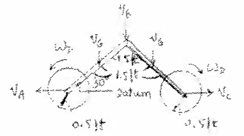
$$[0] + 2(8)(1.5\sin 60°) = 2[\frac{1}{2}(\frac{1}{3})(\frac{8}{32.2})(3)^2\omega^2] + [0]$$

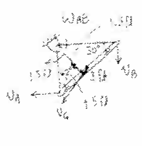
$\omega = 5.28$ rad/s **Ans**

18–58. The assembly consists of two 8-lb bars which are pinconnected to the two 10-lb disks. If the bars are released from rest when $\theta = 60°$, determine their angular velocities at the instant $\theta = 30°$. Assume the disks roll without slipping.



$$\omega_D = \frac{v_A}{0.5}$$





$$v_A = \omega_{AB}(1.5)$$

$$\omega_D = 3\omega_{AB}$$

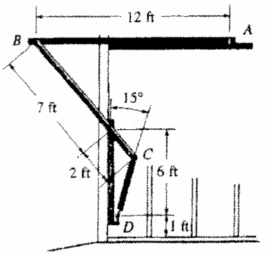
$$v_G = 1.5\omega_{AB}$$

$$T_1 + V_1 = T_2 + V_2$$

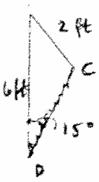
$$\{0 + 0\} + 2[8(1.5\sin 60°)] = 2[\frac{1}{2}\{\frac{1}{2}(\frac{10}{32.2})(0.5)^2\}(3\omega_{AB})^2 + \frac{1}{2}(\frac{10}{32.2})\{\omega_{AB}(1.5)\}^2$$
$$+ \frac{1}{2}(\frac{8}{32.2})(1.5\omega_{AB})^2 + \frac{1}{2}\{\frac{1}{12}(\frac{8}{32.2})(3)^2\}(\omega_{AB})^2] + 2[8(1.5\sin 30°)]$$

$\omega_{AB} = 2.21$ rad/s **Ans**

18-59. The end A of the garage door AB travels along the horizontal track, and the end of member BC is attached to a spring at C. If the spring is originally unstretched, determine the stiffness k so that when the door falls downward from rest in the position shown, it will have zero angular velocity the moment it closes, i.e., when it and BC become vertical. Neglect the mass of member BC and assume the door is a thin plate having a weight of 200 lb and a width and height of 12 ft. There is as similar connection and spring on the other side of the door.



$$(2)^2 = (6)^2 + (CD)^2 - 2(6)(CD)\cos 15°$$



$$CD^2 - 11.591CD + 32 = 0$$

Selecting the smaller root :

$$CD = 4.5352 \text{ ft}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 + 2[\frac{1}{2}(k)(8 - 4.5352)^2] - 200(6)$$

$k = 100$ lb/ft **Ans**