**15-1.** A 20-lb block slides down a 30° inclined plane with an initial velocity of 2 ft/s. Determine the velocity of the block in 3 s if the coefficient of kinetic friction between the block and the plane is $\mu_k = 0.25$.

[handwritten: N; 1 m/s]

$(+\nwarrow)\quad m(v_{y'})_1 + \Sigma\int_{t_1}^{t_2} F_{y'}\,dt = m(v_{y'})_2$

$0 + N(3) - 20\cos 30°(3) = 0 \qquad N = 17.32\ \text{lb}$

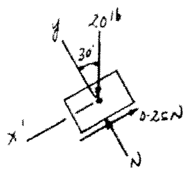
$(+\swarrow)\quad m(v_{x'})_1 + \Sigma\int_{t_1}^{t_2} F_{x'}\,dt = m(v_{x'})_2$

$\frac{20}{32.2}(2) + 20\sin 30°(3) - 0.25(17.32)(3) = \frac{20}{32.2}v$

[handwritten: 32.2 → 9.81; 2 → 1]

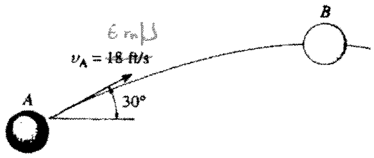
$v = 29.4\ \text{ft/s}$ **Ans**

[handwritten: 9.34 m/s]




**15-2.** A 2-lb ball is thrown in the direction shown with an initial speed $v_A = 18$ ft/s. Determine the time needed for it to reach its highest point $B$ and the speed at which it is traveling at $B$. Use the principle of impulse and momentum for the solution.

[handwritten: 2 N; 6 m/s]




$(+\uparrow)\quad m(v_y)_1 + \Sigma\int F\,dt = m(v_y)_2$

$\frac{2}{32.2}(18\sin 30°) - 2(t) = 0$

$t = 0.2795 = 0.280\ \text{s}$ **Ans**

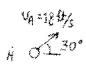
[handwritten: 0.306 s]

$(\xrightarrow{+})\quad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$

$\frac{2}{32.2}(18\cos 30°) + 0 = \frac{2}{32.2}(v_B)$

$v_B = 15.588 = 15.6\ \text{ft/s}$ **Ans**

[handwritten: = 5.20 m/s]




**15-3.** A 5-lb block is given an initial velocity of 10 ft/s up a 45° smooth slope. Determine the time it will take to travel up the slope before it stops.

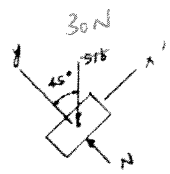
[handwritten: 30 N; 3 m/s]

$(\nearrow +)\quad m(v_{x'})_1 + \Sigma\int_{t_1}^{t_2} F_{x'}\,dt = m(v_{x'})_2$

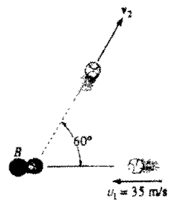
$\frac{5}{32.2}(10) + (-5\sin 45°)t = 0$

$t = 0.439\ \text{s}$ **Ans**

[handwritten: 0.432 s]



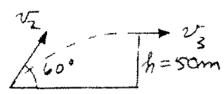
*15-4. The baseball has a horizontal speed of 35 m/s when it is struck by the bat $B$. If it then travels away at an angle of 60° from the horizontal and reaches a maximum height of 50 m, measured from the height of the bat, determine the magnitude of the net impulse of the bat on the ball. The ball has a mass of 400 g. Neglect the weight of the ball during the time the bat strikes the ball.




$(+\uparrow)$ $\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$$0^2 = (v_2 \sin 60°)^2 - 2(9.81)(50 - 0)$$

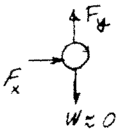
$$v_2 = 36.17 \text{ m/s}$$




$(\overset{+}{\rightarrow})$ $\quad m(v_x)_1 + \Sigma \int F_x \, dt = m(v_x)_2$

$$-0.4(35) + \int F_x \, dt = 0.4(36.17)\cos 60°$$

$$\int F_x \, dt = 21.23$$



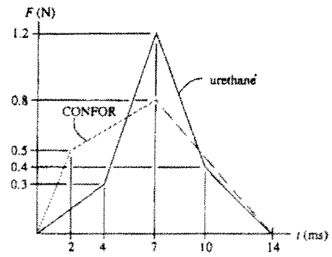

$(+\uparrow)$ $\quad m(v_y)_1 + \Sigma \int F_y \, dt = m(v_y)_2$

$$0 + \int F_y \, dt = 0.4(36.17)\sin 60°$$

$$\int F_y \, dt = 12.53$$

$$\int F \, dt = \sqrt{(21.23)^2 + (12.53)^2} = 24.7 \text{ N} \cdot \text{s} \qquad \textbf{Ans}$$

15-5. The choice of a seating material for moving vehicles depends upon its ability to resist shock and vibration. From the data shown in the graphs, determine the impulses created by a falling weight onto a sample of urethane foam and CONFOR foam.




CONFOR foam :

$$I_C = \int F \, dt = \left[\frac{1}{2}(2)(0.5) + \frac{1}{2}(0.5 + 0.8)(7 - 2) + \frac{1}{2}(0.8)(14 - 7)\right](10^{-3})$$

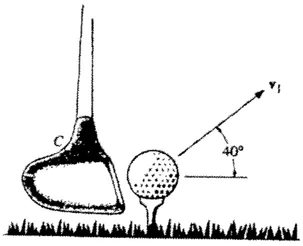
$= 6.55$ N · ms **Ans**

Urethane foam :

$$I_U = \int F \, dt = \left[\frac{1}{2}(4)(0.3) + \frac{1}{2}(1.2 + 0.3)(7 - 4) + \frac{1}{2}(1.2 + 0.4)(10 - 7) + \frac{1}{2}(14 - 10)(0.4)\right](10^{-3})$$

$= 6.05$ N · ms **Ans**

**15-6.** A man hits the 50-g golf ball such that it leaves the tee at an angle of 40° with the horizontal and strikes the ground at the same elevation a distance of 20 m away. Determine the impulse of the club $C$ on the ball. Neglect the impulse caused by the ball's weight while the club is striking the ball.




$(\xrightarrow{+})$ $s_x = (s_0)_x + (v_0)_x t$

$20 = 0 + v\cos 40°(t)$

$(+\uparrow)$ $s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$0 = 0 + v\sin 40°(t) - \frac{1}{2}(9.81)t^2$

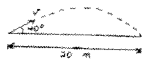
$t = 1.85$ s

$v = 14.115$ m/s

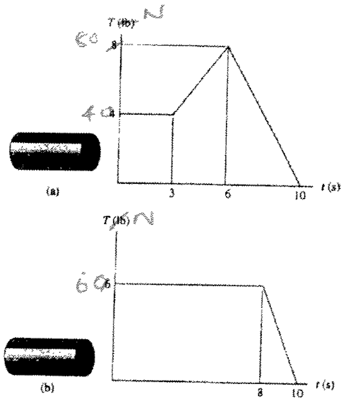
$(+\nearrow)$ $mv_1 + \Sigma\int F\,dt = mv_2$

$0 + \int F\,dt = (0.05)(14.115)$

$\int F\,dt = 0.706$ N·s $\angle 40°$ **Ans**



**15-7.** A solid-fueled rocket can be made using a fuel grain with either a hole (a), or starred cavity (b), in the cross section. From experiment the engine thrust-time curves ($T$ vs. $t$) for the same amount of propellant using these geometries are shown. Determine the total impulse in both cases.




Impulse is area under curve for hole cavity.

$I = \int F\,dt = 4(3) + \frac{1}{2}(8+4)(6-3) + \frac{1}{2}(8)(10-6)$

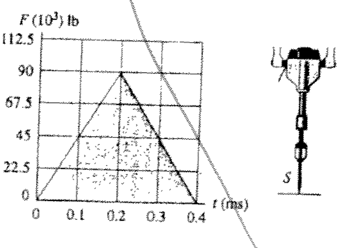
$= 46$ lb·s **Ans**

For starred cavity :

$I = \int F\,dt = 6(8) + \frac{1}{2}(6)(10-8)$

$= 54$ lb·s **Ans**

***15-8.** During operation the breaker hammer develops on the concrete surface a force which is indicated in the graph. To achieve this the 2-lb spike $S$ is fired from rest into the surface at 200 ft/s. Determine the speed of the spike just after rebounding.




$(+\downarrow)$ $mv_1 + \int F\,dt = mv_2$

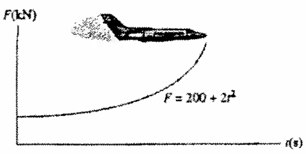
$\frac{2}{32.2}(200) + 2(0.0004) - Area = \frac{-2}{32.2}(v)$

$Area = \frac{1}{2}(90)(10^3)(0.4)(10^{-3}) = 18$ lb·s

Thus,

$v = 89.8$ ft/s **Ans**

**15-9.** The jet plane has a mass of 250 Mg and a horizontal velocity of 100 m/s when $t = 0$. If *both* engines provide a horizontal thrust which varies as shown in the graph, determine the plane's velocity in $t = 15$ s. Neglect air resistance and the loss of fuel during the motion.

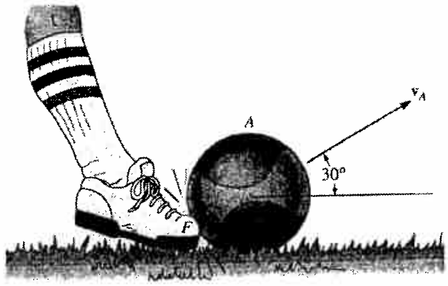



$$(\overset{+}{\rightarrow}) \quad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$250(10^3)(100) + \int_0^{15} 10^3(200 + 2t^2)\,dt = 250(10^3)\,v$$

$v = 121$ m/s **Ans**

**15-10.** A man kicks the 200-g ball such that it leaves the ground at an angle of 30° with the horizontal and strikes the ground at the same elevation a distance of 15 m away. Determine the impulse of his foot $F$ on the ball. Neglect the impulse caused by the ball's weight while it's being kicked.




$$(\overset{+}{\rightarrow}) \quad s_x = (s_0)_x + (v_0)_x t + \frac{1}{2}a_x t^2$$

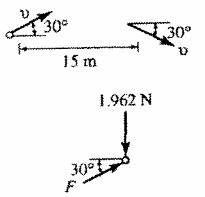
$$15 = 0 + v\cos 30°\, t + 0$$

$$(+\uparrow) \quad v_y = (v_0)_y + a_c t$$

$$-v\sin 30° = v\sin 30° - 9.81t$$

$t = 1.329$ s

$v = 13.04$ m/s

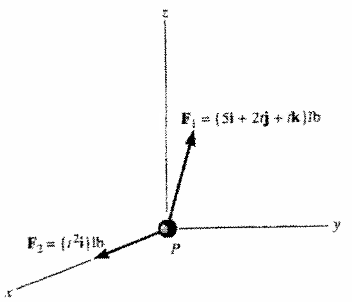



$$(+\nearrow) \quad mv_1 + \Sigma\int F\,dt = mv_2$$

$$0 + \int F\,dt = 0.2(13.04)$$

$I = \int F\,dt = 2.608 = 2.61$ N·s $\angle^{\theta}_{30°}$ **Ans**

**15-11.** The particle $P$ is acted upon by its weight of 3 lb and forces $F_1$ and $F_2$, where $t$ is in seconds. If the particle originally has a velocity of $v_1 = \{3i + 1j + 6k\}$ ft/s, determine its speed after 2 s.




$$mv_1 + \sum\int_0^2 F\,dt = mv_2$$

Resolving into scalar components,

$$\frac{3}{32.2}(3) + \int_0^2 (5 + t^2)\,dt = \frac{3}{32.2}(v_x)$$

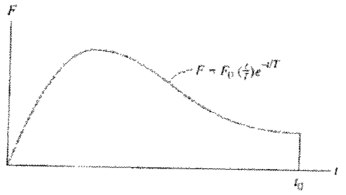
$$\frac{3}{32.2}(1) + \int_0^2 2t\,dt = \frac{3}{32.2}(v_y)$$

$$\frac{3}{32.2}(6) + \int_0^2 (t - 3)\,dt = \frac{3}{32.2}(v_z)$$

$v_x = 138.96$ ft/s $\quad v_y = 43.933$ ft/s $\quad v_z = -36.933$ ft/s

$v = \sqrt{(138.96)^2 + (43.933)^2 + (-36.933)^2} = 150$ ft/s **Ans**

***15-12.** The twitch in a muscle of the arm develops a force which can be measured as a function of time as shown in the graph. If the effective contraction of the muscle lasts for a time $t_0$, determine the impulse developed by the muscle.




$$I = \int F\,dt = \int_0^{t_0} F_0\left(\frac{t}{T}\right)e^{-t/T}\,dt$$

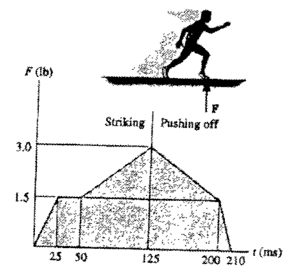
$$I = \frac{F_0}{T}\int_0^{t_0} t e^{-(t/T)}\,dt$$

$$I = -F_0\left[Te^{-t/T}\left(\frac{t}{T}+1\right)\right]_0^{t_0}$$

$$I = -F_0\left[Te^{-t_0/T}\left(\frac{t_0}{T}+1\right)-T\right]$$

$$I = TF_0\left[1-e^{-t_0/T}\left(1+\frac{t_0}{T}\right)\right] \quad \textbf{Ans}$$

**15-13.** From experiments, the time variation of the vertical force on a runner's foot as he strikes and pushes off the ground is shown in the graph. These results are reported for a 1-lb *static* load, i.e., in terms of unit weight. If a runner weighs 175 lb, determine the approximate vertical impulse he exerts on the ground if the impulse occurs in 210 ms.



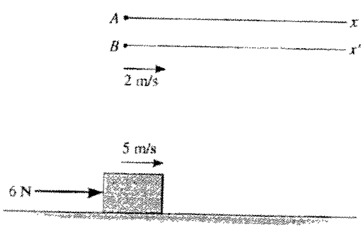

$\int F\,dt = (175 \text{ lb})(\text{area under curve})$

$$\text{Area} = \left[\frac{1}{2}(25)(1.5) + 1.5(50-25) + 1.5(200-50) + \frac{1}{2}(210-200)(1.5) + \frac{1}{2}(3-1.5)(200-50)\right](10^{-3})$$

$$= 401.25(10^{-3})$$

$\int F\,dt = 175(401.25)(10^{-3}) = 70.2 \text{ lb}\cdot\text{s}$ **Ans**

**15-14.** As indicated by the derivation, the principle of impulse and momentum is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which rests on the smooth surface and is subjected to a horizontal force of 6 N. If observer $A$ is in a *fixed* frame $x$, determine the final speed of the block in 4 s if it has an initial speed of 5 m/s measured from the fixed frame. Compare the result with that obtained by an observer $B$, attached to the $x'$ axis that moves at a constant velocity of 2 m/s relative to $A$.

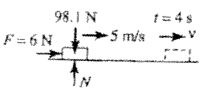



Observer $A$:

$(\xrightarrow{+})\quad mv_1 + \sum\int F\,dt = mv_2$

$10(5) + 6(4) = 10v$

$v = 7.40$ m/s **Ans**



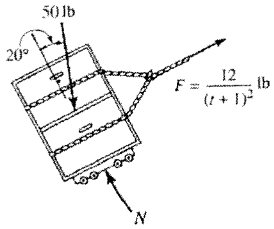

Observer $B$:

$(\xrightarrow{+})\quad mv_1 + \sum\int F\,dt = mv_2$

$10(3) + 6(4) = 10v$

$v = 5.40$ m/s **Ans**

**15-15.** The 50-lb cabinet is subjected to the force $F = 20(t+1)$ lb where $t$ is in seconds. If the cabinet is initially moving down the plane with a velocity of 10 ft/s, determine how long it will take before the cabinet comes to a stop. $F$ always acts parallel to the plane. Neglect the size of the rollers.



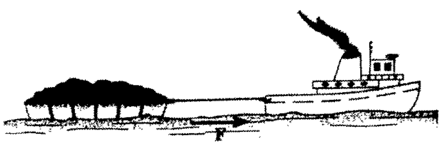

***Principle of Linear Impulse and Momentum:*** Applying Eq. 15-4, we have

$$m(v_{x'})_1 + \sum\int_{t_1}^{t_2} F_{x'}\,dt = m(v_{x'})_2$$

$$\left(\frac{50}{32.2}\right)(10) + \int_0^t 20(t+1)\,dt - 50\sin 20^\circ t = \left(\frac{50}{32.2}\right)(0)$$

$t = 2.95$ s **Ans**

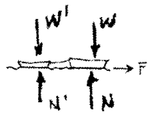
***15-16.** If it takes 35 s for the 50-Mg tugboat to increase its speed uniformly to 25 km/h, starting from rest, determine the force of the rope on the tugboat. The propeller provides the propulsion force **F** which gives the tugboat forward motion, whereas the barge moves freely. Also, determine $F$ acting on the tugboat. The barge has a mass of 75 Mg.




$25\left(\frac{1000}{3600}\right) = 6.944$ m/s

System :

$(\overset{+}{\rightarrow})\quad mv_1 + \Sigma\int F\,dt = mv_2$




$[0+0] + F(35) = (50+75)(10^3)(6.944)$

$F = 24.8$ kN **Ans**

Barge :

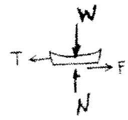
$(\overset{+}{\rightarrow})\quad mv_1 + \Sigma\int F\,dt = mv_2$


$0 + T(35) = (75)(10^3)(6.944)$

$T = 14.881 = 14.9$ kN **Ans**

Also, using this result for $T$,

Tugboat :

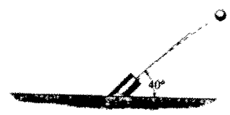



$(\overset{+}{\rightarrow})\quad mv_1 + \Sigma\int F\,dt = mv_2$

$0 + F(35) - (14.881)(35) = (50)(10^3)(6.944)$

$F = 24.8$ kN **Ans**

**15-17.** When the 0.4-lb ball is fired, it leaves the ground at an angle of 40° from the horizontal and strikes the ground at the same elevation a distance of 130 ft away. Determine the impulse given to the ball.




$(\overset{+}{\rightarrow})$ $s = v_0 t$

$130 = v_0 \cos 40°(t)$

$(+\uparrow)$ $v = v_0 - at$

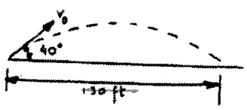
$-v_0 \sin 40° = v_0 \sin 40° - 32.2t$

Solving,

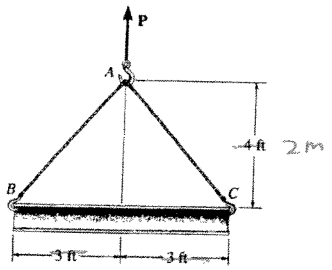
$v_0 = 65.20$ ft/s, $t = 2.60$ s

$m\mathbf{v}_1 + \Sigma \int \mathbf{F}\, dt = m\mathbf{v}_2$

$0 + \Sigma \int F\, dt = \left(\frac{0.4}{32.2}\right)(65.20) = 0.810$ lb · s **Ans**




**15-18.** The uniform beam has a weight of 5000 lb. Determine the average tension in each of the two cables $AB$ and $AC$ if the beam is given an upward speed of 8 ft/s in 1.5 s starting from rest. Neglect the mass of the cables.



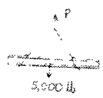

$(+\uparrow)$ $mv_1 + \Sigma \int F\, dt = mv_2$

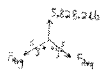
$0 + P_{avg}(1.5) - 5000(1.5) = \frac{5000}{32.2}(8)$

$P_{avg} = 5828.157 = 5828.2$ lb

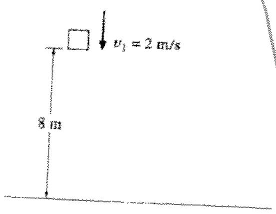
$+\uparrow \Sigma F_y = 0;$ $5828.157 - 2\left(\frac{4}{5}\right)F_{avg} = 0$

$F_{avg} = 3642.598$ lb $= 3.64$ kip **Ans**




**15-19.** The 5-kg block is moving downward at $v_1 = 2$ m/s when it is 8 m from the sandy surface. Determine the impulse of the sand on the block necessary to stop its motion. Neglect the distance the block dents into the sand and assume the block does not rebound. Neglect the weight of the block during the impact with the sand.




Just before impact

$T_1 + \Sigma U_{1-2} = T_2$

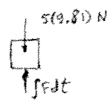
$\frac{1}{2}(5)(2)^2 + 8(5)(9.81) = \frac{1}{2}(5)(v^2)$

$v = 12.687$ m/s

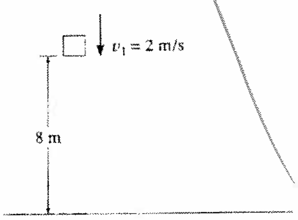
$(+\downarrow)$ $mv_1 + \Sigma \int F\, dt = mv_2$

$5(12.687) - \int F\, dt = 0$

$I = \int F\, dt = 63.4$ N · s **Ans**



**•15-20.** The 5-kg block is falling downward at $v_1 = 2$ m/s when it is 8 m from the sandy surface. Determine the average impulsive force acting on the block by the sand if the motion of the block is stopped in 0.9 s once the block strikes the sand. Neglect the distance the block dents into the sand and assume the block does not rebound. Neglect the weight of the block during the impact with the sand.




Just before impact

$$T_1 + \Sigma U_{1-2} = T_2$$

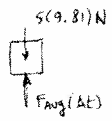
$$\frac{1}{2}(5)(2)^2 + 8(5)(9.81) = \frac{1}{2}(5)(v^2)$$

$$v = 12.69 \text{ m/s}$$

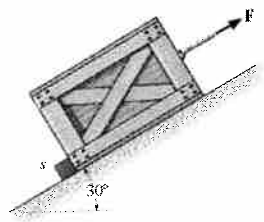
$$(+\downarrow) \quad mv_1 + \Sigma\int F\,dt = mv_2$$

$$5(12.69) - F_{avg}(0.9) = 0$$

$$F_{avg} = 70.5 \text{ N} \qquad \textbf{Ans}$$




**15-21.** A 50-kg crate rests against a stop block *s*, which prevents the crate from moving down the plane. If the coefficients of static and kinetic friction between the plane and the crate are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively, determine the time needed for the force **F** to give the crate a speed of 2 m/s up the plane. The force always acts parallel to the plane and has a magnitude of $F = (300t)$ N, where $t$ is in seconds. *Hint:* First determine the time needed to overcome static friction and start the crate moving.



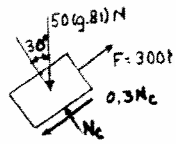

The time needed to overcome friction is determined from statics :

$$\nwarrow\!+ \Sigma F_y = 0; \quad -50(9.81)\cos 30° + N_C = 0$$

$$N_C = 424.79 \text{ N}$$

$$+\nearrow \Sigma F_x = 0; \quad 300t - 0.3(424.79) - 50(9.81)\sin 30° = 0$$

$$t = 1.242 \text{ s}$$




$$(+\nearrow) \quad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$$

$$0 + \int_{1.242}^{t} 300t\,dt - 0.2(424.79)(t - 1.242) - 50(9.81)\sin 30°(t - 1.242) = 50(2)$$

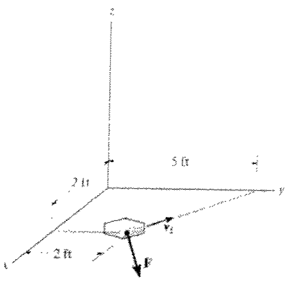
$$150\left[t^2 - (1.242)^2\right] - 84.957t + 105.541 - 245.25t + 304.67 = 100$$

$$150t^2 - 330.21t + 78.720 = 0$$

Solving for the positive root $t > 1.242$ s, yields

$$t = 1.93 \text{ s} \qquad \textbf{Ans}$$

**15-22.** The 2-lb block has an initial velocity of $v_1 = 10$ ft/s in the direction shown. If a force of $\mathbf{F} = \{0.5\mathbf{i} + 0.2\mathbf{j}\}$ lb acts on the block for $t = 5$ s, determine the final speed of the block. Neglect friction.

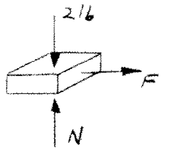



$$v_1 = 10\left(-\frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}\right) = \{-5.547\mathbf{i} + 8.321\mathbf{j}\}\ \text{ft/s}$$

$$m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$$

$$\left(\frac{2}{32.2}\right)(-5.547\mathbf{i} + 8.321\mathbf{j}) + (0.5\mathbf{i} + 0.2\mathbf{j})5 - 2(5)\mathbf{k} + 2(5)\mathbf{k} = \left(\frac{2}{32.2}\right)(v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k})$$

Expand and equate components:




$$-0.3445 + 2.5 = \frac{2}{32.2}v_x$$

$$v_x = 34.70\ \text{ft/s}$$

$$0.5168 + 1 = \frac{2}{32.2}v_y$$

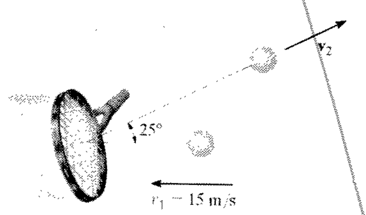
$$v_y = 24.42\ \text{ft/s}$$

$$v_z = 0$$

Thus,

$$v = \sqrt{(34.70)^2 + (24.42)^2 + (0)^2} = 42.4\ \text{ft/s} \qquad \textbf{Ans}$$

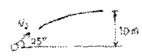
**15-23.** The tennis ball has a horizontal speed of 15 m/s when it is struck by the racket. If it then travels away at an angle of 25° from the horizontal and reaches a maximum altitude of 10 m, measured from the height of the racket, determine the magnitude of the net impulse of the racket on the ball. The ball has a mass of 180 g. Neglect the weight of the ball during the time the racket strikes the ball.




$(+\uparrow)$ $$v_y^2 = (v_0)_y^2 + 2a_c(s_y - (s_0)_y)$$

$$(v_2 \sin 25°)^2 = 0 + 2(9.81)(10 - 0)$$

$$v_2 = 33.14\ \text{m/s}$$




$(\overset{+}{\rightarrow})$ $$m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$$

$$-0.180(15) + \int F_x\,dt = 0.180(33.14\cos 25°)$$

$$\int F_x\,dt = 8.107\ \text{N}\cdot\text{s}$$

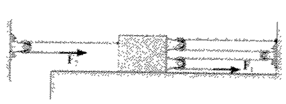
$(+\uparrow)$ $$m(v_y)_1 + \Sigma\int F_y\,dt = m(v_y)_2$$

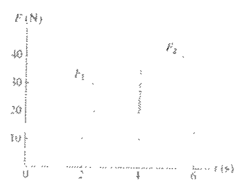
$$0 + \int F_y\,dt = 0.180(33.14\sin 25°)$$

$$\int F_y\,dt = 2.521\ \text{N}\cdot\text{s}$$

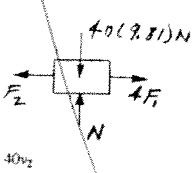
$$I = \int F\,dt = \sqrt{(8.107)^2 + (2.521)^2} = 8.49\ \text{N}\cdot\text{s} \;\angle_{17.3°} \qquad \textbf{Ans}$$

***15-24.** The 40-kg slider block is moving to the right with a speed of 1.5 m/s when it is acted upon by the forces $\mathbf{F}_1$ and $\mathbf{F}_2$. If these loadings vary in the manner shown on the graph, determine the speed of the block at $t = 6$ s. Neglect friction and the mass of the pulleys and cords.




The impulses acting on the block are equal to the areas under the graph.

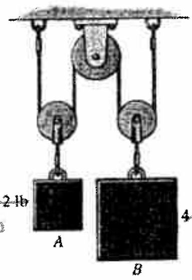



$(\overset{+}{\rightarrow})$ $$m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$$

$$40(1.5) + 4[(30)4 + 10(6-4)] - [10(2) + 20(4-2) + 40(6-4)] = 40v_2$$

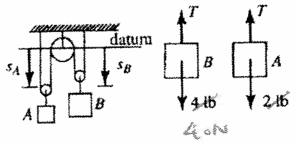
$$v_2 = 12.0\ \text{m/s}(\rightarrow) \qquad \textbf{Ans}$$

**15-25.** Determine the velocities of blocks $A$ and $B$ 2 s after they are released from rest. Neglect the mass of the pulleys and cables.




$2s_A + 2s_B = l$

$v_A = -v_B$




Block $A$:

$(+\downarrow) \quad m(v_y)_1 + \sum \int F_y dt = m(v_y)_2$

$0 - T(2) + 2(2) = \left(\frac{2}{32.2}\right) v_A$

Block $B$:

$(+\downarrow) \quad m(v_y)_1 + \sum \int F_y dt = m(v_y)_2$

$0 + 4(2) - T(2) = \left(\frac{4}{32.2}\right) v_B$

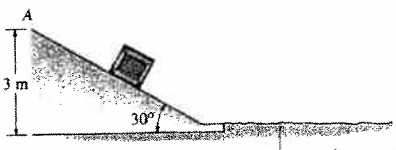
Solving,

$T = 2.67$ lb

$v_B = 21.5$ ft/s $\downarrow$ **Ans**

$v_A = -21.5$ ft/s $= 21.5$ ft/s $\uparrow$ **Ans**

**15-26.** The 5-kg package is released from rest at $A$. It slides down the smooth plane onto the rough surface having a coefficient of kinetic friction of $\mu_k = 0.2$. Determine the total time of travel before the package stops sliding. Neglect the size of the package.




***Potential Energy:*** The datum is set at point $A$. When the package reaches the toe of the inclined plane, its position is 3 m *below* the datum. Its gravitational potential energy is $5(9.81)(-3) = -147.15$ N · m.

***Conservation of Energy:*** Applying Eq. 14-21, we have

$T_1 + V_1 = T_2 + V_2$

$0 + 0 = \frac{1}{2}(5)v^2 + (-147.15)$

$v = 7.672$ m/s

***Principle of Linear Impulse and Momentum:*** The time taken for the package to reach the toe of the inclined plane can be obtained by applying Eq. 15-4.

$m(v_{x'})_1 + \sum \int_{t_1}^{t_2} F_{x'} dt = m(v_{x'})_2$

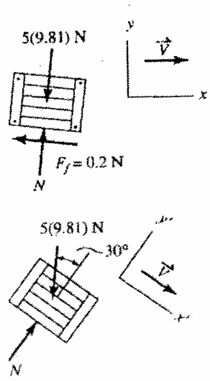
$(\searrow +) \quad 5(0) + 5(9.81)\sin 30° t_1 = 5(7.672)$

$t_1 = 1.564$ s

The time taken for the package to stop when it moves along the horizontal rough surface can be obtained by applying Eq. 15-4.

$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y dt = m(v_y)_2$

$(+\uparrow) \quad 5(0) + N(t_2) - 5(9.81)t_2 = 5(0)$




$N = 49.05$ N

$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x dt = m(v_x)_2$

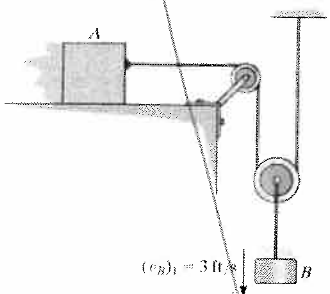
$(\overset{+}{\rightarrow}) \quad 5(7.672) + [-0.2(49.05)t_2] = 5(0)$

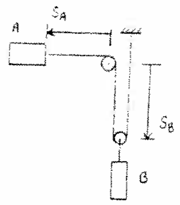
$t = 3.910$ s

Thus, the total package's traveling time is

$t = t_1 + t_2 = 1.564 + 3.910 = 5.47$ s **Ans**

**15–27.** Block $A$ weighs 10 lb and block $B$ weighs 3 lb. If $B$ is moving downward with a velocity $(v_B)_1 = 3$ ft/s at $t = 0$, determine the velocity of $A$ when $t = 1$ s. Assume that the horizontal plane is smooth. Neglect the mass of the pulleys and cords.




$$s_A + 2s_B = l$$

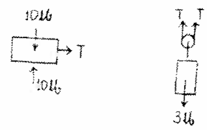
$$v_A = -2v_B$$

$$(\overset{+}{\leftarrow}) \qquad mv_1 + \Sigma\int F\,dt = mv_2$$

$$-\frac{10}{32.2}(2)(3) - T(1) = \frac{10}{32.2}(v_A)_2$$

$$(+\downarrow) \qquad mv_1 + \Sigma\int F\,dt = mv_2$$

$$\frac{3}{32.2}(3) + 3(1) - 2T(1) = \frac{3}{32.2}(-\frac{(v_A)_2}{2})$$



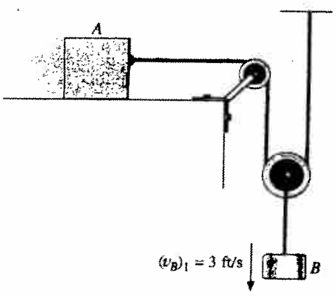

$$-32.2T - 10(v_A)_2 = 60$$

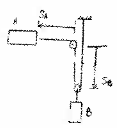
$$-64.4T + 1.5(v_A)_2 = -105.6$$

$$T = 1.40 \text{ lb}$$

$$(v_A)_2 = -10.5 \text{ ft/s} = 10.5 \text{ ft/s} \rightarrow \qquad \textbf{Ans}$$

***15–28.** Block $A$ weighs 10 lb and block $B$ weighs 3 lb. If $B$ is moving downward with a velocity $(\mu_B)_1 = 3$ ft/s at $t = 0$, determine the velocity of $A$ when $t = 1$ s. The coefficient of kinetic friction between the horizontal plane and block $A$ is $\mu_A = 0.15$.



$$s_A + 2s_B = l$$

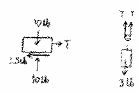
$$v_A = -2v_B$$

$$(\overset{+}{\leftarrow}) \qquad mv_1 + \Sigma\int F\,dt = mv_2$$

$$-\frac{10}{32.2}(2)(3) - T(1) + 0.15(10) = \frac{10}{32.2}(v_A)_2$$

$$(+\downarrow) \qquad mv_1 + \Sigma\int F\,dt = mv_2$$

$$\frac{3}{32.2}(3) + 3(1) - 2T(1) = \frac{3}{32.2}(-\frac{(v_A)_2}{2})$$



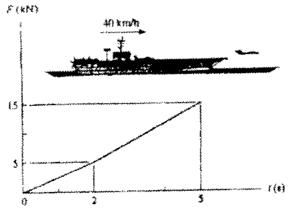
$$-32.2T - 10(v_A)_2 = 11.70$$

$$-64.4T + 1.5(v_A)_2 = -105.6$$

$$T = 1.50 \text{ lb}$$

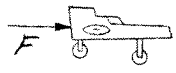
$$(v_A)_2 = -6.00 \text{ ft/s} = 6.00 \text{ ft/s} \rightarrow \qquad \textbf{Ans}$$

**15-29.** A jet plane having a mass of 7 Mg takes off from an aircraft carrier such that the engine thrust varies as shown by the graph. If the carrier is traveling forward with a speed of 40 km/h, determine the plane's airspeed after 5 s.




The impulse exerted on the plane is equal to the area under the graph.

$v_1 = 40\text{ km/h} = 11.11\text{ m/s}$

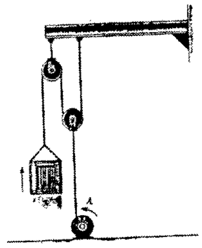



$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \Sigma\int F_x\, dt = m(v_x)_2$$

$$(7)(10^3)(11.11) - \frac{1}{2}(2)(5)(10^3) + \frac{1}{2}(15+5)(5-2)(10^3) = 7(10^3)v_2$$

$v_2 = 16.1\text{ m/s}$ **Ans**

**15-30.** The motor pulls on the cable at $A$ with a force $F = (30 + t^2)$ lb, where $t$ is in seconds. If the 17-lb crate is originally at rest at $t = 0$, determine its speed in $t = 4$ s. Neglect the mass of the cable and pulleys. *Hint:* First find the time needed to begin lifting the crate.



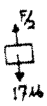

Time to begin lifting crate: $\frac{1}{2}(30 + t^2) = 17$


$t = 2$ s

$$(+\uparrow) \qquad m(v_y)_1 + \Sigma\int F_y\, dt = m(v_y)_2$$

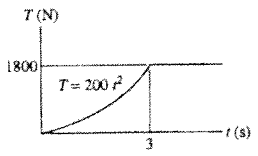
$$0 + \frac{1}{2}\int_2^4 (30 + t^2)\, dt - 17(2) = \left(\frac{17}{32.2}\right)v_2$$

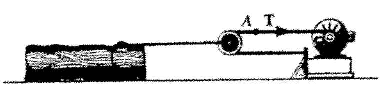
$$\frac{1}{2}\left[30t + \frac{1}{3}t^3\right]_2^4 - 34 = \left(\frac{17}{32.2}\right)v_2$$



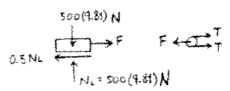

$v_2 = 10.1\text{ ft/s}$ **Ans**

**15-31.** The log has a mass of 500 kg and rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. The winch delivers a horizontal towing force $T$ to its cable at $A$ which varies as shown in the graph. Determine the speed of the log when $t = 5$ s. Originally the tension in the cable is zero. *Hint:* First determine the force needed to begin moving the log.




$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad F - 0.5(500)(9.81) = 0$



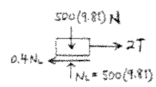

$F = 2452.5$ N

Thus,

$2T = F$

$2(200t^2) = 2452.5$

$t = 2.476$ s to start log moving




$$(\overset{+}{\rightarrow}) \qquad m v_1 + \Sigma\int F\, dt = m v_2$$

$$0 + 2\int_{2.476}^{3} 200t^2\, dt + 2(1800)(5-3) - 0.4(500)(9.81)(5-2.476) = 500v_2$$

$$400\left(\frac{t^3}{3}\right)\Big|_{2.476}^{3} + 2247.91 = 500v_2$$

$v_2 = 7.65\text{ m/s}$ **Ans**

***15-32.** A railroad car having a mass of 15 Mg is coasting at 1.5 m/s on a horizontal track. At the same time another car having a mass of 12 Mg is coasting at 0.75 m/s in the opposite direction. If the cars meet and couple together, determine the speed of both cars just after the coupling. Find the difference between the total kinetic energy before and after coupling has occurred, and explain qualitatively what happened to this energy.

$(\overset{+}{\rightarrow})\quad \Sigma mv_1 = \Sigma mv_2$

$$15\,000(1.5) - 12\,000(0.75) = 27\,000(v_2)$$

$$v_2 = 0.5 \text{ m/s} \qquad \textbf{Ans}$$

$$T_1 = \frac{1}{2}(15\,000)(1.5)^2 + \frac{1}{2}(12\,000)(0.75)^2 = 20.25 \text{ kJ}$$

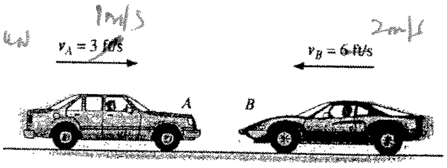
$$T_2 = \frac{1}{2}(27\,000)(0.5)^2 = 3.375 \text{ kJ}$$

$$\Delta T = T_1 - T_2$$

$$= 20.25 - 3.375 = 16.9 \text{ kJ} \qquad \textbf{Ans}$$

This energy is dissipated as noise, shock, and heat during the coupling.

**15-33.** The car $A$ has a weight of 4500 lb and is traveling to the right at 3 ft/s. Meanwhile a 3000-lb car $B$ is traveling at 6 ft/s to the left. If the cars crash head-on and become entangled, determine their common velocity just after the collision. Assume that the brakes are not applied during collision.

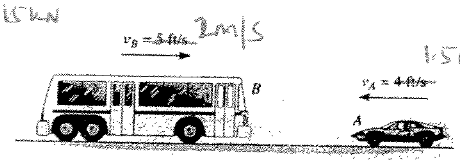



$(\overset{+}{\rightarrow})\quad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$

$$\frac{4500}{32.2}(3) - \frac{3000}{32.2}(6) = \frac{7500}{32.2}v_2$$

$$v_2 = -0.600 \text{ ft/s} = 0.600 \text{ ft/s} \leftarrow \qquad \textbf{Ans}$$

**15-34.** The bus $B$ has a weight of 15 000 lb and is traveling to the right at 5 ft/s. Meanwhile a 3000-lb car $A$ is traveling at 4 ft/s to the left. If the vehicles crash head-on and become entangled, determine their common velocity just after the collision. Assume that the vehicles are free to roll during collision.

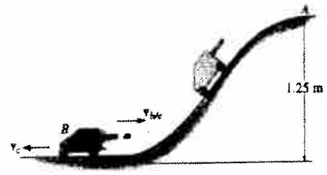



$(\overset{+}{\rightarrow})\quad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v$

$$\frac{15\,000}{32.2}(5) - \frac{3000}{32.2}(4) = \frac{18\,000}{32.2}v$$

$$v = 3.5 \text{ ft/s} \rightarrow \qquad \textbf{Ans}$$

**15-35.** The cart has a mass of 3 kg and rolls freely down the slope. When it reaches the bottom, a spring loaded gun fires a 0.5-kg ball out the back with a horizontal velocity of $v_{b/c} = 0.6$ m/s, measured relative to the cart. Determine the final velocity of the cart.




Datum at $B$ :

$$T_A + V_A = T_B + V_B$$

$$0 + (3 + 0.5)(9.81)(1.25) = \frac{1}{2}(3 + 0.5)(v_B)_2^2 + 0$$

$$v_B = 4.952 \text{ m/s}$$

$(\overset{+}{\leftarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$(3 + 0.5)(4.952) = (3)v_c - (0.5)v_b \qquad (1)$$

$(\overset{+}{\leftarrow})$ $\quad v_b = v_c + v_{b/c}$

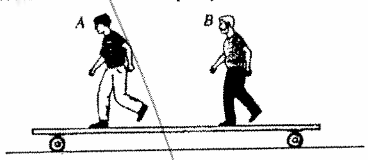
$$-v_b = v_c - 0.6 \qquad (2)$$

Solving Eqs. (1) and (2),

$v_c = 5.04$ m/s $\leftarrow$ **Ans**

$v_b = -4.44$ m/s $= 4.44$ m/s $\leftarrow$

***15-36.** Two men $A$ and $B$, each having a weight of 160 lb, stand on the 200-lb, cart. Each runs with a speed of 3 ft/s measured relative to the cart. Determine the final speed of the cart if (a) $A$ runs and jumps off, then $B$ runs and jumps off the same end, and (b) both run at the same time and jump off at the same time. Neglect the mass of the wheels and assume the jumps are made horizontally.




(a) $A$ jumps first.

$(\overset{+}{\leftarrow})\ 0 + 0 = m_A v_A - (m_C + m_B) v'_C$    However, $v_A = -v'_C + 3$

$$0 = \frac{160}{32.2}(-v'_C + 3) - \frac{360}{32.2} v'_C$$

$$v'_C = 0.9231 \text{ ft/s} \rightarrow$$

And then $B$ jumps

$0 + (m_C + m_B) v'_C = m_B v_B - m_C v_C$    However, $v_B = -v_C + 3$

$$\frac{360}{32.2}(-0.9231) = \frac{160}{32.2}(-v_C + 3) - \frac{200}{32.2} v_C$$

$v_C = 2.26$ ft/s $\rightarrow$ **Ans**

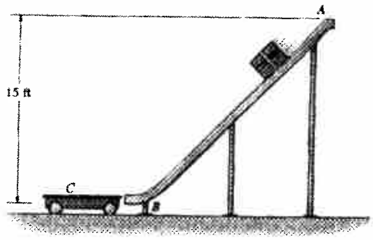
(b) Both men jump at the same time

$(\overset{+}{\leftarrow})\ 0 + 0 = (m_A + m_B) v - m_C v_C$    However, $v = -v_C + 3$

$$0 = \left(\frac{160}{32.2} + \frac{160}{32.2}\right)(-v_C + 3) - \frac{200}{32.2} v_C$$

$v_C = 1.85$ ft/s $\rightarrow$ **Ans**

**15-37.** A 40-lb box slides from rest down the smooth ramp onto the surface of a 20-lb cart. Determine the speed of the box at the instant it stops sliding on the cart. If someone ties the cart to the ramp at *B*, determine the horizontal impulse the box will exert at *C* in order to stop its motion. Neglect friction and the size of the box.




Datum at $B$ :

$$T_A + V_A = T_B + V_B$$

$$0 + 40(15) = \frac{1}{2}\left(\frac{40}{32.2}\right)(v_B)^2 + 0$$

$$v_B = 31.08 \text{ ft/s}$$

Box and cart :

$$(\overset{+}{\leftarrow}) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$0 + \left(\frac{40}{32.2}\right)(31.08) = \left(\frac{40+20}{32.2}\right)v_2$$

$v_2 = 20.7$ ft/s **Ans**

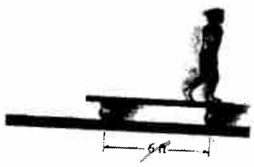
Box :

$$(\overset{+}{\leftarrow}) \qquad m v_1 + \Sigma\int F\,dt = m v_2$$

$$\left(\frac{40}{32.2}\right)(31.08) - \int F\,dt = 0$$

$\int F\,dt = 38.6$ lb · s **Ans**


**15-38.** A 100-lb boy walks forward over the surface of the 60-lb cart with a constant speed of 3 ft/s relative to the cart. Determine the cart's speed and its displacement at the moment he is about to step off. Neglect the mass of the wheels and assume the cart and boy are originally at rest.




$$(\overset{+}{\rightarrow}) \qquad 0 = m_B v_B + m_C v_C, \quad \text{however, } v_B = v_C + v_{B/C} = v_C + 3$$

$$0 = \frac{100}{32.2}(v_C + 3) + \frac{60}{32.2}v_C$$

$v_C = -$ ~~1.875 ft/s~~ $=$ ~~1.88 ft/s~~ $\leftarrow$ **Ans**

$-0.625$ m/s $\quad 0.625$ m/s $\leftarrow$

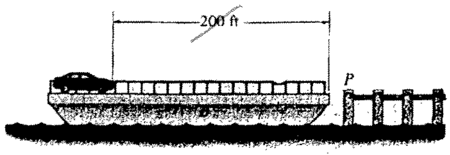
$$s_{B/C} = v_{B/C}\,t$$

$$6 = 3t \quad 2 = 1 \cdot t$$

$$t = 2 \text{ s}$$

$s_C = v_C t =$ ~~1.875~~$(2) =$ ~~3.75 ft~~ **Ans** 0.625 1.25 m

**15-39.** The barge $B$ weighs 30 000 lb and supports an automobile weighing 3000 lb. If the barge is not tied to the pier $P$ and someone drives the automobile to the other side of the barge for unloading, determine how far the barge moves away from the pier. Neglect the resistance of the water.




*Relative Velocity* : The relative velocity of the car with respect to the barge is $v_{c/b}$. Thus, the velocity of the car is

$$(\overset{+}{\rightarrow}) \qquad v_c = -v_b + v_{c/b} \qquad [1]$$

*Conservation of Linear Momentum* : If we consider the car and the barge as a system, then the impulsive force caused by the traction of the tires is *internal* to the system. Therefore, it will cancel out. As the result, the linear momentum is conserved along the $x$ axis.

$$0 = m_c v_c + m_b v_b$$

$$(\overset{+}{\rightarrow}) \qquad 0 + 0 = \left(\frac{3000}{32.2}\right)v_c + \left(\frac{30\,000}{32.2}\right)v_b \qquad [2]$$

Substituting Eq.[1] into [2] yields

$$11v_b - v_{c/b} = 0 \qquad [3]$$

Integrating Eq.[3] becomes

$$(\overset{+}{\rightarrow}) \qquad 11s_b - s_{c/b} = 0 \qquad [4]$$

Here, $s_{c/b} = 200$ ft. Then, from Eq.[4]

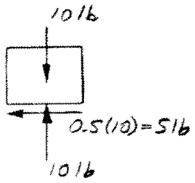
$$11s_b - 200 = 0 \qquad s_b = 18.2\text{ ft} \qquad \textbf{Ans}$$

***15-40.** A 0.03-lb bullet traveling at 1300 ft/s strikes the 10-lb wooden block and exits the other side at 50 ft/s as shown. Determine the speed of the block just after the bullet exits the block, and also determine how far the block slides before it stops. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.5$.


$$(\overset{+}{\rightarrow}) \qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$$

$$\left(\frac{0.03}{32.2}\right)(1300)\left(\frac{12}{13}\right) + 0 = \left(\frac{10}{32.2}\right)v_B + \left(\frac{0.03}{32.2}\right)(50)\left(\frac{4}{5}\right)$$

$$v_B = 3.48\text{ ft/s} \qquad \textbf{Ans}$$

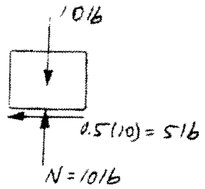



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(3.48)^2 - 5(d) = 0$$

$$d = 0.376\text{ ft} \qquad \textbf{Ans}$$

**15-41.** A 0.03-lb bullet traveling at 1300 ft/s strikes the 10-lb wooden block and exits the other side at 50 ft/s as shown. Determine the speed of the block just after the bullet exits the block. Also, determine the average normal force on the block if the bullet passes through it in 1 ms, and the time the block slides before it stops. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.5$.


$(\overset{+}{\rightarrow})$ $\Sigma m_1 v_1 = \Sigma m_2 v_2$

$$\left(\frac{0.03}{32.2}\right)(1300)\left(\frac{12}{13}\right) + 0 = \left(\frac{10}{32.2}\right)v_B + \left(\frac{0.03}{32.2}\right)(50)\left(\frac{4}{5}\right)$$

$v_B = 3.48$ ft/s **Ans**

$(+\uparrow)$ $mv_1 + \Sigma\int F\,dt = mv_2$

$$-\left(\frac{0.03}{32.2}\right)(1300)\left(\frac{5}{13}\right) - 10(1)(10^{-3}) + N(1)(10^{-3}) = \left(\frac{0.03}{32.2}\right)(50)\left(\frac{3}{5}\right)$$

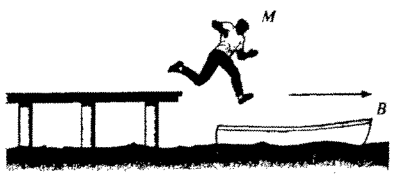
$N = 504$ lb **Ans**

$(\overset{+}{\rightarrow})$ $mv_1 + \Sigma\int F\,dt = mv_2$

$$\left(\frac{10}{32.2}\right)(3.48) - 5(t) = 0$$

$t = 0.216$ s **Ans**

**15-42.** The man $M$ weighs 150 lb and jumps onto the boat $B$ which has a weight of 200 lb. If he has a horizontal component of velocity *relative to the boat* of 3 ft/s, just before he enters the boat, and the boat is traveling $v_B = 2$ ft/s away from the pier when he makes the jump, determine the resulting velocity of the man and boat.




$(\overset{+}{\rightarrow})$ $v_M = v_B + v_{M/B}$

$v_M = 2 + 3$

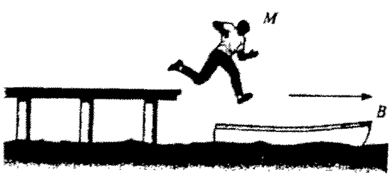
$v_M = 5$ ft/s

$(\overset{+}{\rightarrow})$ $\Sigma m\,v_1 = \Sigma m\,v_2$

$$\frac{150}{32.2}(5) + \frac{200}{32.2}(2) = \frac{350}{32.2}(v_B)_2$$

$(v_B)_2 = 3.29$ ft/s **Ans**

**15-43.** The man $M$ weighs 150 lb and jumps onto the boat $B$ which is originally at rest. If he has a horizontal component of velocity of 3 ft/s just before he enters the boat, determine the weight of the boat if it has a velocity of 2 ft/s once the man enters it.




$(\overset{+}{\rightarrow})$ $v_M = v_B + v_{M/B}$

$v_M = 0 + 3$

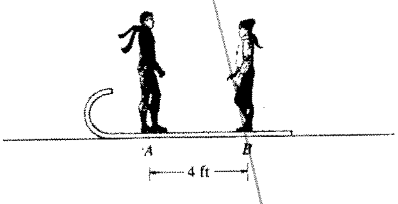
$v_M = 3$ ft/s

$(\overset{+}{\rightarrow})$ $\Sigma m(v_1) = \Sigma m(v_2)$

$$\frac{150}{32.2}(3) + \frac{W_B}{32.2}(0) = \frac{(W_B + 150)}{32.2}(2)$$

$W_B = 75$ lb **Ans**

***15-44.** A boy $A$ having a weight of 80 lb and a girl $B$ having a weight of 65 lb stand motionless at the ends of the toboggan, which has a weight of 20 lb. If $A$ walks to $B$ and stops, and both walk back together to the original position of $A$, determine the final position of the toboggan just after the motion stops. Neglect friction.




$A$ goes to $B$,

$(\overset{+}{\rightarrow})$ $\Sigma m v_1 = \Sigma m v_2$

$$0 = m_A v_A - (m_t + m_B) v_B$$

$$0 = m_A s_A - (m_t + m_B) s_B$$

Assume $B$ moves $x$ to the left, then $A$ moves $(4 - x)$ to the right

$$0 = m_A (4 - x) - (m_t + m_B) x$$

$$x = \frac{4m_A}{m_A + m_B + m_t}$$

$$= \frac{4(80)}{80 + 65 + 20} = 1.939 \text{ ft} \leftarrow$$

$A$ and $B$ go to other end.

$(\overset{+}{\rightarrow})$ $\Sigma m v_1 = \Sigma m v_2$

$$0 = -m_B v - m_A v + m_t v_t$$

$$0 = -m_B s - m_A s + m_t s_t$$

Assume the toboggan moves $x'$ to the right, then $A$ and $B$ move $(4 - x')$ to the left

$$0 = -m_B (4 - x') - m_A (4 - x') + m_t x'$$

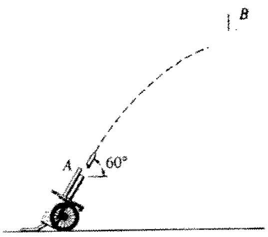
$$x' = \frac{4(m_B + m_A)}{m_A + m_B + m_t}$$

$$= \frac{4(65 + 80)}{80 + 65 + 20} = 3.515 \text{ ft} \rightarrow$$

Net movement of sled is

$(\overset{+}{\rightarrow})$ $x = 3.515 - 1.939 = 1.58 \text{ ft} \rightarrow$ **Ans**

**15-45.** The 10-lb projectile is fired from ground level with an initial velocity of $v_A = 80$ ft/s in the direction shown. When it reaches its highest point $B$ it explodes into two 5-lb fragments. If one fragment travels vertically upward at 12 ft/s, determine the distance between the fragments after they strike the ground. Neglect the size of the gun.




$(\overset{+}{\rightarrow})$ $v_x = (v_0)_x$

$$v_B = 80 \cos 60° = 40 \text{ ft/s}$$

$(+\uparrow)$ $v_y^2 = (v_0)_y^2 + 2a_c(s_y - (s_0)_y)$

$$0 = (80 \sin 60°)^2 + 2(-32.2)(h - 0)$$

$$h = 74.53 \text{ ft}$$

$(\overset{+}{\rightarrow})$ $\Sigma m v_1 = \Sigma m v_2$

$$\frac{10}{32.2}(80 \cos 60°) = \frac{5}{32.2}(v_F)_x$$

$$(v_F)_x = 80 \text{ ft/s} \rightarrow$$

$(+\uparrow)$ $\Sigma m v_1 = \Sigma m v_2$

$$0 = \frac{5}{32.2}(v_F)_y + \frac{5}{32.2}(12)$$

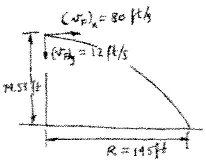
$$(v_F)_y = -12 \text{ ft/s} = 12 \text{ ft/s} \downarrow$$

$(+\downarrow)$ $s_y = (s_0)_y + (v_0)_y t + \frac{1}{2} a_c t^2$

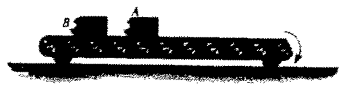
$$74.53 = 0 + 12t + \frac{1}{2}(32.2)t^2$$

$$t = 1.81 \text{ s}$$

$(\overset{+}{\rightarrow})$ $R = 80(1.81) = 145 \text{ ft}$ **Ans**



**15-46.** Two boxes $A$ and $B$, each having a weight of 160 lb, sit on the 500-lb conveyor which is free to roll on the ground. If the belt starts from rest and begins to run with a speed of 3 ft/s, determine the final speed of the conveyor if (a) the boxes are not stacked and $A$ falls off then $B$ falls off, and (b) $A$ is stacked on top of $B$ and both fall off together.




a) Let $v_b$ be the velocity of $A$ and $B$.

$(\overset{+}{\rightarrow})$ $\Sigma m v_1 = \Sigma m v_2$

$$0 = \left(\frac{320}{32.2}\right)(v_b) - \left(\frac{500}{32.2}\right)(v_c)$$

$(\overset{+}{\rightarrow})$ $v_b = v_c + v_{b/c}$

$$v_b = -v_c + 3$$

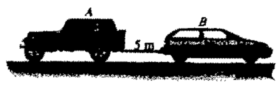
Thus, $v_b = 1.83$ ft/s $\rightarrow$ $v_c = 1.17$ ft/s $\leftarrow$

When a box falls off, it exerts no impulse on the conveyor, and so does not alter the momentum of the conveyor. Thus,

a) $v_c = 1.17$ ft/s $\leftarrow$ **Ans**

b) $v_c = 1.17$ ft/s $\leftarrow$ **Ans**

**15-47.** The winch on the back of the jeep $A$ is turned on and pulls in the tow rope at $v_{rel} = 2$ m/s. If both the 1.25-Mg car $B$ and the 2.5-Mg jeep $A$ are free to roll, determine their velocities at the instant they meet. If the rope is 5 m long, how long will this take?




$(\overset{+}{\rightarrow})$ $0 + 0 = m_A v_A - m_B v_B$ (1)

$$0 = 2.5(10^3)\, v_A - 1.25(10^3)\, v_B$$

However, $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$(\overset{+}{\rightarrow})$ $v_A = -v_B + 2$ (2)

Substituting Eq. (2) into (1) yields:

$v_B = 1.33$ m/s **Ans** $v_A = 0.667$ m/s **Ans**

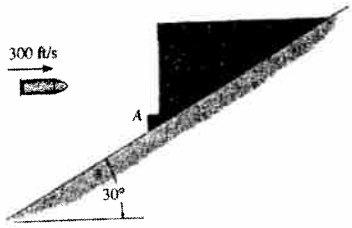
Kinematics:

$(\overset{+}{\rightarrow})$ $s_{A/B} = v_{A/B}\, t$

$$5 = 2t$$

$t = 2.5$ s **Ans**

***15-48.** The 10-kg block is held at rest on the smooth inclined plane by the stop block at $A$. If the 10-g bullet is traveling at 300 m/s when it becomes embedded in the 10-kg block, determine the distance the block will slide up along the plane before momentarily stopping.

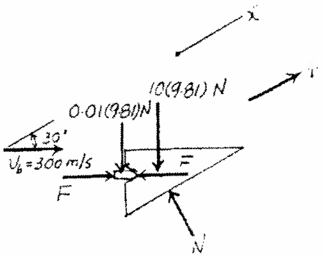



*Conservation of Linear Momentum :* If we consider the block and the bullet as a system, then from the FBD, the *impulsive* force $F$ caused by the impact is *internal* to the system. Therefore, it will cancel out. Also, the weight of the bullet and the block are *nonimpulsive forces*. As the result, linear momentum is conserved along the $x'$ axis.

$$m_b(v_b)_{x'} = (m_b + m_B)v_x$$

$$(+\,)\qquad 0.01(300\cos 30°) = (0.01 + 10)\,v$$

$$v = 0.2595 \text{ m/s}$$




*Conservation of Energy :* The datum is set at the block's initial position. When the block and the embedded bullet is at their highest point they are $h$ *above* the datum. Their gravitational potential energy is $(10+0.01)(9.81)h = 98.1981h$. Applying Eq. 14 − 21, we have

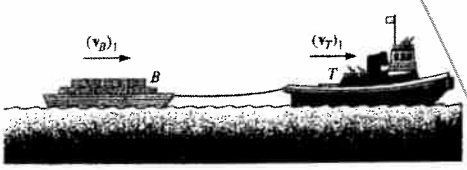
$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(10 + 0.01)(0.2595^2) = 0 + 98.1981h$$

$$h = 0.003433 \text{ m} = 3.43 \text{ mm}$$

$$d = 3.43 / \sin 30° = 6.87 \text{ mm} \qquad \textbf{Ans}$$

**15-49.** A tugboat $T$ having a mass of 19 Mg is tied to a barge $B$ having a mass of 75 Mg. If the rope is "elastic" such that it has a stiffness $k = 600$ kN/m, determine the maximum stretch in the rope during the initial towing. Originally both the tugboat and barge are moving in the same direction with speeds $(v_T)_1 = 15$ km/h and $(v_B)_1 = 10$ km/h, respectively. Neglect the resistance of the water.




$$(v_T)_1 = 15 \text{ km/h} = 4.167 \text{ m/s}$$

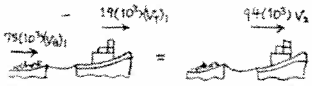
$$(v_B)_1 = 10 \text{ km/h} = 2.778 \text{ m/s}$$

When the rope is stretched to its maximum, both the tug and barge have a common velocity. Hence,

$$(\overset{+}{\rightarrow})\qquad \Sigma m v_1 = \Sigma m v_2$$

$$19\,000(4.167) + 75\,000(2.778) = (19\,000 + 75\,000)v_2$$

$$v_2 = 3.059 \text{ m/s}$$




$$T_1 + V_1 = T_2 + V_2$$

$$T_1 = \frac{1}{2}(19\,000)(4.167)^2 + \frac{1}{2}(75\,000)(2.778)^2 = 454.282 \text{ kJ}$$

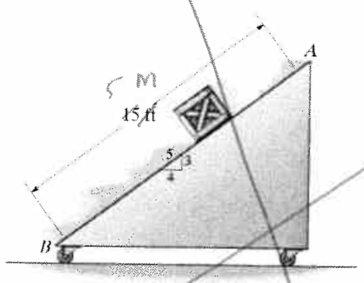
$$T_2 = \frac{1}{2}(19\,000 + 75\,000)(3.059)^2 = 439.661 \text{ kJ}$$

Hence

$$454.282(10^3) + 0 = 439.661(10^3) + \frac{1}{2}(600)(10^3)x^2$$

$$x = 0.221 \text{ m} \qquad \textbf{Ans}$$

**15–50.** The free-rolling ramp has a weight of 120 lb. The crate, whose weight is 80 lb, slides from rest at $A$, 15 ft down the ramp to $B$. Determine the ramp's speed when the crate reaches $B$. Assume that the ramp is smooth, and neglect the mass of the wheels.




$$T_1 + V_1 = T_2 + V_2$$

$$0 + 80\left(\tfrac{3}{5}\right)(15) = \tfrac{1}{2}\left(\tfrac{80}{32.2}\right)v_B^2 + \tfrac{1}{2}\left(\tfrac{120}{32.2}\right)v_r^2 \qquad [1]$$

$(\xrightarrow{+})\ \Sigma m v_1 = \Sigma m v_2$

$$0 + 0 = \tfrac{120}{32.2}v_r - \tfrac{80}{32.2}(v_B)_x$$

$$(v_B)_x = 1.5v_r$$


$$\mathbf{v}_B = \mathbf{v}_r + \mathbf{v}_{B/r}$$

$(\xrightarrow{+})\ -(v_B)_x = v_r - \tfrac{4}{5}v_{B/r} \qquad [2]$

$(+\uparrow)\ (v_B)_y = 0 - \tfrac{3}{5}v_{B/r} \qquad [3]$

Eliminating $v_{B/r}$ from Eqs.[2] and [3] and substituting $(v_B)_x = 1.5v_r$ results in

$$(v_B)_y = 1.875v_r$$

$$v_B^2 = (v_B)_x^2 + (v_B)_y^2 = (1.5v_r)^2 + (1.875v_r)^2 = 5.7656v_r^2 \qquad [4]$$

Substituting Eq.[4] into [1] yields:

$$80\left(\tfrac{3}{5}\right)(15) = \tfrac{1}{2}\left(\tfrac{80}{32.2}\right)(5.7656v_r^2) + \tfrac{1}{2}\left(\tfrac{120}{32.2}\right)v_r^2$$

$v_r = 8.93$ ft/s **Ans**

2.846 m/s

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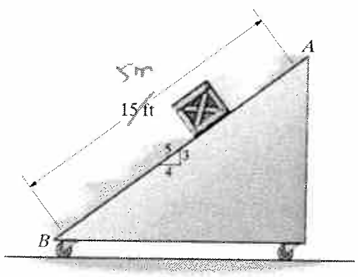
**15–51.** The free-rolling ramp has a weight of 120 lb. If the 80-lb crate is released from rest at $A$, determine the distance the ramp moves when the crate slides 15 ft down the ramp and reaches the bottom $B$.

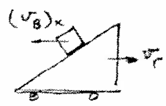



$(\xrightarrow{+})\quad \Sigma m v_1 = \Sigma m v_2$

$$0 = \frac{120}{32.2}v_r - \frac{80}{32.2}(v_B)_x$$

$$(v_B)_x = 1.5\,v_r$$

$$\mathbf{v}_B = \mathbf{v}_r + \mathbf{v}_{B/r}$$




$(\xrightarrow{+})\quad -(v_B)_x = v_r - (v_{B/r})_x$

$$-1.5v_r = v_r - (v_{B/r})_x$$

$$2.5\,v_r = (v_{B/r})_x$$

Integrate

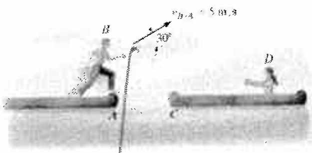
$$2.5\,s_r = (s_{B/r})_x$$

$$2.5\,s_r = \left(\frac{4}{5}\right)(15)$$

$s_r = 4.8$ ft **Ans**

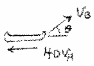
1.6 m

***15–52.** The boy $B$ jumps off the canoe at $A$ with a velocity of 5 m/s relative to the canoe as shown. If he lands in the second canoe $C$, determine the final speed of both canoes after the motion. Each canoe has a mass of 40 kg. The boy's mass is 30 kg, and the girl $D$ has a mass of 25 kg. Both canoes are originally at rest.




$(\overset{+}{\rightarrow})$ $\Sigma m v_1 = \Sigma m v_2$

$$0 + 0 = -40 v_A + 30(v_B)_x$$




$(\overset{+}{\rightarrow})$ $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

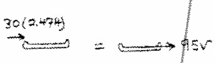
$(\overset{+}{\rightarrow})$ $(v_B)_x = -v_A + 5\cos 30°$

Thus,

$v_A = 1.856 = 1.86$ m/s ← **Ans**

$(v_B)_x = -1.856 + 5\cos 30° = 2.474$ →

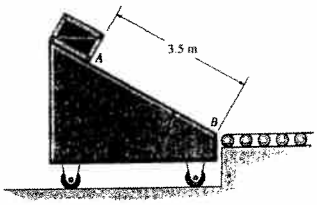
$(\overset{+}{\rightarrow})$ $\Sigma m v_1 = \Sigma m v_2$




$$30(2.474) = (40 + 30 + 25)v$$

$v = 0.781$ m/s **Ans**

**15–53.** The free-rolling ramp has a mass of 40 kg. A 10-kg crate is released from rest at $A$ and slides down 3.5 m to point $B$. If the surface of the ramp is smooth, determine the ramp's speed when the crate reaches $B$. Also, what is the velocity of the crate?

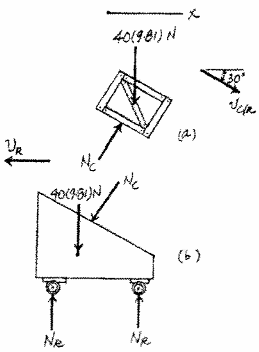



***Conservation of Energy*** : The datum is set at lowest point $B$. When the crate is at point $A$, it is $3.5\sin 30° = 1.75$ m *above* the datum. Its gravitational potential energy is $10(9.81)(1.75) = 171.675$ N·m. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 171.675 = \frac{1}{2}(10)\, v_C^2 + \frac{1}{2}(40)\, v_R^2$$

$$171.675 = 5v_C^2 + 20v_R^2 \qquad [1]$$




***Relative Velocity*** : The velocity of the crate is given by

$$\begin{aligned}\mathbf{v}_C &= \mathbf{v}_R + \mathbf{v}_{C/R}\\ &= -v_R\mathbf{i} + (v_{C/R}\cos 30°\mathbf{i} - v_{C/R}\sin 30°\mathbf{j})\\ &= (0.8660v_{C/R} - v_R)\mathbf{i} - 0.5v_{C/R}\mathbf{j}\end{aligned} \qquad [2]$$

The magnitude of $\mathbf{v}_C$ is

$$\begin{aligned}v_C &= \sqrt{(0.8660v_{C/R} - v_R)^2 + (-0.5v_{C/R})^2}\\ &= \sqrt{v_{C/R}^2 + v_R^2 - 1.732v_R v_{C/R}}\end{aligned} \qquad [3]$$

***Conservation of Linear Momentum*** : If we consider the crate and the ramp as a system, from the FBD, one realizes that the normal reaction $N_C$ (*impulsive force*) is *internal* to the system and will cancel each other. As the result, the linear momentum is conserved along the $x$ axis.

$$0 = m_C(v_C)_x + m_R v_R$$

$(\overset{+}{\rightarrow})$ $$0 = 10(0.8660v_{C/R} - v_R) + 40(-v_R)$$

$$0 = 8.660v_{C/R} - 50v_R \qquad [4]$$

Solving Eqs. [1], [3] and [4] yields

$v_R = 1.101$ m/s $= 1.10$ m/s $\qquad v_C = 5.43$ m/s **Ans**

$v_{C/R} = 6.356$ m/s

From Eq. [1]

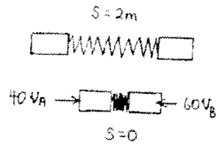
$$\mathbf{v}_C = [0.8660(6.356) - 1.101]\mathbf{i} - 0.5(6.356)\mathbf{j} = \{4.403\mathbf{i} - 3.178\mathbf{j}\}\text{ m/s}$$

Thus, the directional angle $\phi$ of $v_C$ is

$$\phi = \tan^{-1}\frac{3.178}{4.403} = 35.8°$$


**Ans**

**15-54.** Blocks $A$ and $B$ have masses of 40 kg and 60 kg, respectively. They are placed on a smooth surface and the spring connected between them is stretched 2 m. If they are released from rest, determine the speeds of both blocks the instant the spring becomes unstretched.


$(\overset{+}{\rightarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0 + 0 = 40\, v_A - 60\, v_B$$

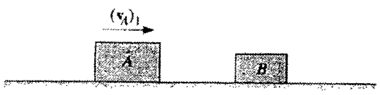
$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(180)(2)^2 = \frac{1}{2}(40)(v_A)^2 + \frac{1}{2}(60)(v_B)^2$$

$v_A = 3.29$ m/s **Ans**

$v_B = 2.19$ m/s **Ans**

**15-55.** Block $A$ has a mass of 3 kg and is sliding on a rough horizontal surface with a velocity $(v_A)_1 = 2$ m/s when it makes a direct collision with block $B$, which has a mass of 2 kg and is originally at rest. If the collision is perfectly elastic ($e = 1$), determine the velocity of each block just after collision and the distance between the blocks when they stop sliding. The coefficient of kinetic friction between the blocks and the plane is $\mu_k = 0.3$.

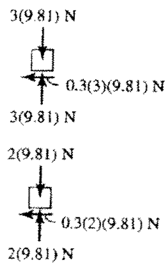



$(\overset{+}{\rightarrow})$ $\quad \sum m v_1 = \sum m v_2$

$$3(2) + 0 = 3(v_A)_2 + 2(v_B)_2$$

$(\overset{+}{\rightarrow})$ $\quad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$$1 = \frac{(v_B)_2 - (v_A)_2}{2 - 0}$$




Solving

$(v_A)_2 = 0.400$ m/s **Ans**

$(v_B)_2 = 2.40$ m/s **Ans**

Block $A$:

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(3)(0.400)^2 - 3(9.81)(0.3)d_A = 0$$

$d_A = 0.0272$ m

Block $B$:

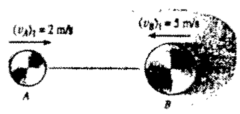
$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(2)(2.40)^2 - 2(9.81)(0.3)d_B = 0$$

$d_B = 0.9786$ m

$d = d_B - d_A = 0.951$ m **Ans**

***15-56.** Disks $A$ and $B$ have a mass of 2 kg and 4 kg, respectively. If they have the velocities shown, and $e = 0.4$, determine their velocities just after direct central impact.




$(\xrightarrow{+})\quad \Sigma m v_1 = \Sigma m v_2$

$$2(2) - 4(5) = 2(v_A)_2 + 4(v_B)_2$$

$(\xrightarrow{+})\quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1};$

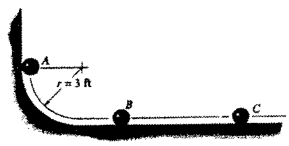
$$0.4 = \frac{(v_B)_2 - (v_A)_2}{(2 - (-5))}$$

Solving:

$(v_B)_2 = -1.73$ m/s $= 1.73$ m/s $\leftarrow$ **Ans**

$(v_A)_2 = -4.53$ m/s $= 4.53$ m/s $\leftarrow$ **Ans**

**15-57.** The three balls each weigh 0.5 lb and have a coefficient of restitution of $e = 0.85$. If ball $A$ is released from rest and strikes ball $B$ and then ball $B$ strikes ball $C$, determine the velocity of each ball after the second collision has occurred. The balls slide without friction.




Ball $A$:
Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (0.5)(3) = \frac{1}{2}\left(\frac{0.5}{32.2}\right)(v_A)_1^2 + 0$$

$(v_A)_1 = 13.90$ ft/s

Balls $A$ and $B$:

$(\xrightarrow{+})\quad \Sigma m v_1 = \Sigma m v_2$

$$\left(\frac{0.5}{32.2}\right)(13.90) + 0 = \left(\frac{0.5}{32.2}\right)(v_A)_2 + \left(\frac{0.5}{32.2}\right)(v_B)_2$$

$(\xrightarrow{+})\quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$$0.85 = \frac{(v_B)_2 - (v_A)_2}{13.90 - 0}$$

Solving:

$(v_A)_2 = 1.04$ ft/s **Ans**

$(v_B)_2 = 12.86$ ft/s

Balls $B$ and $C$:

$(\xrightarrow{+})\quad \Sigma m v_2 = \Sigma m v_3$

$$\left(\frac{0.5}{32.2}\right)(12.86) + 0 = \left(\frac{0.5}{32.2}\right)(v_B)_3 + \left(\frac{0.5}{32.2}\right)(v_C)_3$$

$(\xrightarrow{+})\quad e = \frac{(v_C)_3 - (v_B)_3}{(v_B)_2 - (v_C)_2}$

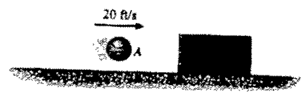
$$0.85 = \frac{(v_C)_3 - (v_B)_3}{12.86 - 0}$$

Solving:

$(v_B)_3 = 0.964$ ft/s **Ans**

$(v_C)_3 = 11.9$ ft/s **Ans**

**15-58.** The 1-lb ball $A$ is thrown so that when it strikes the 10-lb block $B$ it is traveling horizontally at 20 ft/s. If the coefficient of restitution between $A$ and $B$ is $e = 0.6$, and the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the time before block $B$ stops sliding.




$(\overset{+}{\rightarrow})$ $\Sigma m_1 v_1 = \Sigma m_2 v_2$

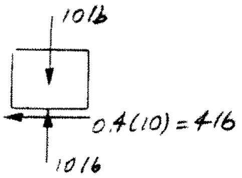
$$\left(\frac{1}{32.2}\right)(20) + 0 = \left(\frac{1}{32.2}\right)(v_A)_2 + \left(\frac{10}{32.2}\right)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 20$$

$(\overset{+}{\rightarrow})$ $$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.6 = \frac{(v_B)_2 - (v_A)_2}{20 - 0}$$

$$(v_B)_2 - (v_A)_2 = 12$$




Thus,

$(v_B)_2 = 2.909$ ft/s $\rightarrow$

$(v_A)_2 = -9.091$ ft/s $= 9.091$ ft/s $\leftarrow$

Block $B$ :

$(\overset{+}{\rightarrow})$ $m v_1 + \Sigma \int F\,dt = m v_2$

$$\left(\frac{10}{32.2}\right)(2.909) - 4t = 0$$

$t = 0.226$ s **Ans**

**15-59.** The 1-lb ball $A$ is thrown so that when it strikes the 10-lb block $B$ it is traveling horizontally at 20 ft/s. If the coefficient of restitution between $A$ and $B$ is $e = 0.6$, and the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the distance block $B$ slides on the plane before stopping.


$(\overset{+}{\rightarrow})$ $\Sigma m_1 v_1 = \Sigma m_2 v_2$

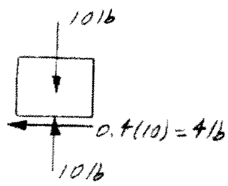
$$\left(\frac{1}{32.2}\right)(20) + 0 = \left(\frac{1}{32.2}\right)(v_A)_2 + \left(\frac{10}{32.2}\right)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 20$$

$(\overset{+}{\rightarrow})$ $$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.6 = \frac{(v_B)_2 - (v_A)_2}{20 - 0}$$

$$(v_B)_2 - (v_A)_2 = 12$$




Thus,

$(v_B)_2 = 2.909$ ft/s $\rightarrow$

$(v_A)_2 = -9.091$ ft/s $= 9.091$ ft/s $\leftarrow$

Block $B$ :

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(2.909)^2 - 4d = 0$$

$d = 0.329$ ft **Ans**

***15-60.** The 1-lb ball $A$ is thrown so that when it strikes the 10-lb block $B$ it is traveling horizontally at 20 ft/s. Determine the average normal force exerted between $A$ and $B$ if the impact occurs in 0.02 s. The coefficient of restitution between $A$ and $B$ is $e = 0.6$.


$(\overset{+}{\rightarrow})$ $\quad \Sigma m_1 v_1 = \Sigma m_2 v_2$

$$\left(\frac{1}{32.2}\right)(20) + 0 = \left(\frac{1}{32.2}\right)(v_A)_2 + \left(\frac{10}{32.2}\right)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 20$$

$(\overset{+}{\rightarrow})$ $\quad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$$0.6 = \frac{(v_B)_2 - (v_A)_2}{20 - 0}$$

$$(v_B)_2 - (v_A)_2 = 12$$

Thus,

$$(v_B)_2 = 2.909 \text{ ft/s} \rightarrow$$

$$(v_A)_2 = -9.091 \text{ ft/s} = 9.091 \text{ ft/s} \leftarrow$$

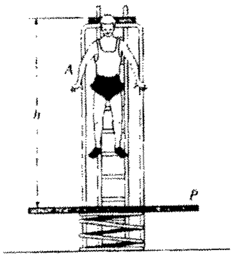
Ball $A$:

$(\overset{+}{\rightarrow})$ $\quad m v_1 + \Sigma \int F\,dt = m v_2$

$$\left(\frac{1}{32.2}\right)(20) - F(0.02) = \left(\frac{1}{32.2}\right)(-9.091)$$

$F = 45.2$ lb **Ans**

**15-61.** The man $A$ has a weight of 175 lb and jumps from rest $h = 8$ ft onto a platform $P$ that has a weight of 60 lb. The platform is mounted on a spring, which has a stiffness $k = 200$ lb/ft. Determine (a) the velocities of $A$ and $P$ just after impact and (b) the maximum compression imparted to the spring by the impact. Assume the coefficient of restitution between the man and the platform is $e = 0.6$, and the man holds himself rigid during the motion.




$$T_0 + V_0 = T_1 + V_1$$

$$0 + 175(8) = \frac{1}{2}\left(\frac{175}{32.2}\right)(v_{m1})^2 + 0$$

$$v_{m1} = 22.698 \text{ ft/s}$$

$(+\downarrow)$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$\frac{175}{32.2}(22.698) + 0 = \frac{175}{32.2}(v_{m2}) + \frac{60}{32.2}(v_{P2})$$

$(+\downarrow)$ $\quad e = \dfrac{v_{P2} - v_{m2}}{v_{m1} - v_{P1}}$

$$0.6 = \frac{v_{P2} - v_{m2}}{22.698 - 0}$$

Solving,

$v_{P2} = 27.04$ ft/s $= 27.0$ ft/s **Ans**

$v_{m2} = 13.4$ ft/s **Ans**

$$60 = 200(x_{eq})$$

$$x_{eq} = 0.3 \text{ ft}$$

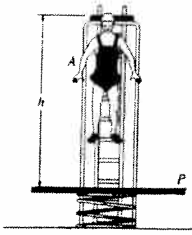
$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}\left(\frac{60}{32.2}\right)(27.04)^2 + \frac{1}{2}(200)(0.3)^2 + 0 = 0 + \frac{1}{2}(200)(x + 0.3)^2 - 60(x)$$

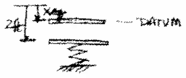
$$100x^2 - 681.206 = 0$$

$x = 2.61$ ft **Ans**

**15-62.** The man $A$ has a weight of 100 lb and jumps from rest onto the platform $P$ that has a weight of 60 lb. The platform is mounted on a spring, which has a stiffness $k = 200$ lb/ft. If the coefficient of restitution between the man and the platform is $e = 0.6$, and the man holds himself rigid during the motion, determine the required height $h$ of the jump if the maximum compression of the spring becomes 2 ft.




For the platform after collision:




$60 = 200(x_{eq})$

$x_{eq} = 0.3$ ft

$T_2 + V_2 = T_3 + V_3$

$\frac{1}{2}\left(\frac{60}{32.2}\right)(v_{p2})^2 + \frac{1}{2}(200)(0.3)^2 + 0 = 0 + \frac{1}{2}(200)(2)^2 - 60(2 - 0.3)$

$v_{p2} = 17.612$ ft/s

$(+\downarrow) \quad e = \frac{v_{p2} - v_{m2}}{v_{m1} - v_{p1}}$

$0.6 = \frac{17.612 - v_{m2}}{v_{m1} - 0}$

$(+\downarrow) \quad \Sigma m v_1 = \Sigma m v_2$

$\frac{100}{32.2}v_{m1} + 0 = \frac{100}{32.2}(v_{m2}) + \frac{60}{32.2}(17.612)$

Solving,

$v_{m1} = 17.612$ m/s

$v_{m2} = 7.045$ m/s

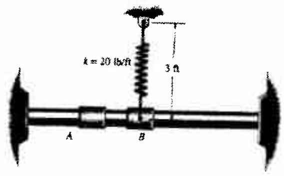
For the man just before striking the platform

$T_0 + V_0 = T_1 + V_1$

$0 + 100h = \frac{1}{2}\left(\frac{100}{32.2}\right)(17.612)^2 + 0$

$h = 4.82$ ft **Ans**

**15-63.** The 10-lb collar $B$ is at rest, and when it is in the position shown the spring is unstretched. If another 1-lb collar $A$ strikes it so that $B$ slides 4 ft on the smooth rod before momentarily stopping, determine the velocity of $A$ just after impact, and the average force exerted between $A$ and $B$ during the impact if the impact occurs in 0.002 s. The coefficient of restitution between $A$ and $B$ is $e = 0.5$.

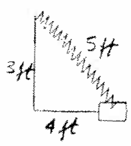



Collar $B$ after impact:

$T_2 + V_2 = T_3 + V_3$

$\frac{1}{2}\left(\frac{10}{32.2}\right)(v_B)_2^2 + 0 = 0 + \frac{1}{2}(20)(5-3)^2$

$(v_B)_2 = 16.05$ ft/s




System:

$(\overset{+}{\rightarrow}) \quad \Sigma m_1 v_1 = \Sigma m_1 v_2$

$\frac{1}{32.2}(v_A)_1 + 0 = \frac{1}{32.2}(v_A)_2 + \frac{10}{32.2}(16.05)$

$(v_A)_1 - (v_A)_2 = 160.5$

$(\overset{+}{\rightarrow}) \quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

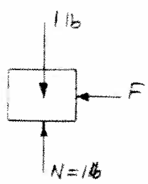
$0.5 = \frac{16.05 - (v_A)_2}{(v_A)_1 - 0}$

$0.5(v_A)_1 + (v_A)_2 = 16.05$

Solving:

$(v_A)_1 = 117.7$ ft/s $= 118$ ft/s $\rightarrow$

$(v_A)_2 = -42.8$ ft/s $= 42.8$ ft/s $\leftarrow$ **Ans**



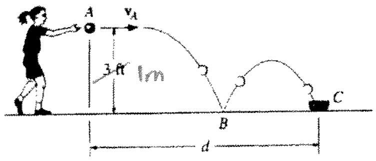

Collar $A$:

$(\overset{+}{\rightarrow}) \quad m v_1 + \Sigma\int F\,dt = m v_2$

$\left(\frac{1}{32.2}\right)(117.7) - F(0.002) = \left(\frac{1}{32.2}\right)(-42.8)$

$F = 2492.2$ lb $= 2.49$ kip **Ans**

***15–64.** If the girl throws the ball with a horizontal velocity of 8 ft/s, determine the distance $d$ so that the ball bounces once on the smooth surface and then lands in the cup at $C$. Take $e = 0.8$.




$(+\downarrow)\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$(v_1)_y^2 = 0 + 2(32.2)(3)$   9.81   1

$(v_1)_y = 13.90 \downarrow$   4.42 ↓

$(+\downarrow)\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$3 = 0 + 0 + \frac{1}{2}(32.2)(t_{AB})^2$   9.81   1

$t_{AB} = 0.43167\text{ s}$   0.45 s

$(+\downarrow)\quad e = \frac{(v_2)_y}{(v_1)_y}$

$0.8 = \frac{(v_2)_y}{13.90}$   4.42

$(v_2)_y = 11.1197 \uparrow$   3.536 ↑

$(+\downarrow)\quad v = v_0 + a_c t$

$11.1197 = -11.1197 + 32.2(t_{BC})$   3.536   3.536   9.81

$t_{BC} = 0.6907\text{ s}$   0.721 s

Total time is $t_{AC} = 1.1224\text{ s}$   1.171 s

Since the $x$ component of momentum is conserved

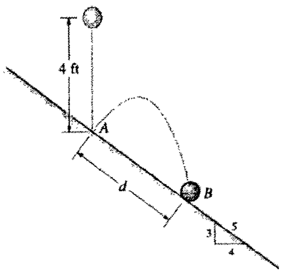
$d = v_A(t_{AC})$

$d = 8(1.1224)$   2   1.171

$d = 8.98\text{ ft}$    **Ans**   2.342 m

---

**15–65.** The 1-lb ball is dropped from rest and falls a distance of 4 ft before striking the smooth plane at $A$. If $e = 0.8$, determine the distance $d$ to where it again strikes the plane at $B$.



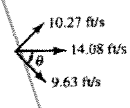

$T_1 + V_1 = T_2 + V_2$

$0 + 0 = \frac{1}{2}(m)(v_A)_1^2 - m(32.2)(4)$

$(v_A)_1 = \sqrt{2(32.2)(4)} = 16.05\text{ ft/s}$


$\searrow+\quad (v_A)_{2x} = \frac{3}{5}(16.05) = 9.63\text{ ft/s}$

$\nearrow+\quad (v_A)_{2y} = 0.8\left(\frac{4}{5}\right)(16.05) = 10.27\text{ ft/s}$




$(v_A)_2 = \sqrt{(9.63)^2 + (10.27)^2} = 14.08\text{ ft/s}$

$\theta = \tan^{-1}\left(\frac{10.27}{9.63}\right) = 46.85°$

$\phi = 46.85° - \tan^{-1}\left(\frac{3}{4}\right) = 9.977°$

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$d\left(\frac{4}{5}\right) = 0 + 14.08\cos 9.977°(t)$

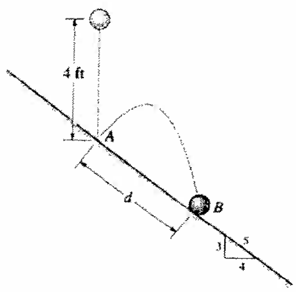
$(+\downarrow)\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$d\left(\frac{3}{5}\right) = 0 - 14.08\sin 9.977°(t) + \frac{1}{2}(32.2)t^2$

$t = 0.798\text{ s}$

$d = 13.8\text{ ft}$    **Ans**

**15–66.** The 1-lb ball is dropped from rest and falls a distance of 4 ft before striking the smooth plane at $A$. If it rebounds and in $t = 0.5$ s again strikes the plane at $B$, determine the coefficient of restitution $e$ between the ball and the plane. Also, what is the distance $d$?




$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}(m)(v_A)_1^2 - m(32.2)(4)$$

$$(v_A)_1 = \sqrt{2(32.2)(4)} = 16.05 \text{ ft/s}$$

$+\searrow \quad (v_A)_{2x'} = \frac{3}{5}(16.05) = 9.63 \text{ ft/s}$

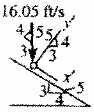
$\nearrow + \quad (v_A)_{2y'} = e\left(\frac{4}{5}\right)(16.05) = 12.84e \text{ ft/s}$

$(\xrightarrow{+}) \quad s = s_0 + v_0 t$

$$\frac{4}{5}(d) = 0 + v_{A2x}(0.5)$$

$(+\downarrow) \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$\frac{3}{5}(d) = 0 - v_{A2y}(0.5) + \frac{1}{2}(32.2)(0.5)^2$$




$(\xrightarrow{+}) \quad 0.5\left[9.63\left(\frac{4}{5}\right) + 12.84e\left(\frac{3}{5}\right)\right] = \frac{4}{5}d$

$(+\uparrow) \quad 0.5\left[-9.63\left(\frac{3}{5}\right) + 12.84e\left(\frac{4}{5}\right)\right] = 4.025 - \frac{3}{5}d$

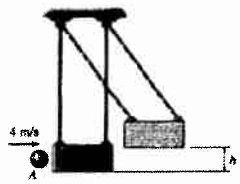
Solving,

$e = 0.502$ **Ans**

$d = 7.23$ ft **Ans**

---

**15-67.** The 2-kg ball is thrown at the suspended 20-kg block with a velocity of 4 m/s. If the coefficient of restitution between the ball and the block is $e = 0.8$, determine the maximum height $h$ to which the block will swing before it momentarily stops.




System :

$(\xrightarrow{+}) \quad \Sigma m_1 v_1 = \Sigma m_2 v_2$

$$(2)(4) + 0 = (2)(v_A)_2 + (20)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 4$$

$(\xrightarrow{+}) \quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$$0.8 = \frac{(v_B)_2 - (v_A)_2}{4 - 0}$$

$$(v_B)_2 - (v_A)_2 = 3.2$$

Solving :

$(v_A)_2 = -2.545$ m/s

$(v_B)_2 = 0.6545$ m/s

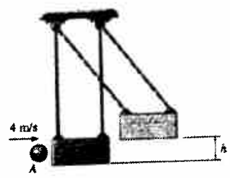
Block :
Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(20)(0.6545)^2 + 0 = 0 + 20(9.81)h$$

$h = 0.0218$ m $= 21.8$ mm **Ans**

***15-68.** The 2-kg ball is thrown at the suspended 20-kg block with a velocity of 4 m/s. If the time of impact between the ball and the block is 0.005 s, determine the average normal force exerted on the block during this time. Take $e = 0.8$.




System :

$(\overset{+}{\rightarrow})$ $\quad \Sigma m_1 v_1 = \Sigma m_2 v_2$

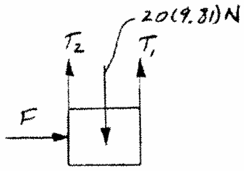
$$(2)(4) + 0 = (2)(v_A)_2 + (20)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 4$$

$(\overset{+}{\rightarrow})$ $\quad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$$0.8 = \frac{(v_B)_2 - (v_A)_2}{4 - 0}$$

$$(v_B)_2 - (v_A)_2 = 3.2$$




Solving :

$(v_A)_2 = -2.545$ m/s

$(v_B)_2 = 0.6545$ m/s

Block :

$(\overset{+}{\rightarrow})$ $\quad m v_1 + \Sigma \int F\, dt = m v_2$

$$0 + F(0.005) = 20(0.6545)$$

$F = 2618$ N $= 2.62$ kN **Ans**

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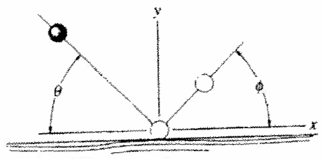
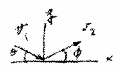
**15-69.** A ball is thrown onto a rough floor at an angle $\theta$. If it rebounds at an angle $\phi$ and the coefficient of kinetic friction is $\mu$, determine the coefficient of restitution $e$. Neglect the size of the ball. *Hint:* Show that during impact, the average impulses in the $x$ and $y$ directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x \Delta t = \mu F_y \Delta t$ or $F_x = \mu F_y$.




$(+\downarrow)$ $\quad e = \dfrac{0 - [-v_2 \sin\phi]}{v_1 \sin\theta - 0} \qquad e = \dfrac{v_2 \sin\phi}{v_1 \sin\theta} \qquad [1]$

$(\overset{+}{\rightarrow})$ $\quad m(v_x)_1 + \int_{t_1}^{t_2} F_x\, dx = m(v_x)_2$

$$m v_1 \cos\theta - F_x \Delta t = m v_2 \cos\phi$$

$$F_x = \frac{m v_1 \cos\theta - m v_2 \cos\phi}{\Delta t} \qquad [2]$$


$(+\downarrow)$ $\quad m(v_y)_1 + \int_{t_1}^{t_2} F_y\, dx = m(v_y)_2$

$$m v_1 \sin\theta - F_y \Delta t = -m v_2 \sin\phi$$

$$F_y = \frac{m v_1 \sin\theta + m v_2 \sin\phi}{\Delta t} \qquad [3]$$

Since $F_x = \mu F_y$, from Eqs.[2] and [3]

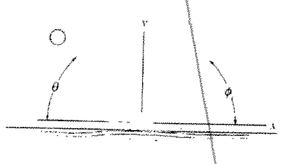
$$\frac{m v_1 \cos\theta - m v_2 \cos\phi}{\Delta t} = \frac{\mu(m v_1 \sin\theta + m v_2 \sin\phi)}{\Delta t}$$

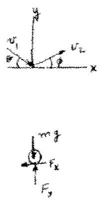
$$\frac{v_2}{v_1} = \frac{\cos\theta - \mu \sin\theta}{\mu \sin\phi + \cos\phi} \qquad [4]$$

Substituting Eq.[4] into [1] yields :

$$e = \frac{\sin\phi}{\sin\theta}\left(\frac{\cos\theta - \mu \sin\theta}{\mu \sin\phi + \cos\phi}\right)$$ **Ans**

**15-70.** A ball is thrown onto a rough floor at an angle of $\theta = 45°$. If it rebounds at the same angle $\phi = 45°$, determine the coefficient of kinetic friction between the floor and the ball. The coefficient of restitution is $e = 0.6$. *Hint:* Show that during impact, the average impulses in the $x$ and $y$ directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x \Delta t = \mu F_y \Delta t$ or $F_x = \mu F_y$.





$(+\downarrow)$ $$e = \frac{0 - [-v_2 \sin\phi]}{v_1 \sin\theta - 0} \qquad e = \frac{v_2 \sin\phi}{v_1 \sin\theta} \qquad [1]$$

$(\overset{+}{\rightarrow})$ $$m(v_x)_1 + \int_{t_1}^{t_2} F_x\, dx = m(v_x)_2$$

$$m v_1 \cos\theta - F_x \Delta t = m v_2 \cos\phi$$

$$F_x = \frac{m v_1 \cos\theta - m v_2 \cos\phi}{\Delta t} \qquad [2]$$

$(+\downarrow)$ $$m(v_y)_1 + \int_{t_1}^{t_2} F_y\, dx = m(v_y)_2$$

$$m v_1 \sin\theta \quad F_y \Delta t = -m v_2 \sin\phi$$

$$F_y = \frac{m v_1 \sin\theta + m v_2 \sin\phi}{\Delta t} \qquad [3]$$

Since $F_x = \mu F_y$, from Eqs.[2] and [3]

$$\frac{m v_1 \cos\theta - m v_2 \cos\phi}{\Delta t} = \frac{\mu( m v_1 \sin\theta + m v_2 \sin\phi)}{\Delta t}$$

$$\frac{v_2}{v_1} = \frac{\cos\theta - \mu \sin\theta}{\mu \sin\phi + \cos\phi} \qquad [4]$$

Substituting Eq.[4] into [1] yields :

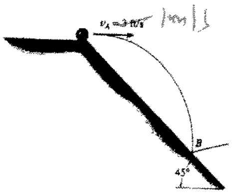
$$e = \frac{\sin\phi}{\sin\theta}\left(\frac{\cos\theta - \mu\sin\theta}{\mu\sin\phi + \cos\phi}\right)$$

$$0.6 = \frac{\sin 45°}{\sin 45°}\left(\frac{\cos 45° - \mu \sin 45°}{\mu \sin 45° + \cos 45°}\right)$$

$$0.6 = \frac{1-\mu}{1+\mu} \qquad \mu = 0.25 \qquad \textbf{Ans}$$

35 10N

**15-71.** The 0.2-lb ball bearing travels over the edge $A$ with a velocity of $v_A = 3$ ft/s. Determine the speed at which it rebounds from the smooth inclined plane at $B$. Take $e = 0.8$. 1 m/s

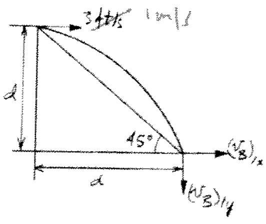


$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t$

$d = 0 + 3t$ 0 + 1 × t

$(+\downarrow)$ $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$d = 0 + 0 + \frac{1}{2}(32.2)t^2$ 9.81



$d = 0.559$ ft 0.204 m

$t = 0.1863$ s 0.204 s

$(\overset{+}{\rightarrow})$ $v = v_0$

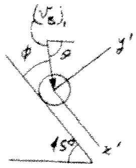
$(v_B)_{1x} = 3$ ft/s $\rightarrow$ 1 m/s

$(+\downarrow)$ $v = v_0 + a_c t$

$(v_B)_{1y} = 0 + (32.2)(0.1863) = 6$ ft/s $\downarrow$ 9.81 × 204 2 m/s

$(v_B)_1 = \sqrt{(3)^2 + (6)^2} = 6.708$ ft/s 2.24 m/s

$\theta = \tan^{-1}\left(\frac{6}{3}\right) = 63.43°$ ✓ $\phi = 63.43° - 45° = 18.43°$ ✓



$(+\searrow)$ $(v_B)_{2x'} = (v_B)_1 \cos\phi = 6.364$ ft/s 2.125 m/s

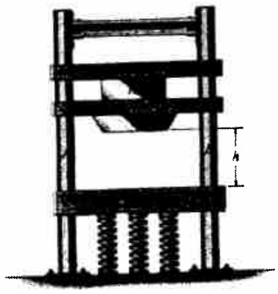
$(+\nearrow)$ $$e = \frac{(v_B)_{2y'} - 0}{0 - (-6.708 \sin 18.43°)} = 0.8$$ 2.24

$(v_B)_{2y'} = 1.697$ ft/s 0.567 m/s

$(v_B)_2 = \sqrt{(6.364)^2 + (1.697)^2} = 6.59$ ft/s **Ans**

2.125 0.567 2.20 m/s

***15-72.** The drop hammer $H$ has a weight of 900 lb and falls from rest $h = 3$ ft onto a forged anvil plate $P$ that has a weight of 500 lb. The plate is mounted on a set of springs which have a combined stiffness of $k_T = 500$ lb/ft. Determine (a) the velocity of $P$ and $H$ just after collision and (b) the maximum compression in the springs caused by the impact. The coefficient of restitution between the hammer and the plate is $e = 0.6$. Neglect friction along the vertical guideposts $A$ and $B$.




Just before impact:
Datum at lowest point.

$$T_0 + V_0 = T_1 + V_1$$

$$0 + 900(3) = \frac{1}{2}\left(\frac{900}{32.2}\right)(v_H)_1^2 + 0$$

$$(v_H)_1 = 13.90 \text{ ft/s}$$

Conservation of momentum will be applied since the force of the springs is nonimpulsive compared to the impact force.

$$(+\downarrow) \quad m_H(v_H)_1 + m_P(v_P)_1 = m_H(v_H)_2 + m_P(v_P)_2$$

$$\left(\frac{900}{32.2}\right)(13.90) + 0 = \left(\frac{900}{32.2}\right)(v_H)_2 + \left(\frac{500}{32.2}\right)(v_P)_2$$

$$13.90 = (v_H)_2 + 0.556(v_P)_2 \qquad (1)$$

$$(+\downarrow) \quad e = \frac{(v_P)_2 - (v_H)_2}{(v_H)_1 - (v_P)_1}$$

$$0.6 = \frac{(v_P)_2 - (v_H)_2}{13.90 - 0}$$

$$8.34 = (v_P)_2 - (v_H)_2 \qquad (2)$$

Solving Eqs. (1) and (2):

$(v_P)_2 = 14.30 = 14.3$ ft/s **Ans**

$(v_H)_2 = 5.96$ ft/s **Ans**

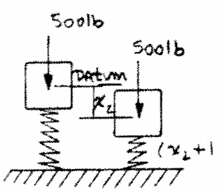
The initial compression in the springs is

$F = kx;\quad 500 = 500x_1 \quad x_1 = 1$ ft

Datum at highest point:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{500}{32.2}\right)(14.30)^2 + \frac{1}{2}(500)(1)^2 = 0 - 500x_2 + \frac{1}{2}(500)(x_2 + 1)^2$$




$$1837 = -500x_2 + 250x_2^2 + 500x_2 + 250$$

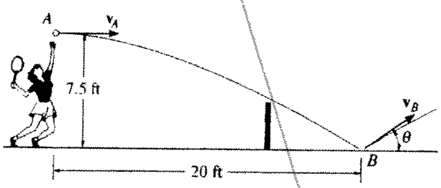
$$1587 = 250x_2^2$$

$$x_2 = 2.52 \text{ ft}$$

Total compression in springs is

$x = x_2 + 1 = 3.52$ ft **Ans**

**15-73.** It was observed that a tennis ball when served horizontally 7.5 ft above the ground strikes the smooth ground at $B$ 20 ft away. Determine the initial velocity $\mathbf{v}_A$ of the ball and the velocity $\mathbf{v}_B$ (and $\theta$) of the ball just after it strikes the court at $B$. Take $e = 0.7$.




$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$20 = 0 + v_A t$

$(+\downarrow)\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$7.5 = 0 + 0 + \frac{1}{2}(32.2)t^2$

$t = 0.682524$

$v_A = 29.303 = 29.3 \text{ ft/s}$ **Ans**

$v_{Bx1} = 29.303 \text{ ft/s}$

$(+\downarrow)\quad v = v_0 + a_c t$

$v_{By1} = 0 + 32.2(0.68252) = 21.977 \text{ ft/s}$

$(\overset{+}{\rightarrow})\quad mv_1 = mv_2$

$v_{B2x} = v_{B1x} = 29.303 \text{ ft/s} \rightarrow$

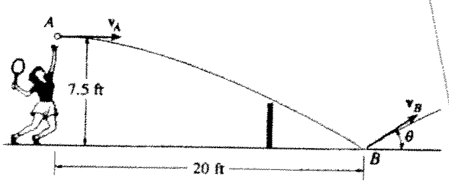
$e = \frac{v_{By2}}{v_{By1}}$

$0.7 = \frac{v_{By2}}{21.977}, \quad v_{By2} = 15.384 \text{ ft/s} \uparrow$

$v_{B2} = \sqrt{(29.303)^2 + (15.384)^2} = 33.1 \text{ ft/s}$ **Ans**

$\theta = \tan^{-1}\frac{15.384}{29.303} = 27.7°$ **Ans**

**15-74.** The tennis ball is struck with a horizontal velocity $\mathbf{v}_A$, strikes the smooth ground at $B$, and bounces upward at $\theta = 30°$. Determine the initial velocity $\mathbf{v}_A$, the final velocity $\mathbf{v}_B$, and the coefficient of restitution between the ball and the ground.




$(+\downarrow)\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$(v_{By})_1^2 = 0 + 2(32.2)(7.5 - 0)$

$v_{By1} = 21.9773 \text{ m/s}$

$(+\downarrow)\quad v = v_0 + a_c t$

$21.9773 = 0 + 32.2\,t$

$t = 0.68252 \text{ s}$

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$20 = 0 + v_A(0.68252)$

$v_A = 29.303 = 29.3 \text{ ft/s}$ **Ans**

$(\overset{+}{\rightarrow})\quad mv_1 = mv_2$

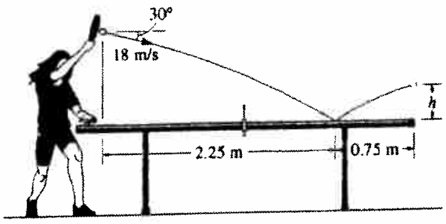
$v_{Bx2} = v_{Bx1} = v_A = 29.303$

$v_{B_2} = 29.303/\cos 30° = 33.8 \text{ ft/s}$ **Ans**

$v_{By2} = 29.303\tan 30° = 16.918 \text{ ft/s}$

$e = \frac{v_{By2}}{v_{By1}} = \frac{16.918}{21.9773} = 0.770$ **Ans**

**15-75.** The ping-pong ball has a mass of 2 g. If it is struck with the velocity shown, determine how high $h$ it rises above the end of the smooth table after the rebound. Take $e = 0.8$.




$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$2.25 = 0 + 18\cos 30° t$

$t = 0.14434$ s

$(v_x)_1 = (v_x)_2 = 18\cos 30° = 15.5885$ m/s

$(+\downarrow)\quad v = v_0 + a_c t$

$(v_y)_1 = 18\sin 30° + 9.81(0.14434)$

$(v_y)_1 = 10.4160$ m/s

$(+\uparrow)\quad e = 0.8 = \dfrac{(v_y)_2}{10.4160}$

$(v_y)_2 = 8.3328$ m/s

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$0.75 = 0 + 15.5885t$

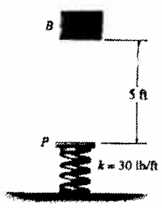
$t = 0.048112$ s

$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$h = 0 + 8.3328(0.048112) - \frac{1}{2}(9.81)(0.048112)^2$

$h = 0.390$ m **Ans**

***15-76.** The 5-lb box $B$ is dropped from rest 5 ft from the top of the 10-lb plate $P$, which is supported by the spring having a stiffness of $k = 30$ lb/ft. If $e = 0.6$ between the box and plate, determine the maximum compression imparted to the spring. Neglect the mass of the spring.




Box :
Datum at $P$.

$T_1 + V_1 = T_2 + V_2$

$0 + 5(5) = \frac{1}{2}\left(\frac{5}{32.2}\right)(v_B)_1^2 + 0$

$(v_B)_1 = 17.94$ ft/s

System :

$(+\downarrow)\quad \Sigma m v_1 = \Sigma m v_2$

$\left(\frac{5}{32.2}\right)(17.94) + 0 = \left(\frac{5}{32.2}\right)(v_B)_2 + \left(\frac{10}{32.2}\right)(v_P)_2$

$(+\downarrow)\quad e = \dfrac{(v_P)_2 - (v_B)_2}{(v_B)_1 - (v_P)_1};\quad 0.6 = \dfrac{(v_P)_2 - (v_B)_2}{17.94 - 0}$

Solving :

$(v_P)_2 = 9.570$ ft/s $\downarrow$

$(v_B)_2 = 1.196$ ft/s $\uparrow$

Initial compression of spring :

$F_s = kx;\quad x_1 = \dfrac{F_s}{k} = \dfrac{10}{30} = 0.333$ ft

Datum at initial position of plate :

$T_1 + V_1 = T_2 + V_2$

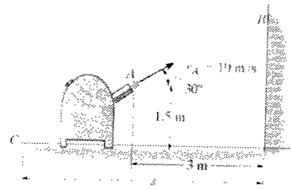
$\frac{1}{2}\left(\frac{10}{32.2}\right)(9.570)^2 + \frac{1}{2}(30)(0.333)^2 = 0 + \frac{1}{2}(30)(x' + 0.333)^2 - 10x'$

$x' = 0.974$ ft

Thus,

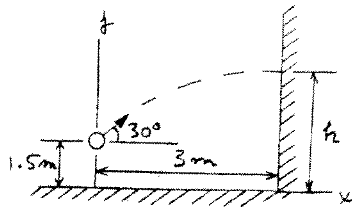
$x_{max} = 0.974 + 0.333 = 1.31$ ft **Ans**

**15-77.** A pitching machine throws the 0.5-kg ball towards the wall with an initial velocity $v_A = 10$ m/s as shown. Determine (a) the velocity at which it strikes the wall at $B$, (b) the velocity at which it rebounds from the wall if $e = 0.5$, and (c) the distance $s$ from the wall to where it strikes the ground at $C$.




(a)

$(v_B)_{x1} = 10\cos 30° = 8.660$ m/s →




$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t$

$3 = 0 + 10\cos 30° t$

$t = 0.3464$ s

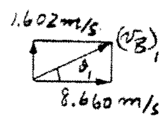
$(+\uparrow)$ $v = v_0 + a_c t$

$(v_B)_{y1} = 10\sin 30° - 9.81(0.3464) = 1.602$ m/s ↑

$(+\uparrow)$ $s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$h = 1.5 + 10\sin 30°(0.3464) - \frac{1}{2}(9.81)(0.3464)^2$

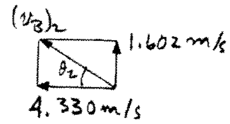
$= 2.643$ m




$(v_B)_1 = \sqrt{(1.602)^2 + (8.660)^2} = 8.81$ m/s **Ans**

$\theta_1 = \tan^{-1}\left(\frac{1.602}{8.660}\right) = 10.5°$ **Ans**

(b)




$(\overset{+}{\rightarrow})$ $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$; $0.5 = \frac{(v_{Bx})_2 - 0}{0 - (8.660)}$

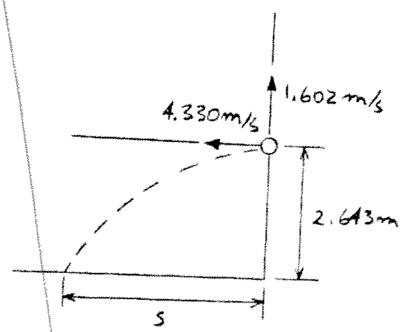
$(v_{Bx})_2 = 4.330$ m/s ←

$(v_{By})_2 = (v_{By})_1 = 1.602$ m/s ↑

$(v_B)_2 = \sqrt{(4.330)^2 + (1.602)^2} = 4.62$ m/s **Ans**

$\theta_2 = \tan^{-1}\left(\frac{1.602}{4.330}\right) = 20.3°$ **Ans**

(c)




$(+\uparrow)$ $s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

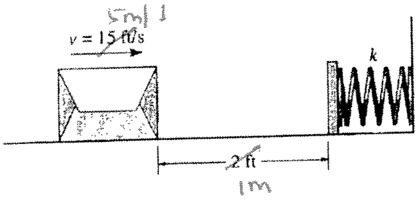
$-2.643 = 0 + 1.602(t) - \frac{1}{2}(9.81)(t)^2$

$t = 0.9153$ s

$(\overset{+}{\leftarrow})$ $s = s_0 + v_0 t$

$s = 0 + 4.330(0.9153) = 3.96$ m **Ans**

**15-78.** The 20-lb box slides on the surface for which $\mu_k = 0.3$. The box has a velocity $v = 15$ ft/s when it is 2 ft from the plate. If it strikes the smooth plate, which has a weight of 10 lb and is held in position by an unstretched spring of stiffness $k = 400$ lb/ft, determine the maximum compression imparted to the spring. Take $e = 0.8$ between the box and the plate. Assume that the plate slides smoothly.




$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{20}{32.2}\right)(15)^2 - (0.3)(20)(2) = \frac{1}{2}\left(\frac{20}{32.2}\right)(v_2)^2$$

$v_2 = 13.65$ ft/s


$(\overset{+}{\rightarrow})\quad \sum m v_1 = \sum m v_2$

$$\left(\frac{20}{32.2}\right)(13.65) = \left(\frac{20}{32.2}\right)v_A + \frac{10}{32.2}v_B$$

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.8 = \frac{v_P - v_A}{13.65}$$

Solving,

$v_P = 16.38$ ft/s, $\quad v_A = 5.46$ ft/s

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(16.38)^2 + 0 = 0 + \frac{1}{2}(400)(s)^2$$

$s = 0.456$ ft **Ans**

**15-79.** The 200-g billiard ball is moving with a speed of 2.5 m/s when it strikes the side of the pool table at $A$. If the coefficient of restitution between the ball and the side of the table is $e = 0.6$, determine the speed of the ball just after striking the table twice, i.e., at $A$, then at $B$. Neglect the size of the ball.

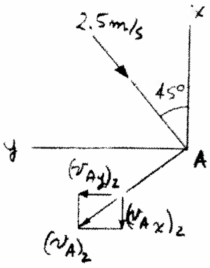

At $A$:

$(v_A)_{y1} = 2.5(\sin 45°) = 1.7678 \text{ m/s} \rightarrow$

$$e = \frac{(v_{A_y})_2}{(v_{A_y})_1}; \quad 0.6 = \frac{(v_{A_y})_2}{1.7678}$$

$(v_{A_y})_2 = 1.061 \text{ m/s} \leftarrow$

$(v_{A_x})_2 = (v_{A_x})_1 = 2.5\cos 45° = 1.7678 \text{ m/s} \downarrow$




At $B$:

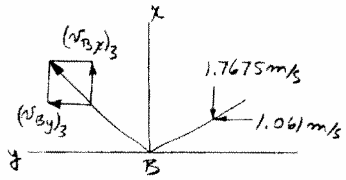
$$e = \frac{(v_{B_x})_3}{(v_{B_x})_2}; \quad 0.6 = \frac{(v_{B_x})_3}{1.7678}$$

$(v_{B_x})_3 = 1.061 \text{ m/s}$

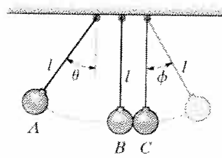
$(v_{B_y})_3 = (v_{A_y})_2 = 1.061 \text{ m/s}$

Hence,

$(v_B)_3 = \sqrt{(1.061)^2 + (1.061)^2} = 1.50 \text{ m/s}$ **Ans**




***15-80.** The three balls each have the same mass $m$. If $A$ is released from rest at $\theta$, determine the angle $\phi$ to which $C$ rises after collision. The coefficient of restitution between each ball is $e$.




$T_1 + V_1 = T_2 + V_2$

$0 + l(1 - \cos\theta)mg = \frac{1}{2}m(v_A)_2^2$

$(v_A)_2 = \sqrt{2(1 - \cos\theta)gl}$

Collision of ball $A$ with $B$:

$(\overset{+}{\rightarrow}) \quad m(v_A)_2 + 0 = mv'_A + mv'_B$

$(\overset{+}{\rightarrow}) \quad e = \dfrac{v'_B - v'_A}{(v_A)_2}$

Solving for $v'_B$:

$v'_B = \frac{1}{2}(1 + e)(v_A)_2$

Collision of ball $B$ with $C$:

$(\overset{+}{\rightarrow}) \quad mv'_B + 0 = mv''_B + mv''_C$

$(\overset{+}{\rightarrow}) \quad e = \dfrac{v''_C - v''_B}{v'_B}$

Solving for $v''_C$,

$v''_C = \frac{1}{2}(1 + e)v'_B = \frac{1}{4}(1 + e)^2(v_A)_2$

$v''_C = \frac{1}{4}(1 + e)^2[2(1 - \cos\theta)gl]^{1/2}$

$T_1 + V_1 = T_2 + V_2$

$\frac{1}{2}m(v''_C)^2 + 0 = 0 + l(1 - \cos\phi)mg$

$\frac{1}{2}(\frac{1}{16})(1 + e)^4(2)(1 - \cos\theta)gl = (1 - \cos\phi)lg$

$(\frac{1 + e}{2})^4(1 - \cos\theta) = 1 - \cos\phi$

$\phi = \cos^{-1}\left[1 - (\frac{1 + e}{2})^4(1 - \cos\theta)\right]$ **Ans**

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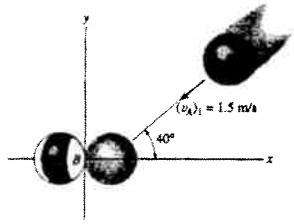
**15-81.** Two smooth billiard balls $A$ and $B$ each have a mass of 200 g. If $A$ strikes $B$ with a velocity $(v_A)_1 = 1.5$ m/s as shown, determine their final velocities just after collision. Ball $B$ is originally at rest and the coefficient of restitution is $e = 0.85$. Neglect the size of each ball.




$(v_{A_x})_1 = -1.5\cos 40° = -1.1491$ m/s

$(v_{A_y})_1 = -1.5\sin 40° = -0.9642$ m/s

$(\overset{+}{\rightarrow})$ $m_A(v_{A_x})_1 + m_B(v_{B_x})_1 = m_A(v_{A_x})_2 + m_B(v_{B_x})_2$

$-0.2(1.1491) + 0 = 0.2(v_{A_x})_2 + 0.2(v_{B_x})_2$

$(\overset{+}{\rightarrow})$ $e = \dfrac{(v_{A_x})_2 - (v_{B_x})_2}{(v_{B_x})_1 - (v_{A_x})_1}$; $\quad 0.85 = \dfrac{(v_{A_x})_2 - (v_{B_x})_2}{1.1491}$

Solving,

$(v_{A_x})_2 = -0.08618$ m/s

$(v_{B_x})_2 = -1.0629$ m/s

$(+\downarrow)$ $m_A(v_{A_y})_1 = m_A(v_{A_y})_2$

$(v_{A_y})_2 = 0.9642$ m/s

For $B$:

$(+\uparrow)$ $m_B(v_{B_y})_1 = m_B(v_{B_y})_2$

Hence, $(v_{B_y})_2 = 0$

$(v_B)_2 = (v_{B_x})_2 = 1.06$ m/s $\leftarrow$ **Ans**

$(v_A)_2 = \sqrt{(-0.08618)^2 + (0.9642)^2} = 0.968$ m/s **Ans**

$(\theta_A)_2 = \tan^{-1}\left(\dfrac{0.08618}{0.9642}\right) = 5.11°$ A **Ans**

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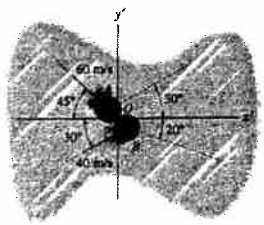
**15-82.** The two hockey pucks $A$ and $B$ each have a mass of 250 g. If they collide at $O$ and are deflected along the colored paths, determine their speeds just after impact. Assume that the icy surface over which they slide is smooth. *Hint:* Since the $y'$ axis is *not* along the line of impact, apply the conservation of momentum along the $x'$ and $y'$ axes.




$(\overset{+}{\rightarrow})$ $\Sigma m(v_{x'})_1 = \Sigma m(v_{x'})_2$

$0.25(60\cos 45°) + 0.25(40\cos 30°) = 0.25(v_A)_2\cos 30° + 0.25(v_B)_2\cos 20°$

$77.067 = 0.8660(v_A)_2 + 0.9397(v_B)_2$

$(+\uparrow)$ $\Sigma m(v_{y'})_1 = \Sigma m(v_{y'})_2$

$-0.25(60\sin 45°) + 0.25(40\sin 30°) = 0.25(v_A)_2\sin 30° - 0.25(v_B)_2\sin 20°$

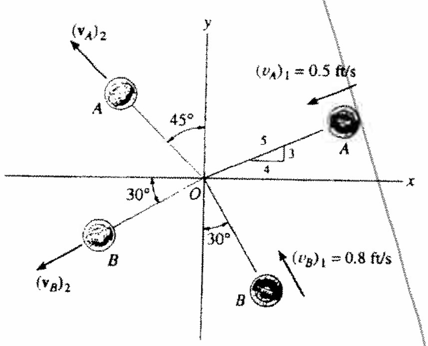
$-22.426 = 0.5(v_A)_2 - 0.3420(v_B)_2$

Solving,

$(v_A)_2 = 6.90$ m/s **Ans**

$(v_B)_2 = 75.7$ m/s **Ans**

**15-83.** Two smooth coins $A$ and $B$, each having the same mass, slide on a smooth surface with the motion shown. Determine the speed of each coin after collision if they move off along the dashed paths. *Hint:* Since the line of impact has not been defined, apply the conservation of momentum along the $x$ and $y$ axes, respectively.




$$\Sigma m v_1 = \Sigma m v_2$$

$(\overset{+}{\rightarrow})$ $\quad -m(0.8)\sin 30° - m(0.5)(\frac{4}{5}) = -m(v_A)_2 \sin 45° - m(v_B)_2 \cos 30°$

$0.8 = 0.707(v_A)_2 + 0.866(v_B)_2$

$(+\uparrow)$ $\quad m(0.8)\cos 30° - m(0.5)(\frac{3}{5}) = m(v_A)_2 \cos 45° - m(v_B)_2 \sin 30°$

$-0.3928 = -0.707(v_A)_2 + 0.5(v_B)_2$

Solving,

$(v_B)_2 = 0.298$ ft/s **Ans**

$(v_A)_2 = 0.766$ ft/s **Ans**

***15-84.** The two disks $A$ and $B$ have a mass of 3 kg and 5 kg, respectively. If they collide with the initial velocities shown, determine their velocities just after impact. The coefficient of restitution is $e = 0.65$.

$(v_{A_x})_1 = 6$ m/s $\quad (v_{A_y})_1 = 0$

$(v_{B_x})_1 = -7\cos 60° = -3.5$ m/s $\quad (v_{B_y})_1 = -7\sin 60° = -6.062$ m/s

$(\overset{+}{\rightarrow})$ $\quad m_A (v_{A_x})_1 + m_B (v_{B_x})_1 = m_A (v_{A_x})_2 + m_B (v_{B_x})_2$

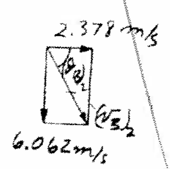
$3(6) - 5(3.5) = 3(v_A)_{x2} + 5(v_B)_{x2}$

$(\overset{+}{\rightarrow})$ $\quad e = \frac{(v_{B_x})_2 - (v_{A_x})_2}{(v_{A_x})_1 - (v_{B_x})_1}; \quad 0.65 = \frac{(v_{B_x})_2 - (v_{A_x})_2}{6 - (-3.5)}$

$(v_{B_x})_2 - (v_{A_x})_2 = 6.175$

Solving,

$(v_{A_x})_2 = -3.80$ m/s $\quad (v_{B_x})_2 = 2.378$ m/s




$(+\uparrow)$ $\quad m_A (v_{A_y})_1 = m_A (v_{A_y})_2$

$(v_{A_y})_2 = 0$

$(+\uparrow)$ $\quad m_B (v_{B_y})_1 = m_B (v_{B_y})_2$

$(v_{B_y})_2 = -6.062$ m/s

$(v_A)_2 = \sqrt{(3.80)^2 + (0)^2} = 3.80$ m/s $\leftarrow$ **Ans**

$(v_B)_2 = \sqrt{(2.378)^2 + (-6.062)^2} = 6.51$ m/s **Ans**

$(\theta_B)_2 = \tan^{-1}\left(\frac{6.062}{2.378}\right) = 68.6°$ **Ans**

**15-85.** Two smooth disks $A$ and $B$ each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine their final velocities just after collision. The coefficient of restitution is $e = 0.75$.

$(\xrightarrow{+})\quad \Sigma m v_1 = \Sigma m v_2$

$$0.5(4)(\tfrac{3}{5}) - 0.5(6) = 0.5(v_B)_{2x} + 0.5(v_A)_{2x}$$

$$(\xrightarrow{+})\quad e = \frac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$$

$$0.75 = \frac{(v_A)_{2x} - (v_B)_{2x}}{4(\frac{3}{5}) - (-6)}$$

$(v_A)_{2x} = 1.35$ m/s $\rightarrow$

$(v_B)_{2x} = 4.95$ m/s $\leftarrow$

$(+\uparrow)\quad m v_1 = m v_2$

$$0.5(\tfrac{4}{5})(4) = 0.5(v_B)_{2y}$$

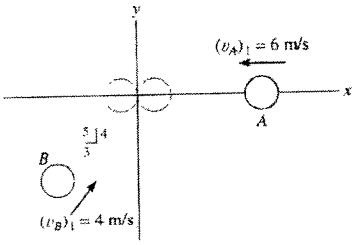
$(v_B)_{2y} = 3.20$ m/s $\uparrow$

$v_A = 1.35$ m/s $\rightarrow$ **Ans**

$v_B = \sqrt{(4.95)^2 + (3.20)^2} = 5.89$ m/s **Ans**

$\theta = \tan^{-1}\frac{3.20}{4.95} = 32.9°$ **Ans**

**15-86.** Two smooth disks $A$ and $B$ each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine the coefficient of restitution between the disks if after collision $B$ travels along a line, 30° counterclockwise from the $y$ axis.



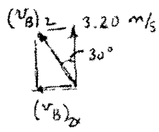

$\Sigma m v_1 = \Sigma m v_2$

$$(\xrightarrow{+})\quad 0.5(4)(\tfrac{3}{5}) - 0.5(6) = -0.5(v_B)_{2x} + 0.5(v_A)_{2x}$$

$$-3.60 = -(v_B)_{2x} + (v_A)_{2x}$$

$$(+\uparrow)\quad 0.5(4)(\tfrac{4}{5}) = 0.5(v_B)_{2y}$$

$(v_B)_{2y} = 3.20$ m/s $\uparrow$



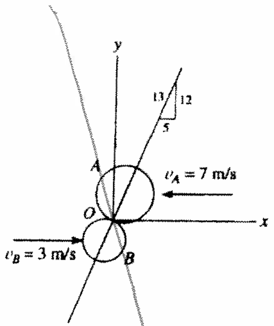

$(v_B)_{2x} = 3.20\tan 30° = 1.8475$ m/s $\leftarrow$

$(v_A)_{2x} = -1.752$ m/s $= 1.752$ m/s $\leftarrow$

$$(\xrightarrow{+})\quad e = \frac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$$

$$e = \frac{-1.752 - (-1.8475)}{4(\frac{3}{5}) - (-6)} = 0.0113$$ **Ans**

**15-87.** Two smooth disks $A$ and $B$ have the initial velocities shown just before they collide at $O$. If they have masses $m_A = 8$ kg and $m_B = 6$ kg, determine their speeds just after impact. The coefficient of restitution is $e = 0.5$.

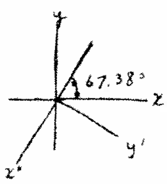



$+\swarrow \Sigma m v_1 = \Sigma m v_2$

$$-6(3\cos 67.38°) + 8(7\cos 67.38°) = 6(v_B)_{x'} + 8(v_A)_{x'}$$

$$(+\swarrow) \quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.5 = \frac{(v_B)_{x'} - (v_A)_{x'}}{7\cos 67.38° + 3\cos 67.38°}$$




Solving,

$(v_B)_{x'} = 2.14$ m/s

$(v_A)_{x'} = 0.220$ m/s

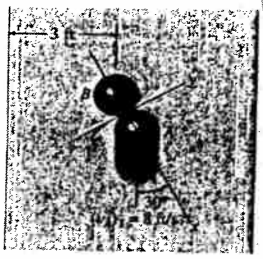
$(v_B)_{y'} = 3\sin 67.38° = 2.769$ m/s

$(v_A)_{y'} = -7\sin 67.38° = -6.462$ m/s

$v_B = \sqrt{(2.14)^2 + (2.769)^2} = 3.50$ m/s **Ans**

$v_A = \sqrt{(0.220)^2 + (6.462)^2} = 6.47$ m/s **Ans**

***15-88.** The "stone" $A$ used in the sport of curling slides over the ice track and strikes another "stone" $B$ as shown. If each "stone" is smooth and has a weight of 47 lb, and the coefficient of restitution between the "stones" is $e = 0.8$, determine their speeds just after collision. Initially $A$ has a velocity of 8 ft/s and $B$ is at rest. Neglect friction.




Line of impact ($x$ – axis) :

$\Sigma m v_1 = \Sigma m v_2$

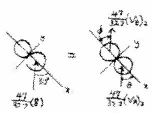
$$(+\searrow) \quad 0 + \frac{47}{32.2}(8)\cos 30° = \frac{47}{32.2}(v_B)_{2x} + \frac{47}{32.2}(v_A)_{2x}$$

$$(+\searrow) \quad e = 0.8 = \frac{(v_B)_{2x} - (v_A)_{2x}}{8\cos 30° - 0}$$

Solving :

$(v_A)_{2x} = 0.6928$ ft/s

$(v_B)_{2x} = 6.235$ ft/s




Plane of impact ($y$ – axis) :

Stone $A$ :

$m v_1 = m v_2$

$$(+\nearrow) \quad \frac{47}{32.2}(8)\sin 30° = \frac{47}{32.2}(v_A)_{2y}$$

$(v_A)_{2y} = 4$

Stone $B$ :

$m v_1 = m v_2$

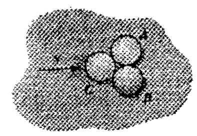
$$(+\nearrow) \quad 0 = \frac{47}{32.2}(v_B)_{2y}$$

$(v_B)_{2y} = 0$

$(v_A)_2 = \sqrt{(0.6928)^2 + (4)^2} = 4.06$ ft/s **Ans**

$(v_B)_2 = \sqrt{(0)^2 + (6.235)^2} = 6.235 = 6.24$ ft/s **Ans**

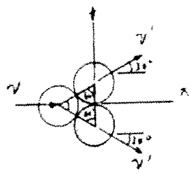
**15-89.** The two billiard balls $A$ and $B$ are originally in contact with one another when a third ball $C$ strikes each of them at the same time as shown. If ball $C$ remains at rest after the collision, determine the coefficient of restitution. All the balls have the same mass. Neglect the size of each ball.



Conservation of " $x$" momentum :

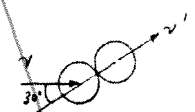
$(\overset{+}{\rightarrow})$ $\quad mv = 2mv'\cos 30°$

$$v = 2v'\cos 30° \qquad (1)$$



Coefficient of restitution :

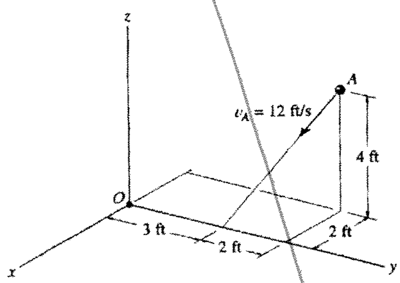
$$(+\nearrow) \quad e = \frac{v'}{v\cos 30°} \qquad (2)$$



Substituting Eq. (1) into Eq. (2) yields :

$$e = \frac{v'}{2v'\cos^2 30°} = \frac{2}{3}$$ **Ans**

**15-90.** Determine the angular momentum of the 2-lb particle $A$ about point $O$. Use a Cartesian vector solution.




$$m\mathbf{v}_A = \frac{2}{32.2}(12)\left(\frac{2}{\sqrt{24}}\mathbf{i} - \frac{2}{\sqrt{24}}\mathbf{j} - \frac{4}{\sqrt{24}}\mathbf{k}\right)$$

$$= \{0.3043\mathbf{i} - 0.3043\mathbf{j} - 0.6086\mathbf{k}\} \text{ slug}\cdot\text{ft/s}$$

$$(\mathbf{H}_A)_O = \mathbf{r}_A \times m\mathbf{v}_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 5 & 4 \\ 0.3043 & -0.3043 & -0.6086 \end{vmatrix}$$

$$= \{-1.83\mathbf{i} - 0.913\mathbf{k}\} \text{ slug}\cdot\text{ft}^2/\text{s}$$ **Ans**

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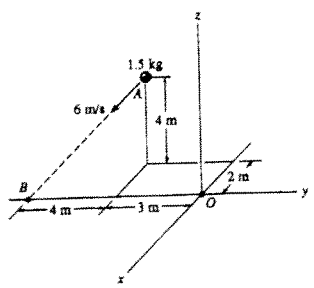
**15-91.** Determine the angular momentum $\mathbf{H}_O$ of the particle about point $O$.



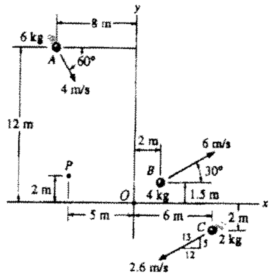

$$\mathbf{r}_{OB} = \{-7\mathbf{j}\} \text{ m} \qquad \mathbf{v}_A = 6\left(\frac{2\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}}{\sqrt{2^2 + (-4)^2 + (-4)^2}}\right) = \{2\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}\} \text{ m/s}$$

$$\mathbf{H}_O = \mathbf{r}_{OB} \times m\mathbf{v}_A$$

$$\mathbf{H}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -7 & 0 \\ 1.5(2) & 1.5(-4) & 1.5(-4) \end{vmatrix} = \{42\mathbf{i} + 21\mathbf{k}\} \text{ kg}\cdot\text{m}^2/\text{s}$$ **Ans**

44

***15-92.** Determine the angular momentum $\mathbf{H}_O$ of each of the particles about point $O$.



$(H_A)_O = 8(6)(4\sin 60°) - 12(6)(4\cos 60°) = 22.3\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans**

$(H_B)_O = -1.5(4)(6\cos 30°) + 2(4)(6\sin 30°) = -7.18\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans**

$(H_C)_O = -2(2)\left(\frac{12}{13}\right)(2.6) - 6(2)\left(\frac{5}{13}\right)(2.6) = -21.6\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans**

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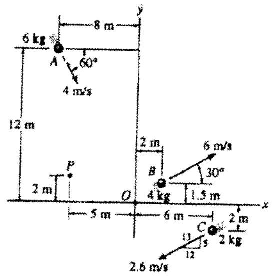
**15-93.** Determine the angular momentum $\mathbf{H}_P$ of each of the particles about point $P$.



$(H_A)_P = -6(4\cos 60°)(10) + 6(4\sin 60°)(3) = -57.6\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans**

$(H_B)_P = 4(6\cos 30°)(0.5) + 4(6\sin 30°)(7) = 94.4\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans**

$(H_C)_P = -2(2.6)\left(\frac{5}{13}\right)(11) - 2(2.6)\left(\frac{12}{13}\right)(4) = -41.2\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans**

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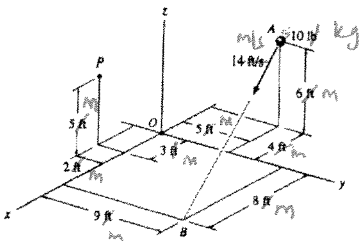
**15-94.** Determine the angular momentum $\mathbf{H}_O$ of the particle about point $O$.



$\mathbf{r}_{OB} = \{8\mathbf{i} + 9\mathbf{j}\}$ ~~ft~~ m $\qquad \mathbf{v}_A = 14\left(\frac{12\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}}{\sqrt{12^2 + 4^2 + (-6)^2}}\right) = \{12\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}\}$ ~~ft/s~~ m/s

$\mathbf{H}_O = \mathbf{r}_{OB} \times m\mathbf{v}_A$

$$\mathbf{H}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 9 & 0 \\ \left(\frac{10}{32.2}\right)(12) & \left(\frac{10}{32.2}\right)(4) & \left(\frac{10}{32.2}\right)(-6) \end{vmatrix}$$

~~$= \{-16.8\mathbf{i} + 14.9\mathbf{j} - 23.6\mathbf{k}\}$ slug · ft²/s~~ **Ans**

$-540i + 480j - 760k$ kg m²/s

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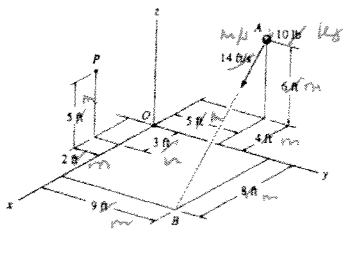
**15-95.** Determine the angular momentum $\mathbf{H}_P$ of the particle about point $P$.



$\mathbf{r}_{PB} = \{5\mathbf{i} + 11\mathbf{j} - 5\mathbf{k}\}$ ~~ft~~ m

$\mathbf{v}_A = 14\left(\frac{12\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}}{\sqrt{(12)^2 + (4)^2 + (-6)^2}}\right) = \{12\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}\}$ ~~ft/s~~ m/s

$$\mathbf{H}_P = \mathbf{r}_{PB} \times m\mathbf{v}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 11 & -5 \\ \left(\frac{10}{32.2}\right)(12) & \left(\frac{10}{32.2}\right)(4) & \left(\frac{10}{32.2}\right)(-6) \end{vmatrix}$$

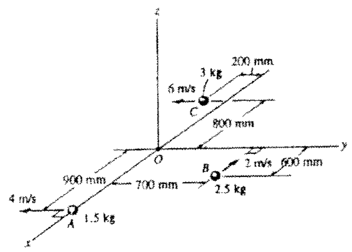
~~$= \{-14.3\mathbf{i} - 9.32\mathbf{j} - 34.8\mathbf{k}\}$ slug · ft²/s~~ **Ans**

~~$-460i + 300j -$ 1120k kg m²/s~~



$-460i - 300j - 1120k$ kg m²/s

***15-96.** Determine the total angular momentum $\mathbf{H}_O$ for the system of three particles about point $O$. All the particles are moving in the $x$–$y$ plane.

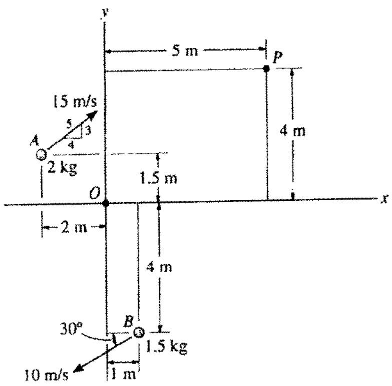



$\mathbf{H}_O = \Sigma \mathbf{r} \times m\mathbf{v}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 0 & 0 \\ 0 & -1.5(4) & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 & 0.7 & 0 \\ -2.5(2) & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.8 & -0.2 & 0 \\ 0 & 3(-6) & 0 \end{vmatrix}$$

$= \{12.5\mathbf{k}\}\ \text{kg} \cdot \text{m}^2/\text{s}$ **Ans**

**15-97.** Determine the angular momentum $\mathbf{H}_O$ of each of the two particles about point $O$. Use a scalar solution.



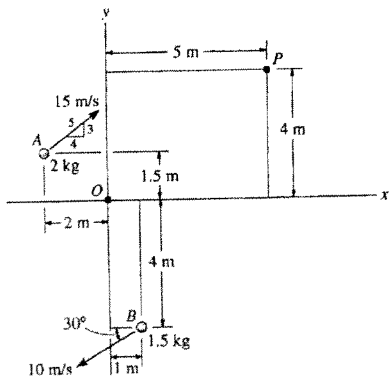

$\circlearrowleft +(H_A)_O = -2(15)\left(\frac{4}{5}\right)(1.5) - 2(15)\left(\frac{3}{5}\right)(2)$

$= -72.0\ \text{kg} \cdot \text{m}^2/\text{s} = 72.0\ \text{kg} \cdot \text{m}^2/\text{s} \circlearrowright$ **Ans**

$\circlearrowleft +(H_B)_O = -1.5(10)(\cos 30°)(4) - 1.5(10)(\sin 30°)(1)$

$= -59.5\ \text{kg} \cdot \text{m}^2/\text{s} = 59.5\ \text{kg} \cdot \text{m}^2/\text{s} \circlearrowright$ **Ans**

**15-98.** Determine the angular momentum $\mathbf{H}_P$ of each of the two particles about point $P$. Use a scalar solution.



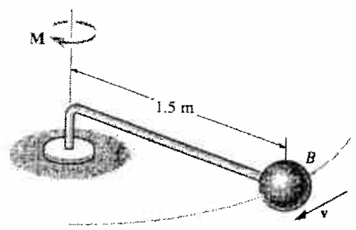

$\circlearrowleft +(H_A)_P = 2(15)\left(\frac{4}{5}\right)(2.5) - 2(15)\left(\frac{3}{5}\right)(7)$

$= -66.0\ \text{kg} \cdot \text{m}^2/\text{s} = 66.0\ \text{kg} \cdot \text{m}^2/\text{s} \circlearrowright$ **Ans**

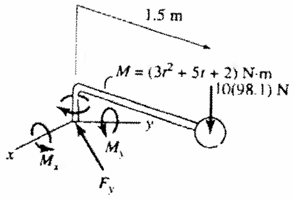
$\circlearrowleft +(H_B)_P = -1.5(10)(\cos 30°)(8) + 1.5(10)(\sin 30°)(4)$

$= -73.9\ \text{kg} \cdot \text{m}^2/\text{s} = 73.9\ \text{kg} \cdot \text{m}^2/\text{s} \circlearrowright$ **Ans**

**15-99.** The ball $B$ has a mass of 10 kg and is attached to the end of a rod whose mass may be neglected. If the rod is subjected to a torque $M = (3t^2 + 5t + 2)$ N · m, where $t$ is in seconds, determine the speed of the ball when $t = 2$ s. The ball has a speed $v = 2$ m/s when $t = 0$.




***Principle of Angular Impluse and Momentum:*** Applying Eq. 15-22, we have

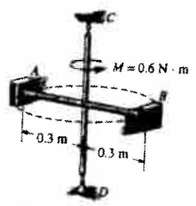



$$(H_z)_1 + \sum \int_{t_1}^{t_2} M_z dt = (H_z)_2$$

$$1.5(10)(2) + \int_0^{2s} (3t^2 + 5t + 2)dt = 1.5(10)v$$

$v = 3.47$ m/s **Ans**

***15-100.** The two blocks $A$ and $B$ each have a mass of 400 g. The blocks are fixed to the horizontal rods, and their initial velocity is 2 m/s in the direction shown. If a couple moment of $M = 0.6$ N·m is applied about $CD$ of the frame, determine the speed of the blocks when $t = 3$ s. The mass of the frame is negligible, and it is free to rotate about $CD$. Neglect the size of the blocks.

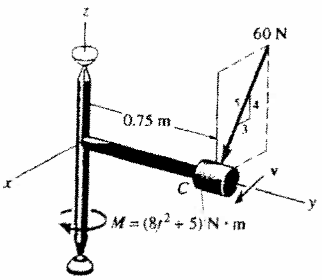


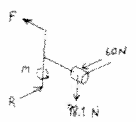

$$(H_O)_1 + \Sigma \int_{t_1}^{t_2} M_O dt = (H_O)_2$$

$$2[0.3(0.4)(2)] + 0.6(3) = 2[0.3(0.4)v]$$

$v = 9.50$ m/s **Ans**

**15-101.** The small cylinder $C$ has a mass of 10 kg and is attached to the end of a rod whose mass may be neglected. If the frame is subjected to a couple $M = (8t^2 + 5)$ N · m, where $t$ is in seconds, and the cylinder is subjected to a force of 60 N, which is always directed as shown, determine the speed of the cylinder when $t = 2$ s. The cylinder has a speed $v_0 = 2$ m/s when $t = 0$.



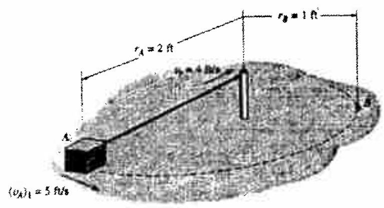

$$(H_z)_1 + \Sigma \int M_z\, dt = (H_z)_2$$

$$(10)(2)(0.75) + 60(2)(\tfrac{3}{5})(0.75) + \int_0^2 (8t^2 + 5)dt = 10v(0.75)$$

$$69 + [\tfrac{8}{3}t^3 + 5t]_0^2 = 7.5v$$

$v = 13.4$ m/s **Ans**

**15-102.** A box having a weight of 8 lb is moving around in a circle of radius $r_A = 2$ ft with a speed of $(v_A)_1 = 5$ ft/s while connected to the end of a rope. If the rope is pulled inward with a constant speed of $v_r = 4$ ft/s, determine the speed of the box at the instant $r_B = 1$ ft. How much work is done after pulling in the rope from $A$ to $B$? Neglect friction and the size of the box.




$(H_z)_A = (H_z)_B;\quad \left(\frac{8}{32.2}\right)(2)(5) = \left(\frac{8}{32.2}\right)(1)(v_B)_{tangent}$

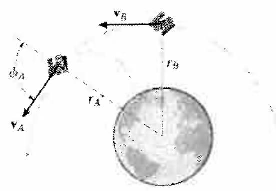
$(v_B)_{tangent} = 10$ ft/s

$v_B = \sqrt{(10)^2 + (4)^2} = 10.77 = 10.8$ ft/s **Ans**

$\sum U_{AB} = T_B - T_A \qquad U_{AB} = \frac{1}{2}\left(\frac{8}{32.2}\right)(10.77)^2 - \frac{1}{2}\left(\frac{8}{32.2}\right)(5)^2$

$U_{AB} = 11.3$ ft · lb **Ans**

**15–103.** An earth satellite of mass 700 kg is launched into a free-flight trajectory about the earth with an initial speed of $v_A = 10$ km/s when the distance from the center of the earth is $r_A = 15$ Mm. If the launch angle at this position is $\phi_A = 70°$, determine the speed $v_B$ of the satellite and its closest distance $r_B$ from the center of the earth. The earth has a mass $M_e = 5.976(10^{24})$ kg. *Hint:* Under these conditions, the satellite is subjected only to the earth's gravitational force, $F = GM_e m_s/r^2$, Eq. 13–1. For part of the solution, use the conservation of energy.




$(H_O)_1 = (H_O)_2$

$m_s(v_A \sin\phi_A)r_A = m_s(v_B)r_B$

$700[10(10^3)\sin 70°](15)(10^6) = 700(v_B)(r_B) \qquad (1)$

$T_A + V_A = T_B + V_B$

$$\frac{1}{2}m_s(v_A)^2 - \frac{GM_e m_s}{r_A} = \frac{1}{2}m_s(v_B)^2 - \frac{GM_e m_s}{r_B}$$

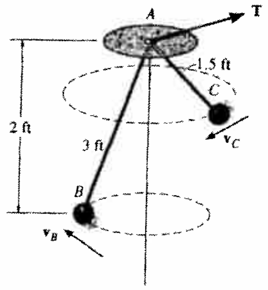
$$\frac{1}{2}(700)[10(10^3)]^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{[15(10^6)]} = \frac{1}{2}(700)(v_B)^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{r_B} \qquad (2)$$

Solving,

$v_B = 10.2$ km/s **Ans**

$r_B = 13.8$ Mm **Ans**

***15-104.** The ball $B$ has a weight of 5 lb and is originally rotating in a circle. As shown, the cord $AB$ has a length of 3 ft and passes through the hole $A$, which is 2 ft above the plane of motion. If 1.5 ft of cord is pulled through the hole, determine the speed of the ball when it moves in a circular path at $C$.




*Equation of Motion :* When the ball is travelling around the first circular path, $\theta = \sin^{-1}\frac{2}{3} = 41.81°$ and $r_1 = 3\cos 41.81° = 2.236$. Applying Eq. 13 – 8, we have

$$\Sigma F_b = 0: \qquad T_1\left(\frac{2}{3}\right) - 5 = 0 \qquad T_1 = 7.50 \text{ lb}$$

$$\Sigma F_n = ma_n: \qquad 7.50\cos 41.81° = \frac{5}{32.2}\left(\frac{v_1^2}{2.236}\right)$$

$$v_1 = 8.972 \text{ ft/s}$$

When the ball is traveling around the second circular path, $r_2 = 1.5\cos\phi$. Applying Eq. 13 – 8, we have

$$\Sigma F_b = 0: \qquad T_2 \sin\phi - 5 = 0 \qquad [1]$$

$$\Sigma F_n = ma_n: \qquad T_2\cos\phi = \frac{5}{32.2}\left(\frac{v_2^2}{1.5\cos\phi}\right) \qquad [2]$$

*Conservation of Angular Momentum :* Since no force acts on the ball along the tangent of the circular path, the angular momentum is conserved about $z$ axis. Applying Eq. 15 – 23, we have

$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$$

$$r_1 m v_1 = r_2 m v_2$$

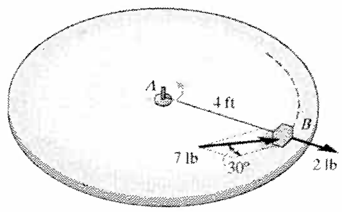
$$2.236\left(\frac{5}{32.2}\right)(8.972) = 1.5\cos\phi\left(\frac{5}{32.2}\right)v_2 \qquad [3]$$

Solving Eqs.[1], [2] and [3] yields

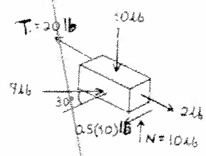
$$\phi = 13.8768° \qquad T_2 = 20.85 \text{ lb}$$

$$v_2 = 13.8 \text{ ft/s} \qquad \textbf{Ans}$$

**15-105.** The 10-lb block rests on a surface for which $\mu_k = 0.5$. It is acted upon by a radial force of 2 lb and a horizontal force of 7 lb, always directed at 30° from the tangent to the path as shown. If the block is initially moving in a circular path with a speed $v_1 = 2$ ft/s at the instant the forces are applied, determine the time required before the tension in cord $AB$ becomes 20 lb. Neglect the size of the block for the calculation.




$$\Sigma F_n = ma_n;$$




$$20 - 7\sin 30° - 2 = \frac{10}{32.2}\left(\frac{v^2}{4}\right)$$

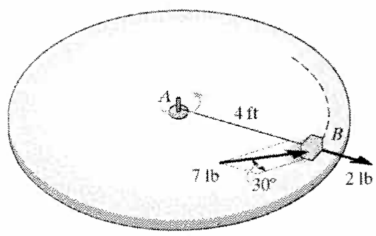
$$v = 13.67 \text{ ft/s}$$

$$(H_A)_1 + \Sigma\int M_A\,dt = (H_A)_2$$

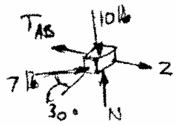
$$\left(\frac{10}{32.2}\right)(2)(4) + (7\cos 30°)(4)(t) - 0.5(10)(4)t = \frac{10}{32.2}(13.67)(4)$$

$$t = 3.41 \text{ s} \qquad \textbf{Ans}$$

**15–106.** The 10-lb block is originally at rest on the smooth surface. It is acted upon by a radial force of 2 lb and a horizontal force of 7 lb, always directed at 30° from the tangent to the path as shown. Determine the time required to break the cord, which requires a tension $T = 30$ lb. What is the speed of the block when this occurs? Neglect the size of the block for the calculation.




$\Sigma F_n = ma_n;$




$$30 - 7\sin 30° - 2 = \frac{10}{32.2}\left(\frac{v^2}{4}\right)$$

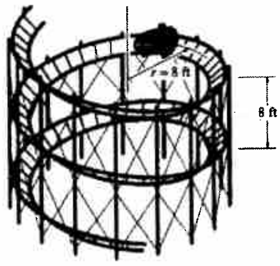
$v = 17.764$ ft/s

$$(H_A)_1 + \Sigma\int M_A\,dt = (H_A)_2$$

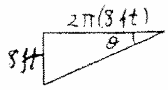
$$0 + (7\cos 30°)(4)(t) = \frac{10}{32.2}(17.764)(4)$$

$t = 0.910$ s **Ans**

**15-107.** The 800-lb roller-coaster car starts from rest on the track having the shape of a cylindrical helix. If the helix descends 8 ft for every one revolution, determine the time required for the car to attain a speed of 60 ft/s. Neglect friction and the size of the car.

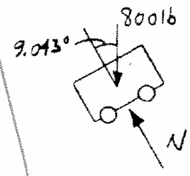



$$\theta = \tan^{-1}\left(\frac{8}{2\pi(8)}\right) = 9.043°$$




$\Sigma F_y = 0; \quad N - 800\cos 9.043° = 0$

$N = 790.1$ lb

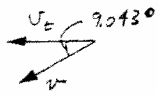



$$v = \frac{v_t}{\cos 9.043°}$$

$$60 = \frac{v_t}{\cos 9.043°}$$

$v_t = 59.254$ ft/s

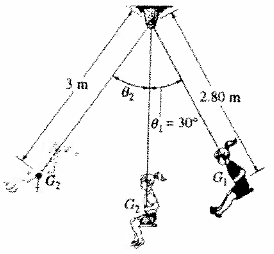
$$H_1 + \int M\,dt = H_2$$




$$0 + \int_0^t 8(790.1\sin 9.043°)dt = \frac{800}{32.2}(8)(59.254)$$

$t = 11.9$ s **Ans**

***15-108.** A child having a mass of 50 kg holds her legs up as shown as she swings downward from rest at $\theta_1 = 30°$. Her center of mass is located at point $G_1$. When she is at the bottom position $\theta = 0°$, she *suddenly* lets her legs come down, shifting her center of mass to position $G_2$. Determine her speed in the upswing due to this sudden movement and the angle $\theta_2$ to which she swings before momentarily coming to rest. Treat the child's body as a particle.




First before $\theta = 0°$:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2.80(1 - \cos 30°)(50)(9.81) = \frac{1}{2}(50)(v_1)^2 + 0$$

$$v_1 = 2.713 \text{ m/s}$$

$$H_1 = H_2$$

$$50(2.713)(2.80) = 50(v_2)(3)$$

$$v_2 = 2.532 = 2.53 \text{ m/s} \qquad \textbf{Ans}$$

Just after $\theta = 0°$:

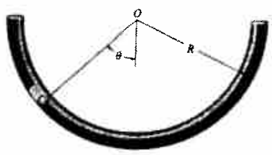
$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}(50)(2.532)^2 + 0 = 0 + 50(9.81)(3)(1 - \cos\theta_2)$$

$$0.1089 = 1 - \cos\theta_2$$

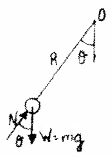
$$\theta_2 = 27.0° \qquad \textbf{Ans}$$

**15-109.** A small particle having a mass $m$ is placed inside the semicircular tube. The particle is placed at the position shown and released. Apply the principle of angular momentum about point $O$ ($\Sigma M_O = \dot{H}_O$), and show that the motion of the particle is governed by the differential equation $\ddot{\theta} + (g/R)\sin\theta = 0$.




$$\curvearrowleft + \Sigma M_O = \frac{dH_O}{dt}; \qquad -Rmg\sin\theta = \frac{d}{dt}(mvR)$$

$$g\sin\theta = -\frac{dv}{dt} = -\frac{d^2s}{dt^2}$$

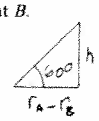


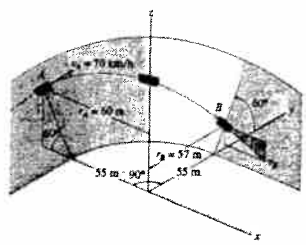

But, $s = R\theta$

Thus, $g\sin\theta = -R\ddot{\theta}$

or, $\ddot{\theta} + \left(\frac{g}{R}\right)\sin\theta = 0$ **Q.E.D.**

**15-110.** A toboggan and rider, having a total mass of 150 kg, enter horizontally tangent to a 90° circular curve with a velocity of $v_A = 70$ km/h. If the track is flat and banked at an angle of 60°, determine the speed $v_B$ and the angle $\theta$ of "descent," measured from the horizontal in a vertical $x$–$z$ plane, at which the toboggan exits at $B$. Neglect friction in the calculation.




$$v_A = 70 \text{ km/h} = 19.44 \text{ m/s}$$

$$(H_A)_z = (H_B)_z$$

$$150(19.44)(60) = 150(v_B)\cos\theta(57) \qquad (1)$$

Datum at $B$:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(150)(19.44)^2 + 150(9.81)h = \frac{1}{2}(150)(v_B)^2 + 0 \qquad (2)$$

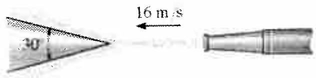
Since $h = (r_A - r_B)\tan 60° = (60 - 57)\tan 60° = 5.196$

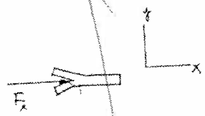
Solving Eq. (1) and Eq.(2):

$$v_B = 21.9 \text{ m/s} \qquad \textbf{Ans}$$

$$\theta = 20.9° \qquad \textbf{Ans}$$

**15–111.** Water is discharged at 16 m/s against the fixed cone diffuser. If the opening diameter of the nozzle is 40 mm, determine the horizontal force exerted by the water on the diffuser. $\rho_w = 1\ \text{Mg/m}^3$.



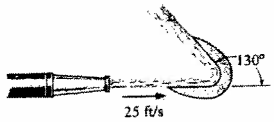

$$Q = Av = \frac{\pi}{4}(0.04)^2(16) = 0.02011\ \text{m}^3/\text{s}$$

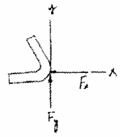
$$\frac{dm}{dt} = \rho Q = (1000)(0.02011) = 20.11\ \text{kg/s}$$

$$\xrightarrow{+} \Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax}); \qquad F_x = 20.11[-16\cos 15° - (-16)]$$

$$F_x = 11.0\ \text{N} \qquad \textbf{Ans}$$

***15–112.** A jet of water having a cross-sectional area of 4 in² strikes the fixed blade with a speed of 25 ft/s. Determine the horizontal and vertical components of force which the blade exerts on the water. $\gamma_w = 62.4\ \text{lb/ft}^3$.




$$Q = Av = \left(\frac{4}{144}\right)(25) = 0.6944\ \text{ft}^3/\text{s}$$

$$\frac{dm}{dt} = \rho Q = \left(\frac{62.4}{32.2}\right)(0.6944) = 1.3458\ \text{slug/s}$$

$v_{Ax} = 25$ ft/s $\qquad v_{Ay} = 0$

$v_{Bx} = -25\cos 50°$ ft/s $\qquad v_{By} = 25\sin 50°$ ft/s

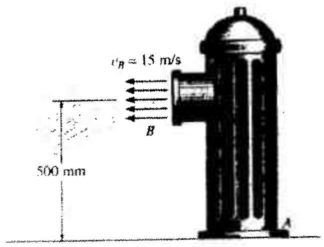
$$\xrightarrow{+} \Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax}); \qquad -F_x = 1.3458[-25\cos 50° - 25]$$

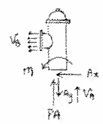
$$F_x = 55.3\ \text{lb} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = \frac{dm}{dt}(v_{By} - v_{Ay}); \qquad F_y = 1.3458(25\sin 50° - 0)$$

$$F_y = 25.8\ \text{lb} \qquad \textbf{Ans}$$

**15–113.** Water is flowing from the 150-mm-diameter fire hydrant with a velocity $v_B = 15$ m/s. Determine the horizontal and vertical components of force and the moment developed at the base joint $A$, if the static (gauge) pressure at $A$ is 50 kPa. The diameter of the fire hydrant at $A$ is 200 mm. $\rho_w = 1\ \text{Mg/m}^3$.




$$\frac{dm}{dt} = \rho v_A A_B = 1000(15)(\pi)(0.075)^2$$

$$\frac{dm}{dt} = 265.07\ \text{kg/s}$$

$$v_A = \left(\frac{dm}{dt}\right)\frac{1}{\rho A_A} = \frac{265.07}{1000(\pi)(0.1)^2}$$

$$v_A = 8.4375\ \text{m/s}$$

$$\xleftarrow{+} \Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax})$$

$$A_x = 265.07(15 - 0) = 3.98\ \text{kN} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = \frac{dm}{dt}(v_{By} - v_{Ay})$$

$$-A_y + 50(10^3)(\pi)(0.1)^2 = 265.07(0 - 8.4375)$$

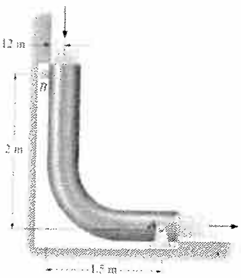
$$A_y = 3.81\ \text{kN} \qquad \textbf{Ans}$$

$$\circlearrowleft + \Sigma M_A = \frac{dm}{dt}(d_{AB}v_B - d_{AA}v_A)$$

$$M = 265.07(0.5(15) - 0)$$

$$M = 1.99\ \text{kN}\cdot\text{m} \qquad \textbf{Ans}$$

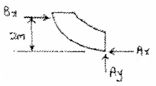
**15–114.** The chute is used to divert the flow of water $Q = 0.6\ \text{m}^3/\text{s}$. If the water has a cross-sectional area of $0.05\ \text{m}^2$ determine the force components at the pin $A$ and roller $B$ necessary for equilibrium. Neglect both the weight of the chute and the weight of the water on the chute. $\rho_w = 1\ \text{Mg/m}^3$.




$\frac{dm}{dt} = \rho_w Q = (1000)(0.6)$

$\frac{dm}{dt} = 600\ \text{kg/s}$

$v_B = v_A = \frac{Q}{A} = \frac{0.6}{0.05} = 12\ \text{m/s}$




$\xrightarrow{+} \Sigma F_x = \frac{dm}{dt}(v_{Ax} - v_{Bx})$

$B_x - A_x = 600(12 - 0) = 7200$

$+\uparrow \Sigma F_y = \frac{dm}{dt}(v_{Ay} - v_{By})$

$A_y = 600(0 - (-12)) = 7.20\ \text{kN}$ **Ans**

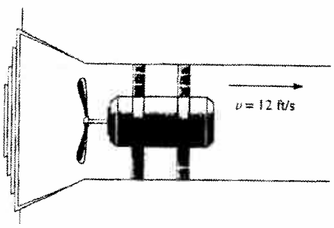
$\circlearrowleft + \Sigma M_A = \frac{dm}{dt}(-d_{AB} v_B)$

$B_x(2) = 600[(1.5 - 0.12)(12)]$

$B_x = 4.97\ \text{kN}$ **Ans**

Thus, $A_x = 2.23\ \text{kN}$ **Ans**

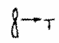
**15-115.** The fan draws air through a vent with a speed of 12 ft/s. If the cross-sectional area of the vent is 2 ft², determine the horizontal thrust on the blade. The specific weight of the air is $\gamma_a = 0.076\ \text{lb/ft}^3$.




$\frac{dm}{dt} = \rho v A$

$= \frac{0.076}{32.2}(12)(2)$

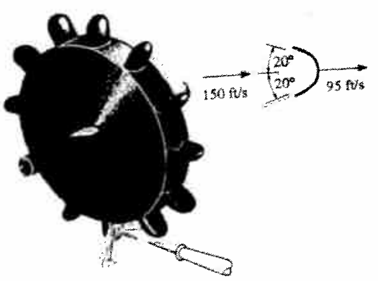
$= 0.05665\ \text{slug/s}$

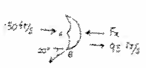



$\Sigma F = \frac{dm}{dt}(v_B - v_A)$

$T = 0.05665(12 - 0) = 0.680\ \text{lb}$ **Ans**

***15-116.** The buckets on the *Pelton wheel* are subjected to a 2-in-diameter jet of water, which has a velocity of 150 ft/s. If each bucket is traveling at 95 ft/s when the water strikes it, determine the power developed by the wheel. $\gamma_w = 62.4\ \text{lb/ft}^3$.




$v_A = 150 - 95 = 55\ \text{ft/s} \rightarrow$

$(\xrightarrow{+})(v_B)_x = -55\cos 20° + 95 = 43.317\ \text{ft/s} \rightarrow$

$\xrightarrow{+} \Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax})$

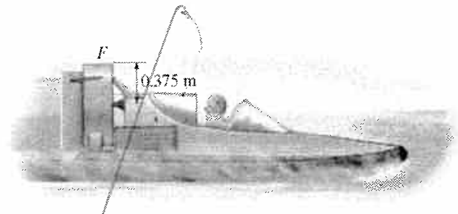
$F_x = (\frac{62.4}{32.2})(\pi)(\frac{1}{12})^2(55)(-43.317 - (-55))$

$F_x = 27.1667\ \text{lb}$

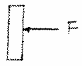
$P = 27.1667(95) = 2580.835\ \text{ft·lb/s}$

$P = 4.69\ \text{hp}$ **Ans**

**15–117.** The 200-kg boat is powered by a fan $F$ which develops a slipstream having a diameter of 0.75 m. If the fan ejects air with a speed of 14 m/s, measured relative to the boat, determine the initial acceleration of the boat if it is initially at rest. Assume that air has a constant density $\rho_a = 1.22\ \text{kg/m}^3$ and that the entering air is essentially at rest. Neglect the drag resistance of the water.

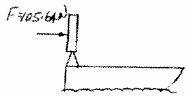



$$Q = Av = \frac{\pi}{4}(0.75)^2(14) = 6.1850\ \text{m}^3/\text{s}$$




$$\frac{dm}{dt} = \rho Q = (1.22)(6.1850) = 7.5457\ \text{kg/s}$$

$$\overset{+}{\leftarrow} \Sigma F_x = \frac{dm}{dt}(v_{B_x} - v_{A_x}); \quad F = 7.5457[14 - 0] \quad F = 105.64\ \text{N}$$




$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \quad 105.64 = 200a \quad a = 0.528\ \text{m/s}^2 \quad \textbf{Ans}$$

**15–118.** The rocket car has a mass of 3 Mg (empty) and carries 150 kg of fuel. If the fuel is consumed at a constant rate of 4 kg/s and ejected from the car with a relative velocity of 250 m/s, determine the maximum speed attained by the car starting from rest. The drag resistance due to the atmosphere is $F_D = (60v^2)$ N, where $v$ is the speed measured in m/s.

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

At time $t$ the mass of the car is

$$m_0 - ct$$

$$\text{Where } c = \frac{dm_e}{dt} = 4\ \text{kg/s}$$

Set $F = kv^2$, then

$$-kv^2 = (m_0 - ct)\frac{dv}{dt} - v_{D/e}c$$

$$\int_0^v \frac{dv}{(cv_{D/e} - kv^2)} = \int_0^t \frac{dt}{(m_0 - ct)}$$

$$\frac{1}{2\sqrt{cv_{D/e}k}}\ln\left(\frac{\sqrt{\frac{cv_{D/e}}{k}} + v}{\sqrt{\frac{cv_{D/e}}{k}} - v}\right)\Bigg|_0^v = -\frac{1}{c}\ln(m_0 - ct)\Bigg|_0^t$$

$$\frac{1}{2\sqrt{cv_{D/e}k}}\ln\left(\frac{\sqrt{\frac{cv_{D/e}}{k}} + v}{\sqrt{\frac{cv_{D/e}}{k}} - v}\right) = -\frac{1}{c}\ln\left(\frac{m_0 - ct}{m_0}\right)$$

Maximum speed occurs at the instant the fuel runs out.

$$t = \frac{150}{4} = 37.5\ \text{s}$$

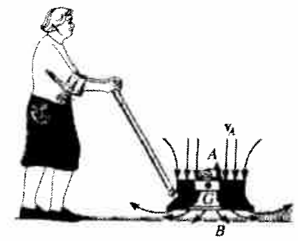
Thus;

$$\frac{1}{2\sqrt{4(250)(60)}}\ln\left(\frac{\sqrt{\frac{4(250)}{60}} + v}{\sqrt{\frac{4(250)}{60}} - v}\right) = -\frac{1}{6}\ln\left(\frac{3150 - 4(37.5)}{3150}\right)$$

Solving,

$$v = 3.93\ \text{m/s} \qquad \textbf{Ans}$$

**15-119.** A power lawn mower hovers very close over the ground. This is done by drawing air in at a speed of 6 m/s through an intake unit $A$, which has a cross-sectional area of $A_A = 0.25\ \text{m}^2$, and then discharging it at the ground, $B$, where the cross-sectional area is $A_B = 0.35\ \text{m}^2$. If air at $A$ is subjected only to atmospheric pressure, determine the air pressure which the lawn mower exerts on the ground when the weight of the mower is freely supported and no load is placed on the handle. The mower has a mass of 15 kg with center of mass at $G$. Assume that air has a constant density of $\rho_a = 1.22\ \text{kg/m}^3$.

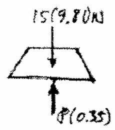



$$\frac{dm}{dt} = \rho A_A v_A = 1.22(0.25)(6) = 1.83\ \text{kg/s}$$

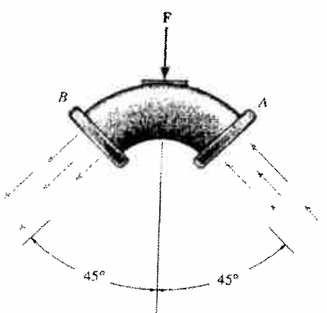
$$+\uparrow \Sigma F_y = \frac{dm}{dt}((v_B)_y - (v_A)_y)$$

$$p(0.35) - 15(9.81) = 1.83(0 - (-6))$$

$$p = 452\ \text{Pa} \qquad \textbf{Ans}$$




***15-120.** The elbow for a 5-in-diameter buried pipe is subjected to a static pressure of 10 lb/in². The speed of the water passing through it is $v = 8$ ft/s. Assuming the pipe connection at $A$ and $B$ do not offer any vertical force resistance on the elbow, determine the resultant vertical force **F** that the soil must then exert on the elbow in order to hold it in equilibrium. Neglect the weight of the elbow and the water within it. $\gamma_w = 62.4\ \text{lb/ft}^3$.




***Equations of Steady Flow*** : Here, $Q = vA = 8\left[\frac{\pi}{4}\left(\frac{5}{12}\right)^2\right] = 1.091\ \text{ft}^3/\text{s}$. Then, the mass flow rate is $\frac{dm}{dt} = \rho_w Q = \frac{62.4}{32.2}(1.091) = 2.114\ \text{slug/s}$

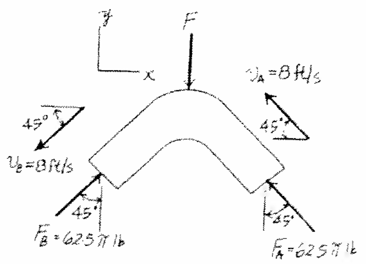
Also, the force induced by the water pressure at $A$ is $F = pA = 10\left[\frac{\pi}{4}(5^2)\right] = 62.5\pi$ lb. Applying Eq. 15–26, we have

$$\Sigma F_y = \frac{dm}{dt}(v_{B_y} - v_{A_y});$$

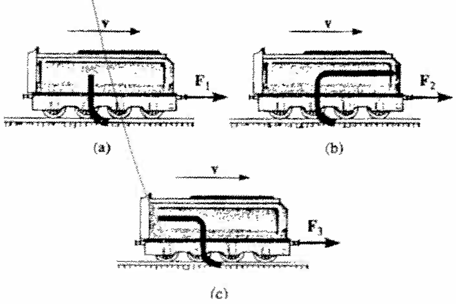
$$-F + 2(62.5\pi\cos 45°) = 2.114(-8\sin 45° - 8\sin 45°)$$
$$F = 302\ \text{lb} \qquad \textbf{Ans}$$

$$\Sigma F_x = \frac{dm}{dt}(v_{B_x} - v_{A_x});$$

$$62.5\pi\sin 45° - 62.5\pi\sin 45° = 2.114[-8\cos 45° - (-8\cos 45°)]$$
$$0 = 0 \qquad (\textit{Check!})$$




**15-121.** The car is used to scoop up water that is lying in a trough at the tracks. Determine the force needed to pull the car forward at constant velocity **v** for each of the three cases. The scoop has a cross-sectional area $A$ and the density of water is $\rho_w$.




The system consists of the car and the scoop. In all cases

$$\Sigma F_t = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

$$F = 0 - v(\rho)(A)v$$

$$F = v^2\rho A \qquad \textbf{Ans}$$

**15–122.** A rocket has an empty weight of 500 lb and carries 300 lb of fuel. If the fuel is burned at the rate of 15 lb/s and ejected with a relative velocity of 4400 ft/s, determine the maximum speed attained by the rocket starting from rest. Neglect the effect of gravitation on the rocket.

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

At a time $t$, $m = m_0 - ct$, where $c = \frac{dm_e}{dt}$. In space the weight of the rocket is zero.

$$0 = (m_0 - ct)\frac{dv}{dt} - v_{D/e}c$$

$$\int_0^v dv = \int_0^t \left(\frac{cv_{D/e}}{m_0 - ct}\right)dt$$

$$v = v_{D/e}\ln\left(\frac{m_0}{m_0 - ct}\right) \qquad [1]$$

The maximum speed occurs when all the fuel is consumed, that is, when $t = \frac{300}{15} = 20$ s.

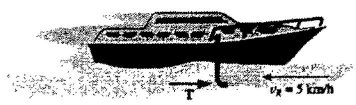
Here, $m_0 = \frac{500+300}{32.2} = 24.8447$ slug, $c = \frac{15}{32.2} = 0.4658$ slug/s, $v_{D/e} = 4400$ ft/s.

Substitute the numerical values into Eq.[1] :

$$v_{max} = 4400\ln\left(\frac{24.8447}{24.8447 - 0.4658(20)}\right)$$

$v_{max} = 2068$ ft/s **Ans**

**15-123.** The boat has a mass of 180 kg and is traveling forward on a river with a constant velocity of 70 km/h, measured *relative* to the river. The river is flowing in the opposite direction at 5 km/h. If a tube is placed in the water, as shown, and it collects 40 kg of water in the boat in 80 s, determine the horizontal thrust $T$ on the tube that is required to overcome the resistance to the water collection. $\rho_w = 1$ Mg/m$^3$.




$$\frac{dm}{dt} = \frac{40}{80} = 0.5 \text{ kg/s}$$

$$v_{D/i} = (70)\left(\frac{1000}{3600}\right) = 19.444 \text{ m/s}$$

$$\Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$T = 0 + 19.444(0.5) = 9.72$ N **Ans**

***15-124.** The second stage of the two-stage rocket weighs 2000 lb (empty) and is launched from the first stage with a velocity of 3000 mi/h. The fuel in the second stage weighs 1000 lb. If it is consumed at the rate of 50 lb/s and ejected with a relative velocity of 8000 ft/s, determine the acceleration of the second stage just after the engine is fired. What is the rocket's acceleration after before all the fuel is consumed? Neglect the effect of gravitation.

Initially,

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\left(\frac{dm_e}{dt}\right)$$

$$0 = \frac{3000}{32.2}a - 8000\left(\frac{50}{32.2}\right)$$

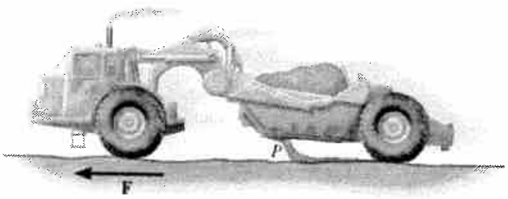
$a = 133 \text{ ft/s}^2$ **Ans**

Finally,

$$0 = \frac{2000}{32.2}a - 8000\left(\frac{50}{32.2}\right)$$

$a = 200 \text{ ft/s}^2$ **Ans**

**15–125.** The earthmover initially carries 10 m³ of sand having a density of 1520 kg/m³. The sand is unloaded horizontally through a 2.5-m² dumping port $P$ at a rate of 900 kg/s measured relative to the port. If the earthmover maintains a constant resultant tractive force $F = 4$ kN at its front wheels to provide forward motion, determine its acceleration when half the sand is dumped. When empty, the earthmover has a mass of 30 Mg. Neglect any resistance to forward motion and the mass of the wheels. The rear wheels are free to roll.




When half the sand remains,

$$m = 30\,000 + \frac{1}{2}(10)(1520) = 37\,600 \text{ kg}$$

$$\frac{dm}{dt} = 900 \text{ kg/s} = \rho\, v_{D/e} A$$

$$900 = 1520(v_{D/e})(2.5)$$

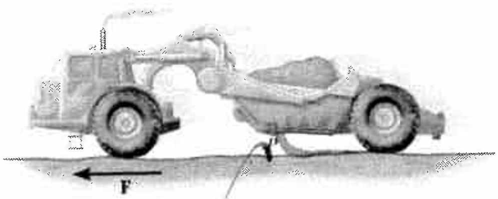
$$v_{D/e} = 0.237 \text{ m/s}$$

$$\overset{+}{\leftarrow} \Sigma F = m\frac{dv}{dt} - \frac{dm}{dt}v_{D/e}$$

$$4000 = 37\,600a - 900(0.237)$$

$a = 0.112 \text{ m/s}^2 = 112 \text{ mm/s}^2$ **Ans**

**15–126.** The earthmover initially carries 10 m³ of sand having a density of 1520 kg/m³. The sand is unloaded horizontally through a 2.5-m² dumping port $P$ at a rate of 900 kg/s measured relative to the port. Determine the resultant tractive force **F** at its front wheels if the acceleration of the earthmover is 0.1 m/s² when half the sand is dumped. When empty, the earthmover has a mass of 30 Mg. Neglect any resistance to forward motion and the mass of the wheels. The rear wheels are free to roll.




When half the sand remains,

$$m = 30\,000 + \frac{1}{2}(10)(1520) = 37\,600 \text{ kg}$$

$$\frac{dm}{dt} = 900 \text{ kg/s} = \rho\, v_{D/e} A$$

$$900 = 1520(v_{D/e})(2.5)$$

$$v_{D/e} = 0.237 \text{ m/s}$$

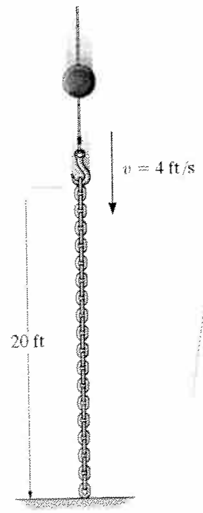
$$a = \frac{dv}{dt} = 0.1$$

$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} - \frac{dm}{dt}v$$

$$F = 37\,600(0.1) - 900(0.237)$$

$F = 3.55$ kN **Ans**

**15–127.** If the chain is lowered at a constant speed determine the normal reaction exerted on the floor as a function of time. The chain has a weight of 5 lb/ft and a total length of 20 ft.




At time $t$, the weight of the chain on the floor is $W = mg(vt)$

$$\frac{dv}{dt} = 0, \qquad m_i = m(vt)$$

$$\frac{dm_i}{dt} = mv$$

$$\Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

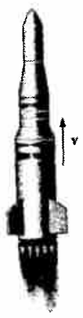
$$R - mg(vt) = 0 + v(mv)$$

$$R = m(gvt + v^2)$$

$$R = \frac{5}{32.2}(32.2(4)(t) + (4)^2)$$

$R = (20t + 2.48)$ lb **Ans**

***15-128.** The rocket has a mass of 65 Mg including the fuel. Determine the constant rate at which the fuel must be burned so that its thrust gives the rocket a speed of 200 ft/s in 10 s starting from rest. The fuel is expelled from the rocket at a relative speed of 3000 ft/s. Neglect the effects of air resistance and assume that $g$ is constant.




*A System That Loses Mass* : Here, $W = \left(m_0 - \frac{dm_e}{dt}t\right)g$. Applying Eq. 15-29, we have

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt};$$

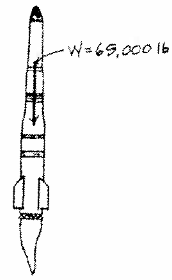
$$-\left(m_0 - \frac{dm_e}{dt}t\right)g = \left(m_0 - \frac{dm_e}{dt}t\right)\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

$$\frac{dv}{dt} = \frac{v_{D/e}\frac{dm_e}{dt}}{m_0 - \frac{dm_e}{dt}t} - g$$

$$\int_0^v dv = \int_0^t \left(\frac{v_{D/e}\frac{dm_e}{dt}}{m_0 - \frac{dm_e}{dt}t} - g\right)dt$$

$$v = \left[-v_{D/e}\ln\left(m_0 - \frac{dm_e}{dt}t\right) - gt\right]\Big|_0^t$$

$$v = v_{D/e}\ln\left(\frac{m_0}{m_0 - \frac{dm_e}{dt}t}\right) - gt \qquad [1]$$




Substitute Eq. [1] with $m_0 = \frac{65\,000}{32.2} = 2018.63$ slug, $v_{D/e} = 3000$ ft/s, $v = 200$ ft/s and $t = 10$ s, we have

$$200 = 3000\ln\left[\frac{2018.63}{2018.63 - \frac{dm_e}{dt}(10)}\right] - 32.2(10)$$

$$e^{0.174} = \frac{2018.63}{2018.63 - \frac{dm_e}{dt}(10)}$$

$$\frac{dm_e}{dt} = 32.2 \text{ slug/s} \qquad \textbf{Ans}$$

**15-129.** The rocket has an initial mass $m_0$, including the fuel. For practical reasons desired for the crew, it is required that it maintain a constant upward acceleration $a_0$. If the fuel is expelled from the rocket at a relative speed $v_{e/r}$ determine the rate at which the fuel should be consumed to maintain the motion. Neglect air resistance, and assume that the gravitational acceleration is constant.


$$a_0 = \frac{dv}{dt}$$

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

$$-mg = ma_0 - v_{e/r}\frac{dm}{dt}$$

$$v_{e/r}\frac{dm}{m} = (a_0 + g)\,dt \qquad (1)$$

Since $v_{e/r}$ is constant, integrating, with $t = 0$ when $m = m_0$ yields

$$v_{e/r}\ln\left(\frac{m}{m_0}\right) = (a_0 + g)t$$

$$\frac{m}{m_0} = e^{[(a_0+g)/v_{e/r}]t}$$

The time rate of fuel consumption is determined from Eq. (1).

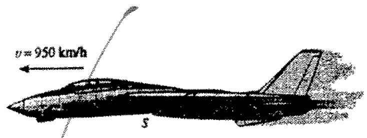
$$\frac{dm}{dt} = m\left(\frac{(a_0+g)}{v_{e/r}}\right)$$

$$\frac{dm}{dt} = m_0\left(\frac{(a_0+g)}{v_{e/r}}\right)e^{[(a_0+g)/v_{e/r}]t} \qquad \textbf{Ans}$$

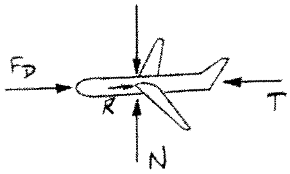
Note : $v_{e/r}$ must be considered a negative quantity.

**15-130.** The 12-Mg jet airplane has a constant speed of 950 km/h when it is flying along a horizontal straight line. Air enters the intake scoops $S$ at the rate of 50 $m^3/s$. If the engine burns fuel at the rate of 0.4 kg/s and the gas (air and fuel) is exhausted relative to the plane with a speed of 450 m/s, determine the resultant drag force exerted on the plane by air resistance. Assume that air has a constant density of 1.22 $kg/m^3$. *Hint:* Since mass both enters and exits the plane, Eqs. 15–29 and 15–30 must be combined to yield

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}$$




$$\Sigma F_s = m\frac{dv}{dt} - \frac{dm_e}{dt}(v_{D/E}) + \frac{dm_i}{dt}(v_{D/i}) \qquad (1)$$




$v = 950$ km/h $= 0.2639$ km/s, $\quad \frac{dv}{dt} = 0$

$v_{D/E} = 0.45$ km/s

$v_{D/i} = 0.2639$ km/s

$\frac{dm_i}{dt} = 50(1.22) = 61.0$ kg/s

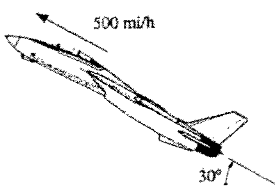
$\frac{dm_e}{dt} = 0.4 + 61.0 = 61.4$ kg/s

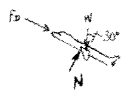
Forces $T$ and $R$ are incorporated into Eq.(1) as the last two terms in the equation.

$(\overset{+}{\leftarrow})\ -F_D = 0 - (0.45)(61.4) + (0.2639)(61)$

$F_D = 11.5$ kN **Ans**

**15-131.** The jet is traveling at a speed of 500 mi/h, 30° with the horizontal. If the fuel is being spent at 3 lb/s, and the engine takes in air at 400 lb/s, whereas the exhaust gas (air and fuel) has a relative speed of 32 800 ft/s, determine the acceleration of the plane at this instant. The drag resistance of the air is $F_D = (0.7v^2)$ lb, where the speed is measured in ft/s. The jet has a weight of 15 000 lb. *Hint:* See Prob. 15–130.




$\frac{dm_i}{dt} = \frac{400}{32.2} = 12.42$ slug/s

$\frac{dm_e}{dt} = \frac{403}{32.2} = 12.52$ slug/s

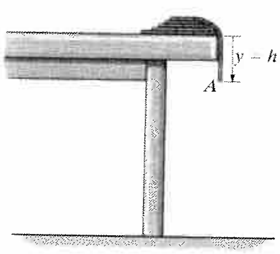
$v = v_{D/i} = 500$ mi/h $= 733.3$ ft/s

$\nwarrow + \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}$

$-(15\,000)\sin 30° - 0.7(733.3)^2 = \frac{15\,000}{32.2}\frac{dv}{dt} - 32\,800(12.52) + 733.3(12.42)$

$a = \frac{dv}{dt} = 37.5\ ft/s^2$ **Ans**

***15–132.** The rope has a mass $m'$ per unit length. If the end length $y = h$ is draped off the edge of the table, and released, determine the velocity of its end $A$ for any position $y$, as the rope uncoils and begins to fall.




$$+\downarrow \Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

At a time $t$, $m = m'y$ and $\frac{dm_i}{dt} = \frac{m'dy}{dt} = m'v$. Here, $v_{D/i} = v$, $\frac{dv}{dt} = g$.

$$m'gy = m'y\frac{dv}{dt} + v(m'v)$$

$$gy = y\frac{dv}{dt} + v^2 \qquad \text{since } v = \frac{dy}{dt}, \text{ then } dt = \frac{dy}{v}$$

$$gy = vy\frac{dv}{dy} + v^2$$

Multiply both sides by $2ydy$

$$2gy^2dy = 2vy^2dv + 2yv^2dy$$

$$\int 2gy^2dy = \int d\left(v^2y^2\right)$$

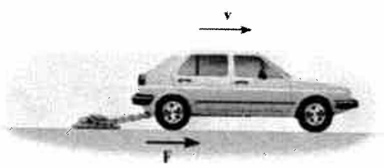
$$\tfrac{2}{3}gy^3 + C = v^2y^2$$

$$v = 0 \text{ at } y = h \qquad \tfrac{2}{3}gh^3 + C = 0 \qquad C = -\tfrac{2}{3}gh^3$$

$$\tfrac{2}{3}gy^3 - \tfrac{2}{3}gh^3 = v^2y^2$$

$$v = \sqrt{\frac{2}{3}g\left(\frac{y^3 - h^3}{y^2}\right)}$$ **Ans**

**15-133.** The car has a mass $m_0$ and is used to tow the smooth chain having a total length $l$ and a mass per unit of length $m'$. If the chain is originally piled up, determine the tractive force $F$ that must be supplied by the rear wheels of the car, necessary to maintain a constant speed $v$ while the chain is being drawn out.



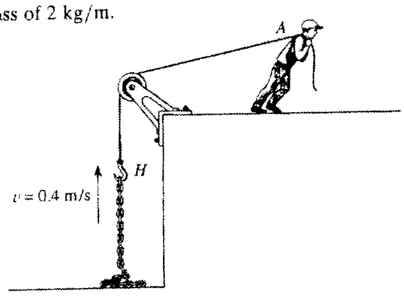

$$\overset{+}{\rightarrow} \Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

At a time $t$, $m = m_0 + ct$, where $c = \frac{dm_i}{dt} = \frac{m'dx}{dt} = m'v$.

Here, $v_{D/i} = v$, $\frac{dv}{dt} = 0$.

$$F = (m_0 - m'vt)(0) + v(m'v) = m'v^2$$ **Ans**

**15-134.** Determine the magnitude of force **F** as a function of time, which must be applied to the end of the cord at $A$ to raise the hook $H$ with a constant speed $v = 0.4$ m/s. Initially the chain is at rest on the ground. Neglect the mass of the cord and the hook. The chain has a mass of 2 kg/m.




$$\frac{dv}{dt} = 0, \qquad y = vt$$

$$m_i = my = mvt$$

$$\frac{dm_i}{dt} = mv$$

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} + v_{D/i}\left(\frac{dm_i}{dt}\right)$$

$$F - mgvt = 0 + v(mv)$$

$$F = m(gvt + v^2)$$

$$= 2[9.81(0.4)t + (0.4)^2]$$

$$F = (7.85t + 0.320)\text{ N} \qquad \textbf{Ans}$$