14-1. A woman having a mass of 70 kg stands in an elevator which has a downward acceleration of 4 m/s^2 starting from rest. Determine the work done by her weight and the work of the normal force which the floor exerts on her when the elevator descends 6 m. Explain why the work of these forces is different.

$+\downarrow \Sigma F_y = ma_y: \quad 70(9.81) - N_P = 70(4)$

$N_P = 406.7 \text{ N}$

$U_W = 6(686.7) = 4.12 \text{ kJ}$ **Ans**

$U_{N_P} = -6(406.7) = -2.44 \text{ kJ}$ **Ans**

The difference accounts for a change in kinetic energy. **Ans**

Note: $v^2 = v_0^2 + 2a_c(s - s_0)$

$v^2 = 0 + 2(4)(6 - 0)$

$v = 6.928 \text{ m/s}$

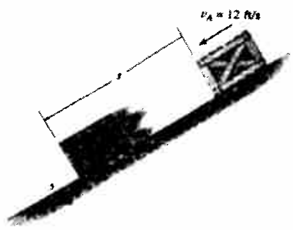
$\Delta T = \frac{1}{2}(70)(6.928)^2 = 1.68 \text{ kJ}$

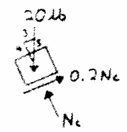
Also, $T_1 + \Sigma U_{1-2} = T_2$

$\Delta T = \Sigma U_{1-2} = 4.12 - 2.44 = 1.68 \text{ kJ}$

200 N 4 m/s

14-2. The ~~20-lb~~ crate has a velocity of v_A = ~~12 ft~~/s when it is at A. Determine its velocity after it slides s = ~~6 ft~~ 2 m down the plane. The coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.2$.





$T_1 + \Sigma U_{1-2} = T_2$

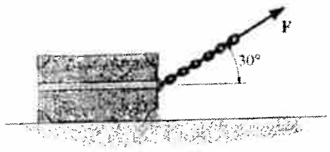
$\frac{1}{2}\left(\frac{20}{32.2}\right)(12)^2 + 20\left(\frac{3}{5}\right)(6) - \left[0.2(20)\left(\frac{4}{5}\right)\right]6 = \frac{1}{2}\left(\frac{20}{32.2}\right)v_2^2$

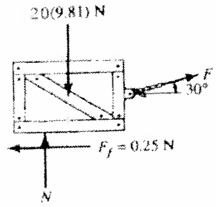
$v_2 = 17.7 \text{ ft/s}$ Ans

$\frac{1}{2}\left(\frac{200}{9.81}\right)(4)^2 + 200\left(\frac{3}{5}\right)(2) - \left(0.2(200)\frac{4}{5}\right)2 = \frac{1}{2} \times \frac{200}{9.81} V_2^2$

$V_2 = 5.76 \text{ m/s}$

14-3. The 20-kg crate is subjected to a force having a constant direction and a magnitude $F = 100$ N, where s is measured in meters. When $s = 4$ m, the crate is moving to the right with a speed of 8 m/s. Determine its speed when $s = 25$ m. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.25$.





Equation of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.25$ N. Applying Eq. 13-7, we have

$$+\uparrow \sum F_y = ma_y; \quad N + 100\sin 30° - 20(9.81) = 2(0)$$

$$N = 146.2 \text{ N}$$

Principle of Work and Energy: The horizontal component of force F which acts in the direction of displacement does *positive* work, whereas the friction force F_f does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of force F and the weight of the crate do not displace hence do no work. Applying Eq. 14-7, we have

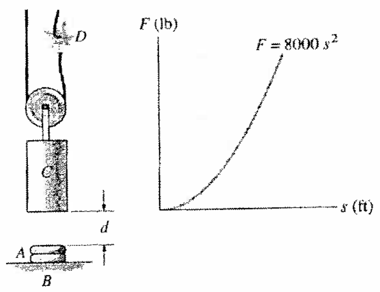
$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(20)(8^2) + 100\cos 30°(25-15)$$

$$-0.25(146.2)(25-15) = \frac{1}{2}(20)v^2$$

$v = 7.07$ m/s **Ans**

•14-4. The "air spring" A is used to protect the support structure B and prevent damage to the conveyor-belt tensioning weight C in the event of a belt failure D. The force developed by the spring as a function of its deflection is shown by the graph. If the weight is 50 lb and is suspended a height $d = 1.5$ ft above the top of the spring, determine the maximum deformation of the spring in the event the conveyor belt fails. Neglect the mass of the pulley and belt.



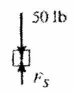
$$T_1 + \sum U_{1-2} = T_2$$

$$0 + \left[50(1.5+s) - \int_0^s 8000 s^2\,ds\right] = 0$$

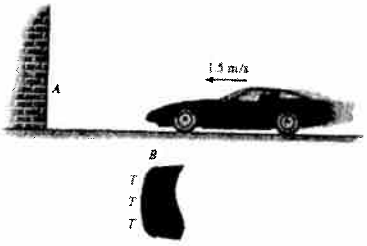
$$50(1.5+s) - \frac{8000 s^3}{3} = 0$$

$$8000 s^3 - 150 s - 225 = 0$$

$s = 0.3246$ ft $= 3.90$ in. **Ans**



14-5. A car is equipped with a bumper B designed to absorb collisions. The bumper is mounted to the car using pieces of flexible tubing T. Upon collision with a rigid barrier at A, a constant horizontal force $\mathbf{F}$ is developed which causes a car deceleration of $3g = 29.43\ \text{m/s}^2$ (the highest safe deceleration for a passenger without a seatbelt). If the car and passenger have a total mass of 1.5 Mg and the car is initially coasting with a speed of 1.5 m/s, determine the magnitude of $\mathbf{F}$ needed to stop the car and the deformation x of the bumper tubing.

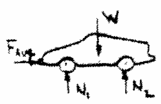


The average force needed to decelerate the car is

$\xrightarrow{+} \Sigma F_x = ma_x;\quad F_{avg} = (1500)(29.43) = 44\,145 = 44.1\ \text{kN}$ **Ans**

The deformation is

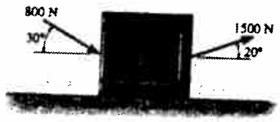
$T_1 + \Sigma U_{1-2} = T_2$



$\frac{1}{2}(1500)(1.5)^2 - (44\,145)(x) = 0$

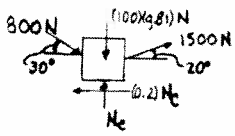
$x = 0.0382 = 38.2\ \text{mm}$ **Ans**

14-6. The 100-kg crate is subjected to forces $F_1 = 800$ N and $F_2 = 1.5$ kN, as shown. If it is originally at rest, determine the distance it slides in order to attain a speed of $v = 6$ m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



$+\uparrow \Sigma F_y = 0;\quad N_C - 800(\sin 30°) - 100(9.81) + 1500(\sin 20°) = 0$

$N_C = 867.97\ \text{N}$



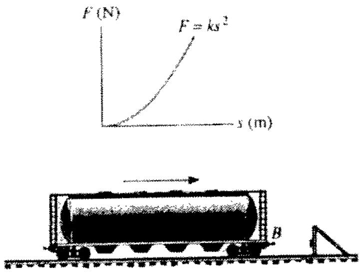
$T_1 + \Sigma U_{1-2} = T_2$

$0 + (800\cos 30°)s - 0.2(867.97)s + (1500\cos 20°)s = \frac{1}{2}(100)(6)^2$

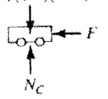
$s(1928.8) = 1800$

$s = 0.933\ \text{m}$ **Ans**

14-7. Design considerations for the bumper B on the 5-Mg train car require use of a nonlinear spring having the load-deflection characteristics shown in the graph. Select the proper value of k so that the maximum deflection of the spring is limited to 0.2 m when the car, traveling at 4 m/s, strikes the rigid stop. Neglect the mass of the car wheels.



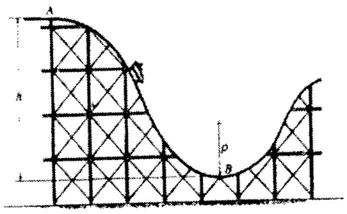
$$\frac{1}{2}(5000)(4)^2 - \int_0^{0.2} ks^2\,ds = 0$$



$$40\,000 - k\frac{(0.2)^3}{3} = 0$$

$k = 15.0\ \text{MN/m}^2$ **Ans**

***14-8.** Determine the required height h of the roller coaster so that when it is essentially at rest at the crest of the hill it will reach a speed of 100 km/h when it comes to the bottom. Also, what should be the minimum radius of curvature ρ for the track at B so that the passengers do not experience a normal force greater than $4\,mg = (39.24\,m)$ N? Neglect the size of the car and passenger.

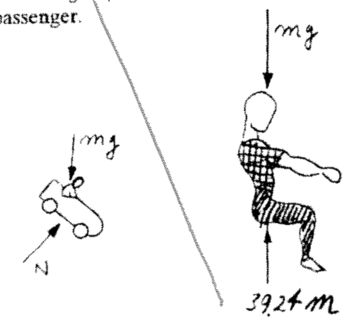


$$100\ \text{km/h} = \frac{100(10^3)}{3600} = 27.778\ \text{m/s}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + m(9.81)h = \frac{1}{2}m(27.778)^2$$

$h = 39.3$ m **Ans**



$+\uparrow \Sigma F_n = ma_n;\quad 39.24m - mg = m\left(\frac{(27.778)^2}{\rho}\right)$

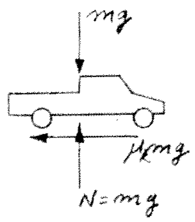
$\rho = 26.2$ m **Ans**

14-9. When the driver applies the brakes of a light truck traveling 40 km/h, it skids 3 m before stopping. How far will the truck skid if it is traveling 80 km/h when the brakes are applied?

$$40\ \text{km/h} = \frac{40(10^3)}{3600} = 11.11\ \text{m/s} \qquad 80\ \text{km/h} = 22.22\ \text{m/s}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m(11.11)^2 - \mu_k mg(3) = 0$$



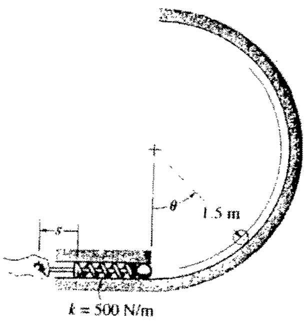
$$\mu_k g = 20.576$$

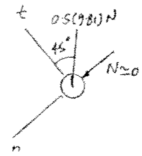
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m(22.22)^2 - (20.576)m(d) = 0$$

$d = 12$ m **Ans**

14-10. The 0.5-kg ball of negligible size is fired up the vertical circular track using the spring plunger. The plunger keeps the spring compressed 0.08 m when $s = 0$. Determine how far s it must be pulled back and released so that the ball will begin to leave the track when $\theta = 135°$.





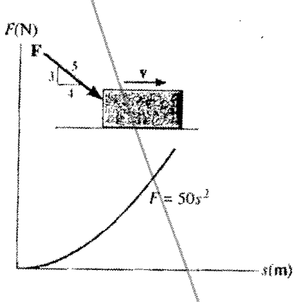
$$\Sigma F_n = ma_n; \quad 0.5(9.81)\sin 45° = 0.5\left(\frac{v^2}{1.5}\right) \quad v^2 = 10.41\ \text{m}^2/\text{s}^2$$

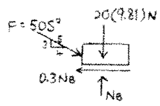
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left\{\left(\tfrac{1}{2}(500)(s+0.08)^2 - \tfrac{1}{2}(500)(0.08)^2\right) - 0.5(9.81)(1.5 + 1.5\sin 45°)\right\} = \tfrac{1}{2}(0.5)(10.41)$$

$s = 0.1789$ m = 179 mm **Ans**

■14-11. The force **F**, acting in a constant direction on the 20-kg block, has a magnitude which varies with the position s of the block. Determine how far the block slides before its velocity becomes 5 m/s. When $s = 0$ the block is moving to the right at 2 m/s. The coefficient of kinetic friction between the block and surface is $\mu_k = 0.3$.





$$+\uparrow \Sigma F_y = 0: \quad N_B - 20(9.81) - \frac{3}{5}(50\,s^2) = 0$$

$$N_B = 196.2 + 30\,s^2$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(20)(2)^2 + \frac{4}{5}\int_0^s 50\,s^2\,ds - 0.3(196.2)(s) - 0.3\int_0^s 30\,s^2\,ds = \frac{1}{2}(20)(5)^2$$

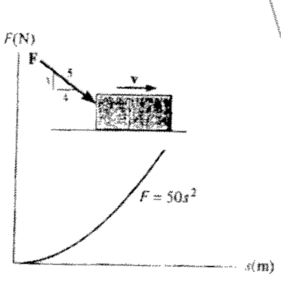
$$40 + 13.33\,s^3 - 58.86\,s - 3\,s^3 = 250$$

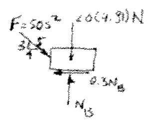
$$s^3 - 5.6961\,s - 20.323 = 0$$

Solving for the real root yields

$s = 3.41$ m **Ans**

***14-12.** The force **F**, acting in a constant direction on the 20-kg block, has a magnitude which varies with position s of the block. Determine the speed of the block after it slides 3 m. When $s = 0$ the block is moving to the right at 2 m/s. The coefficient of kinetic friction between the block and surface is $\mu_k = 0.3$.





$$+\uparrow \Sigma F_y = 0: \quad N_B - 20(9.81) - \frac{3}{5}(50\,s^2) = 0$$

$$N_B = 196.2 + 30\,s^2$$

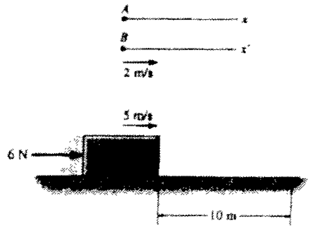
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(20)(2)^2 + \frac{4}{5}\int_0^3 50\,s^2\,ds - 0.3(196.2)(3) - 0.3\int_0^3 30\,s^2\,ds = \frac{1}{2}(20)(v)^2$$

$$40 + 360 - 176.58 - 81 = 10\,v^2$$

$v = 3.77$ m/s **Ans**

14-18. As indicated by the derivation, the principle of work and energy is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which rests on the smooth surface and is subjected to a horizontal force of 6 N. If observer A is in a *fixed* frame x, determine the final speed of the block if it has an initial speed of 5 m/s and travels 10 m, both directed to the right and measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis and moving at a constant velocity of 2 m/s relative to A. *Hint:* The distance the block travels will first have to be computed for observer B before applying the principle of work and energy.



Observer A :

$$T_1 + \Sigma U_{1-2} = T_2$$

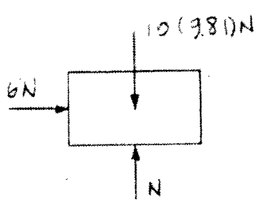
$$\frac{1}{2}(10)(5)^2 + 6(10) = \frac{1}{2}(10)v_2^2$$

$v_2 = 6.08$ m/s Ans

Observer B :

$$F = ma$$

$$6 = 10a \qquad a = 0.6\ \text{m/s}^2$$



$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$10 = 0 + 5t + \frac{1}{2}(0.6)t^2$$

$$t^2 + 16.67t - 33.33 = 0$$

$$t = 1.805\ \text{s}$$

At $v = 2$ m/s, $s' = 2(1.805) = 3.609$ m

Block moves $10 - 3.609 = 6.391$ m

Thus

$$T_1 + \Sigma U_{1-2} = T_2$$

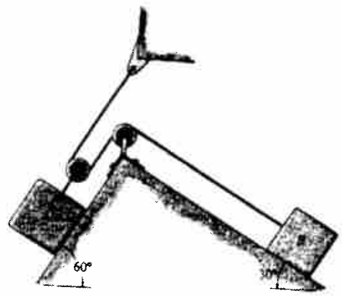
$$\frac{1}{2}(10)(3)^2 + 6(6.391) = \frac{1}{2}(10)v_2^2$$

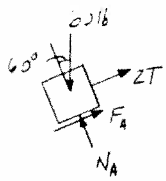
$v_2 = 4.08$ m/s Ans

Note that this result is 2m/s less than that observed by A.

300 N

14-14. Determine the velocity of the ~~60-lb~~ block A if the two blocks are released from rest and the 40-lb block B moves 2 ft up the incline. The coefficient of kinetic friction between both blocks and the inclined planes is $\mu_k = 0.10$.



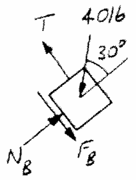


Block A :

$+\nwarrow \Sigma F_y = ma_y;\quad N_A - 60\cos 60° = 0$

$N_A = 30$ lb

$F_A = 0.1(30) = 3$ lb

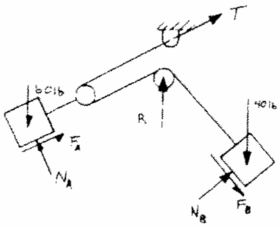


Block B :

$+\nearrow \Sigma F_y = ma_y;\quad N_B - 40\cos 30° = 0$

$N_B = 34.64$ lb

$F_B = 0.1(34.64) = 3.464$ lb



Use the system of both blocks. N_A, N_B, T and R do no work.

$T_1 + \Sigma U_{1-2} = T_2$

$(0+0) + 60\sin 60°|\Delta s_A| - 40\sin 30°|\Delta s_B| - 3|\Delta s_A| - 3.464|\Delta s_B| = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{40}{32.2}\right)v_B^2$

$2s_A + s_B = l$

$2\Delta s_A = -\Delta s_B$

When $|\Delta s_B| = 2$ ft, $|\Delta s_A| = 1$ ft

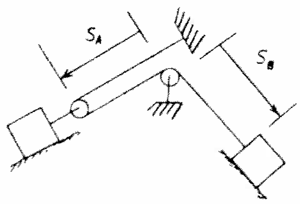
Also,

$2v_A = -v_B$

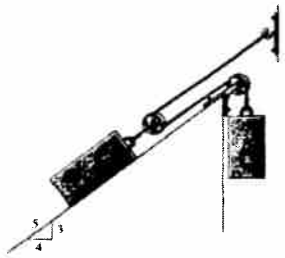
Substituting and solving,

$v_A = 0.771$ ft/s **Ans**

$v_B = -1.54$ ft/s



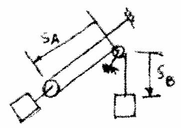
14-15. Block *A* has a weight of ~~60 lb~~ 300 N and block *B* has a weight of ~~10 lb~~ 50 N. Determine the speed of block *A* after it moves ~~5 ft~~ 2 m down the plane, starting from rest. Neglect friction and the mass of the cord and pulleys.

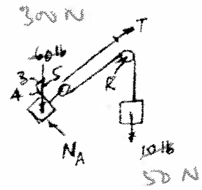


$$2s_A + s_B = l$$

$$2\Delta s_A + \Delta s_B = 0$$

$$2v_A + v_B = 0$$





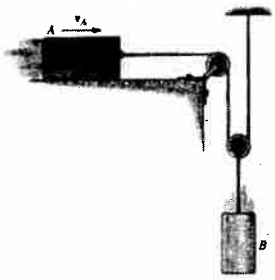
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 60\left(\frac{3}{5}\right)(5) - 10(10) = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)(2v_A)^2$$

$v_A = 7.18$ ft/s **Ans**

$0 + 300\left(\frac{3}{5}\right)2 - 50 \times 4 = \frac{1}{2}\left(\frac{300}{9.81}\right)v_A^2 + \frac{1}{2}\left(\frac{50}{9.81}\right)(2v_a)^2 \Rightarrow v_a = 2.51$ m/s

14-16. The 3-lb block *A* rests on a surface for which the coefficient of kinetic friction is $\mu_k = 0.3$. Determine the distance the 8-lb cylinder *B* must descend so that *A* has a speed of $v_A = 5$ ft/s starting from rest.



$+\uparrow \Sigma F_y = 0;\quad N_A - 3 = 0$

$N_A = 3$ lb

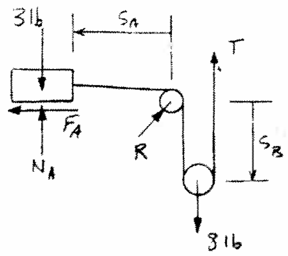
$F_A = 0.3(3) = 0.9$ lb

$s_A + 2s_B = l$

$\Delta s_A = -2\Delta s_B$

$v_A = -2v_B$

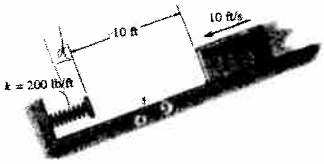
$T_1 + \Sigma U_{1-2} = T_2$



$$(0 + 0) + (8)(\Delta s_B) - 0.9(2\Delta s_B) = \frac{1}{2}\left(\frac{8}{32.2}\right)\left(\frac{5}{2}\right)^2 + \frac{1}{2}\left(\frac{3}{32.2}\right)(5)^2$$

$\Delta s_B = 0.313$ ft **Ans**

14-17. The 100-lb block slides down the inclined plane for which the coefficient of kinetic friction is $\mu_k = 0.25$. If it is moving at 10 ft/s when it reaches point A, determine the maximum deformation of the spring needed to momentarily arrest the motion.



$+\nwarrow \Sigma F_y = 0; \quad N - \frac{4}{5}(100) = 0 \quad N = 80\text{ lb}$

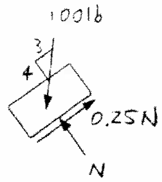
$T_1 + \Sigma U_{1-2} = T_2$

$\frac{1}{2}\left(\frac{100}{32.2}\right)(10)^2 - 0.25(80)(10+d) - \frac{1}{2}(200)(d)^2 + 100(10+d)\left(\frac{3}{5}\right) = 0$

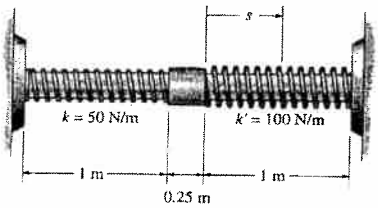
$-100d^2 + 40d + 555.280 = 0$

Use the positive root :

$d = 2.56$ ft **Ans**



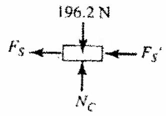
14-18. The collar has a mass of 20 kg and rests on the smooth rod. Two springs are attached to it and the ends of the rod as shown. Each spring has an uncompressed length of 1 m. If the collar is displaced $s = 0.5$ m and released from rest, determine its velocity at the instant it returns to the point $s = 0$.



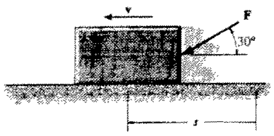
$T_1 + \Sigma U_{1-2} = T_2$

$0 + \frac{1}{2}(50)(0.5)^2 + \frac{1}{2}(100)(0.5)^2 = \frac{1}{2}(20)v_C^2$

$v_C = 1.37$ m/s **Ans**



14-19. The 2-kg block is subjected to a force having a constant direction and a magnitude $F = [300/(1+s)]$ N, where s is in meters. When $s = 4$ m, the block is moving to the left with a speed of 8 m/s. Determine its speed when $s = 12$ m. The coefficient of kinetic friction between the block and the ground is $\mu_k = 0.25$.



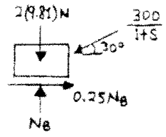
$$+\uparrow \Sigma F_y = 0; \qquad N_B = 2(9.81) + \left(\frac{300}{1+s}\right)\sin 30^\circ$$

$$T_1 + \Sigma U_{1-2} = T_2$$

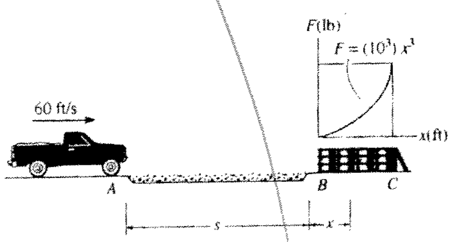
$$\frac{1}{2}(2)(8)^2 - 0.25[2(9.81)(12-4)] - 0.25\int_4^{12} \frac{300 \sin 30^\circ}{1+s}\,ds + \int_4^{12}\left(\frac{300}{1+s}\right)\cos 30^\circ ds = \frac{1}{2}(2)v_2^2$$

$$v_2^2 = 24.76 - 37.5\ln\left(\frac{1+12}{1+4}\right) + 259.81\ln\left(\frac{1+12}{1+4}\right)$$

$v_2 = 15.4$ m/s **Ans**



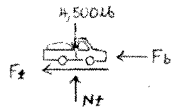
***14-20.** The motion of a truck is arrested using a bed of loose stones AB and a set of crash barrels BC. If experiments show that the stones provide a rolling resistance of 160 lb per wheel and the crash barrels provide a resistance as shown in the graph, determine the distance x the 4500-lb truck penetrates the barrels if the truck is coasting at 60 ft/s when it approaches A. Take $s = 50$ ft and neglect the size of the truck.



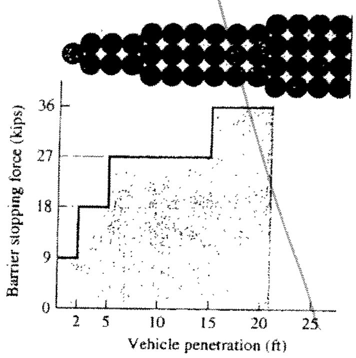
$$\frac{1}{2}\left(\frac{4500}{32.2}\right)(60)^2 - 4(160)(50) - \int_0^x (10)^3 x^3\,dx = 0$$

$$219\,552.80 - \frac{10^3}{4}x^4 = 0$$

$x = 5.44$ ft **Ans**

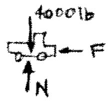


14-21. The crash cushion for a highway barrier consists of a nest of barrels filled with an impact-absorbing material. The barrier stopping force is measured versus the vehicle penetration into the barrier. Determine the distance a car having a weight of 4000 lb will penetrate the barrier if it is originally traveling at 55 ft/s when it strikes the first barrel.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{4000}{32.2}\right)(55)^2 - Area = 0$$



$Area = 187.89$ kip·ft

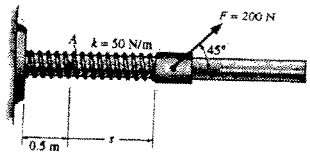
$2(9) + (5-2)(18) + x(27) = 187.89$

$x = 4.29\text{ ft} < (15-5)\text{ ft}$ **(O.K!)**

Thus

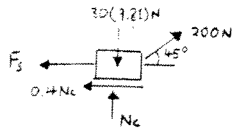
$s = 5\text{ ft} + 4.29\text{ ft} = 9.29\text{ ft}$ **Ans**

14-22. The collar has a mass of 30 kg and is supported on the rod having a coefficient of kinetic friction $\mu_k = 0.4$. The attached spring has an unstretched length of 0.2 m and a stiffness $k = 50$ N/m. Determine the speed of the collar after the applied force $F = 200$ N causes it to be displaced $s = 1.5$ m from point A. When $s = 0$ the collar is held at rest.



$+\uparrow \Sigma F_y = 0;\quad 200\sin 45° - 30(9.81) + N_C = 0$

$N_C = 152.88$ N

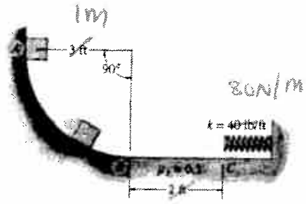


$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 200\cos 45°(1.5) - 0.4(152.88)(1.5) - \left[\frac{1}{2}(50)(1.8)^2 - \frac{1}{2}(50)(0.3)^2\right] = \frac{1}{2}(30)v^2$$

$v = 1.67$ m/s **Ans**

14-23. The 5-lb block is released from rest at A and slides down the smooth circular surface AB. It then continues to slide along the horizontal rough surface until it strikes the spring. Determine how far it compresses the spring before stopping.



$$+\uparrow \Sigma F_y = 0; \quad -5 + N_b = 0$$

$$N_b = 5 \text{ lb}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

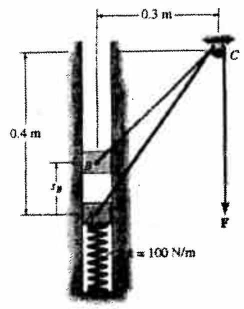
$$0 + 5(3) - 0.2(5)(2+s) - \frac{1}{2}(40)(s)^2 = 0$$

$$-20s^2 - s + 13 = 0$$

Solving for the positive root :

$s = 0.782$ ft **Ans**

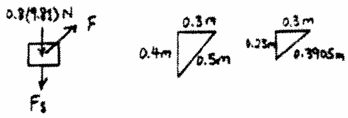
***14-24.** The block has a mass of 0.8 kg and moves within the smooth vertical slot. If it starts from rest when the *attached* spring is in the unstretched position at A, determine the *constant* vertical force F which must be applied to the cord so that the block attains a speed $v_B = 2.5$ m/s when it reaches B; $s_B = 0.15$ m. Neglect the size and mass of the pulley. *Hint:* The work of **F** can be determined by finding the difference Δl in cord lengths AC and BC and using $U_F = F\,\Delta l$.



$$l_{AC} = \sqrt{(0.3)^2 + (0.4)^2} = 0.5 \text{ m}$$

$$l_{BC} = \sqrt{(0.4-0.15)^2 + (0.3)^2} = 0.3905 \text{ m}$$

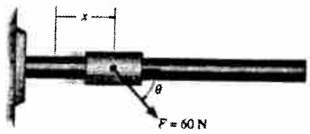
$$T_A + \Sigma U_{A-B} = T_B$$



$$0 + F(0.5 - 0.3905) - \frac{1}{2}(100)(0.15)^2 - (0.8)(9.81)(0.15) = \frac{1}{2}(0.8)(2.5)^2$$

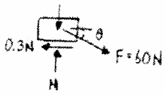
$F = 43.9$ N **Ans**

14-25. The collar has a mass of 5 kg and is moving at 8 m/s when $x = 0$ and a force of $F = 60$ N is applied to it. The direction θ of this force varies such that $\theta = 10x$, where x is in meters and θ is clockwise, measured in degrees. Determine the speed of the collar when $x = 3$ m. The coefficient of kinetic friction between the collar and the rod is $\mu_k = 0.3$.



$+\uparrow \Sigma F_y = 0; \quad N - 5(9.81) - 60\sin\theta = 0$

$N = 60\sin\theta + 49.05$



$T_1 + \Sigma U_{1-2} = T_2$

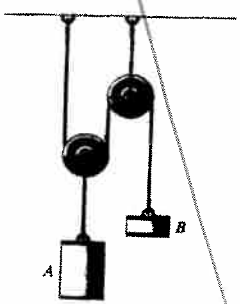
$$\frac{1}{2}(5)(8)^2 + \int_0^3 60\cos\theta\,dx - 0.3\int_0^3(60\sin\theta + 49.05)\,dx = \frac{1}{2}(5)v^2$$

$$160 + 60\int_0^3 \cos 10x\,dx - 18\int_0^3 \sin 10x\,dx - 14.715\int_0^3 dx = 2.5v^2$$

$$160 + 60\left[\frac{\sin 10x}{10}\right]_0^3 + 18\left[\frac{\cos 10x}{10}\right]_0^3 - 44.145 = 2.5v^2$$

$v = 6.89$ m/s **Ans**

14-26. Cylinder A has a weight of 60 lb and block B has a weight of 10 lb. Determine the distance A must descend from rest before it obtains a speed of 8 ft/s. Also, what is the tension in the cord supporting block A? Neglect the mass of the cord and pulleys.



$2s_A + s_B = l$

$2\Delta s_A = -\Delta s_B$

$2v_A = -v_B$

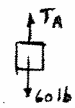
For $v_A = 8$ ft/s, $v_B = -16$ ft/s

For the system:

$T_1 + \Sigma U_{1-2} = T_2$

$$[0+0] + [60(s_A) - 10(2s_A)] = \frac{1}{2}\left(\frac{60}{32.2}\right)(8)^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)(-16)^2$$

$s_A = 2.484 = 2.48$ ft **Ans**



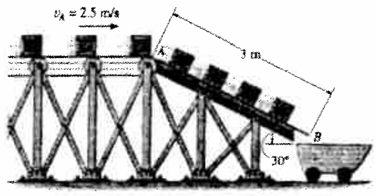
For block A:

$T_1 + \Sigma U_{1-2} = T_2$

$$0 + 60(2.484) - T_A(2.484) = \frac{1}{2}\left(\frac{60}{32.2}\right)(8)^2$$

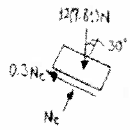
$T_A = 36.0$ lb **Ans**

14-27. The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's velocity is $v_A = 2.5$ m/s, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is $\mu_k = 0.3$, determine the speed at which each crate slides off the ramp at B. Assume that no tipping occurs.



$+\nearrow \Sigma F_y = ma_y;\quad N_C - 12(9.81)\cos 30° = 0$

$N_C = 102.0$ N

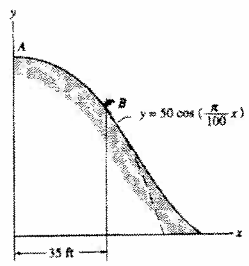


$T_A + \Sigma U_{A-B} = T_B$

$\frac{1}{2}(12)(2.5)^2 + 12(9.81)(3\sin 30°) - 0.3(102.0)(3) = \frac{1}{2}(12)v_B^2$

$v_B = 4.52$ m/s **Ans**

***14-28.** When the 150-lb skier is at point A he has a speed of 5 ft/s. Determine his speed when he reaches point B on the smooth slope. For this distance the slope follows the cosine curve shown. Also, what is the normal force on his skis at B and his rate of increase in speed? Neglect friction and air resistance.



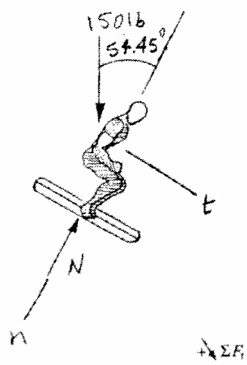
$y = 50\cos\left(\frac{\pi}{100}\right)x\Big|_{x=35} = 22.70$ ft

$\frac{dy}{dx} = \tan\theta = -50\left(\frac{\pi}{100}\right)\sin\left(\frac{\pi}{100}\right)x = -\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{100}\right)x\Big|_{x=35} = -1.3996$

$\theta = -54.45°$

$\frac{d^2y}{dx^2} = -\left(\frac{\pi^2}{200}\right)\cos\left(\frac{\pi}{100}\right)x\Big|_{x=35} = -0.02240$

$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (-1.3996)^2\right]^{\frac{3}{2}}}{|-0.02240|} = 227.179$



$T_A + \Sigma U_{A-B} = T_B$

$\frac{1}{2}\left(\frac{150}{32.2}\right)(5)^2 + 150(50 - 22.70) = \frac{1}{2}\left(\frac{150}{32.2}\right)v_B^2$

$v_B = 42.227$ ft/s $= 42.2$ ft/s **Ans**

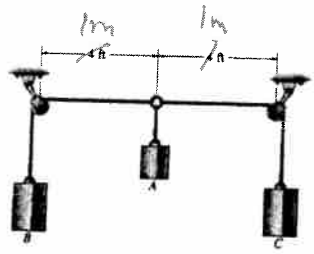
$+\swarrow \Sigma F_n = ma_n;\quad -N + 150\cos 54.45° = \left(\frac{150}{32.2}\right)\left(\frac{(42.227)^2}{227.179}\right)$

$N = 50.6$ lb **Ans**

$+\searrow \Sigma F_t = ma_t;\quad 150\sin 54.45° = \left(\frac{150}{32.2}\right)a_t$

$a_t = 26.2$ ft/s^2 **Ans**

14-29. When the 12-lb block A is released from rest it lifts the two 15-lb weights B and C. Determine the maximum distance A will fall before its motion is momentarily stopped. Neglect the weight of the cord and the size of the pulleys.



Consider the entire system :

$$l = \sqrt{y^2 + 4^2}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$(0+0+0) + 12y - 2(15)\left(\sqrt{y^2+4^2} - 4\right) = (0+0+0)$$

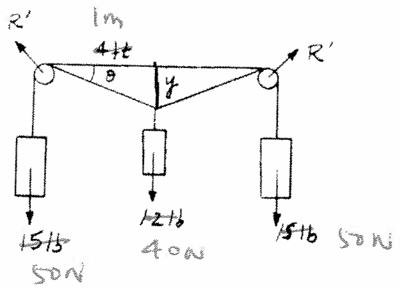
$$0.4y = \sqrt{y^2+16} - 4$$

$$(0.4y+4)^2 = y^2 + 16$$

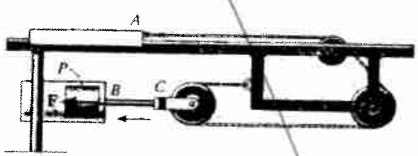
$$-0.84y^2 + 3.20y + 16 = 16$$

$$-0.84y + 3.20 = 0$$

$y = 3.81$ ft **Ans**



14-30. The catapulting mechanism is used to propel the 10-kg slider A to the right along the smooth track. The propelling action is obtained by drawing the pulley attached to rod BC rapidly to the left by means of a piston P. If the piston applies a constant force $F = 20$ kN to rod BC such that it moves it 0.2 m, determine the speed attained by the slider if it was originally at rest. Neglect the mass of the pulleys, cable, piston, and rod BC.

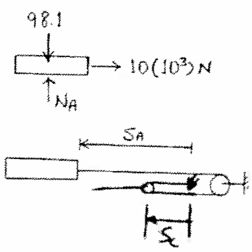


$$2s_C + s_A = l$$

$$2\Delta s_C + \Delta s_A = 0$$

$$2(0.2) = -\Delta s_A$$

$$-0.4 = \Delta s_A$$

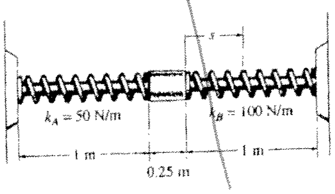


$$T_1 + \Sigma U_{1-2} = T_2$$

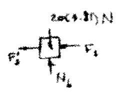
$$0 + (10\,000)(0.4) = \frac{1}{2}(10)(v_A)^2$$

$v_A = 28.3$ m/s **Ans**

14-31. The collar has a mass of 20 kg and slides along the smooth rod. Two springs are attached to it and the ends of the rod as shown. If each spring has an uncompressed length of 1 m and the collar has a speed of 2 m/s when $s = 0$, determine the maximum compression of each spring due to the back-and-forth (oscillating) motion of the collar.



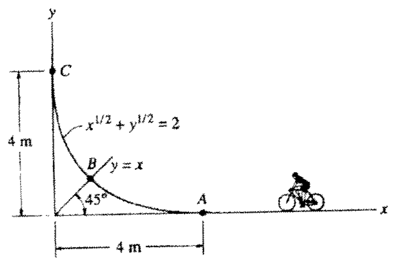
$$T_1 + \Sigma U_{1-2} = T_2$$



$$\frac{1}{2}(20)(2)^2 - \frac{1}{2}(50)(s)^2 - \frac{1}{2}(100)(s)^2 = 0$$

$s = 0.730$ m **Ans**

***14-32.** The cyclist travels to point A, pedaling until he reaches a speed $v_A = 8$ m/s. He then coasts freely up the curved surface. Determine the normal force he exerts on the surface when he reaches point B. The total mass of the bike and man is 75 kg. Neglect friction, the mass of the wheels, and the size of the bicycle.



$$x^{\frac{1}{2}} + y^{\frac{1}{2}} = 2$$

$$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^{-\frac{1}{2}}}{y^{-\frac{1}{2}}}$$

For $y = x$,

$$2x^{\frac{1}{2}} = 2$$

$x = 1,\ y = 1$ (Point B)

Thus,

$$\tan\theta = \frac{dy}{dx} = -1$$

$$\theta = -45^\circ$$

$$\frac{dy}{dx} = (-x^{-\frac{1}{2}})(y^{\frac{1}{2}})$$

$$\frac{d^2y}{dx^2} = y^{\frac{1}{2}}(\frac{1}{2}x^{-\frac{3}{2}}) - x^{-\frac{1}{2}}(\frac{1}{2})(y^{-\frac{1}{2}})(\frac{dy}{dx})$$

$$\frac{d^2y}{dx^2} = \frac{1}{2}y^{\frac{1}{2}}x^{-\frac{3}{2}} + \frac{1}{2}(\frac{1}{x})$$

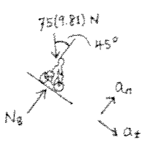
For $x = y = 1$

$$\frac{dy}{dx} = -1, \quad \frac{d^2y}{dx^2} = 1$$

$$\rho = \frac{[1+(-1)^2]^{3/2}}{1} = 2.828 \text{ m}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(75)(8^2) - 75(9.81)(1) = \frac{1}{2}(75)(v_B^2)$$

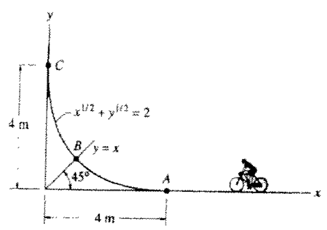


$$v_B^2 = 44.38$$

$$\nearrow + \Sigma F_n = m\,a_n; \quad N_B - 9.81(75)\cos 45^\circ = 75(\frac{44.38}{2.828})$$

$N_B = 1.70$ kN **Ans**

14-33. The cyclist travels to point A, pedaling until he reaches a speed $v_A = 4$ m/s. He then coasts freely up the curved surface. Determine how high he reaches up the surface before he comes to a stop. Also, what are the resultant normal force on the surface at this point and his acceleration? The total mass of the bike and man is 75 kg. Neglect friction, the mass of the wheels, and the size of the bicycle.



$$x^{\frac{1}{2}} + y^{\frac{1}{2}} = 2$$

$$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^{-\frac{1}{2}}}{y^{-\frac{1}{2}}}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(75)(4)^2 - 75(9.81)(y) = 0$$

$y = 0.81549 \text{ m} = 0.815 \text{ m}$ **Ans**

$$x^{1/2} + (0.81549)^{1/2} = 2$$

$$x = 1.2033 \text{ m}$$

$$\tan\theta = \frac{dy}{dx} = \frac{(1.2033)^{-1/2}}{(0.81549)^{-1/2}} = -0.82323$$

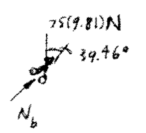
$$\theta = -39.46°$$

$\nearrow + \Sigma F_n = m a_n: \quad N_b - 9.81(75)\cos 39.46° = 0$

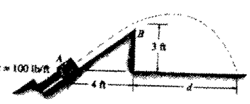
$N_b = 568 \text{ N}$ **Ans**

$+\searrow \Sigma F_t = m a_t: \quad 75(9.81)\sin 39.46° = 75 a_t$

$a = a_t = 6.23 \text{ m/s}^2$ **Ans**



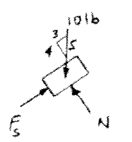
14-34. The 10-lb block is pressed against the spring so as to compress it 2 ft when it is at A. If the plane is smooth, determine the distance d, measured from the wall, to where the block strikes the ground. Neglect the size of the block.



$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + \frac{1}{2}(100)(2)^2 - (10)(3) = \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2$$

$$v_B = 33.09 \text{ ft/s}$$



$(\overset{+}{\rightarrow}) \quad s = s_0 + v_0 t$

$$d = 0 + 33.09\left(\frac{4}{5}\right)t$$

$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

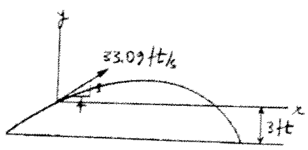
$$-3 = 0 + (33.09)\left(\frac{3}{5}\right)t + \frac{1}{2}(-32.2)t^2$$

$$16.1t^2 - 19.853t - 3 = 0$$

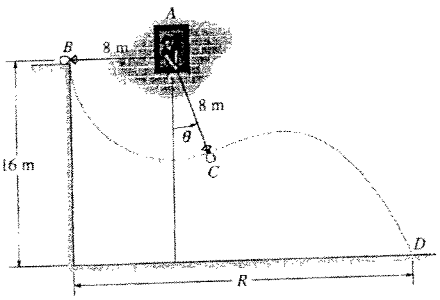
Solving for the positive root,

$$t = 1.369 \text{ s}$$

$d = 33.09\left(\frac{4}{5}\right)(1.369) = 36.2 \text{ ft}$ **Ans**

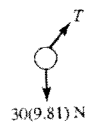


14-35. The man at the window A wishes to throw the 30-kg sack on the ground. To do this he allows it to swing from rest at B to point C, when he releases the cord at $\theta = 30°$. Determine the speed at which it strikes the ground and the distance R.



$$T_B + \Sigma U_{B-C} = T_C$$

$$0 + 30(9.81)8\cos 30° = \frac{1}{2}(30)v_C^2$$



$$v_C = 11.659 \text{ m/s}$$

$$T_B + \Sigma U_{B-D} = T_D$$

$$0 + 30(9.81)(16) = \frac{1}{2}(30)v_D^2$$

$$v_D = 17.7 \text{ m/s} \qquad \textbf{Ans}$$

During free flight:

$$(+\downarrow)\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$16 = 8\cos 30° - 11.659\sin 30° t + \frac{1}{2}(9.81)t^2$$

$$t^2 - 1.18848\,t - 1.8495 = 0$$

Solving for the positive root:

$$t = 2.0784 \text{ s}$$

$$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$$

$$s = 8\sin 30° + 11.659\cos 30°(2.0784)$$

$$s = 24.985 \text{ m}$$

Thus,

$$R = 8 + 24.985 = 33.0 \text{ m} \qquad \textbf{Ans}$$

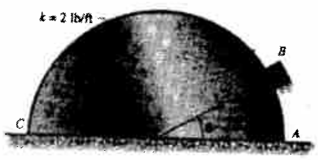
Also,

$$(v_D)_x = 11.659\cos 30° = 10.097 \text{ m/s}$$

$$(+\downarrow)\ (v_D)_y = -11.659\sin 30° + 9.81(2.0784) = 14.5617 \text{ m/s}$$

$$v_D = \sqrt{(10.097)^2 + (14.5617)^2} = 17.7 \text{ m/s} \qquad \textbf{Ans}$$

***14-36.** A 2-lb block rests on the smooth semicylindrical surface. An elastic cord having a stiffness $k = 2$ lb/ft is attached to the block at B and to the base of the semicylinder at point C. If the block is released from rest at $A(\theta = 0°)$, determine the unstretched length of the cord so the block begins to leave the semicylinder at the instant $\theta = 45°$. Neglect the size of the block.



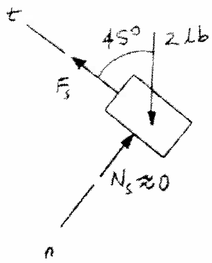
$+\swarrow \Sigma F_n = ma_n; \quad 2\sin 45° = \frac{2}{32.2}\left(\frac{v^2}{1.5}\right)$

$v = 5.844$ ft/s

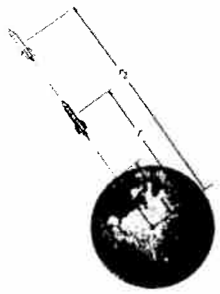
$T_1 + \Sigma U_{1-2} = T_2$

$0 + \frac{1}{2}(2)[\pi(1.5) - l_0]^2 - \frac{1}{2}(2)\left[\frac{3\pi}{4}(1.5) - l_0\right]^2 - 2(1.5\sin 45°) = \frac{1}{2}\left(\frac{2}{32.2}\right)(5.844)^2$

$l_0 = 2.77$ ft **Ans**



14-37. A rocket of mass m is fired vertically from the surface of the earth, i.e., at $r = r_1$. Assuming no mass is lost as it travels upward, determine the work it must do against gravity to reach a distance r_2. The force of gravity is $F = GM_e m/r^2$ (Eq. 13–1), where M_e is the mass of the earth and r the distance between the rocket and the center of the earth.

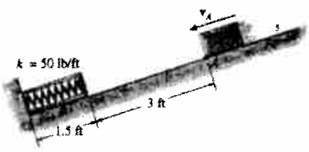


$F = G\frac{M_e m}{r^2}$

$U_{1-2} = \int F\,dr = GM_e m \int_{r_1}^{r_2} \frac{dr}{r^2}$

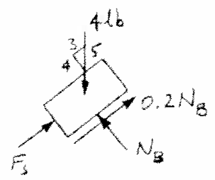
$= GM_e m\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ **Ans**

14-38. The spring has a stiffness $k = 50$ lb/ft and an *unstretched length* of 2 ft. As shown, it is confined by the plate and wall using cables so that its length is 1.5 ft. A 4-lb block is given a speed v_A when it is at A, and it slides down the incline having a coefficient of kinetic friction $\mu_k = 0.2$. If it strikes the plate and pushes it forward 0.25 ft before stopping, determine its speed at A. Neglect the mass of the plate and spring.



$\nwarrow + \Sigma F_y = 0; \quad N_B - 4\left(\frac{4}{5}\right) = 0$

$N_B = 3.20$ lb

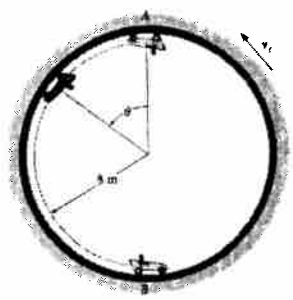


$T_1 + \Sigma U_{1-2} = T_2$

$$\frac{1}{2}\left(\frac{4}{32.2}\right)v_A^2 + (3+0.25)\left(\frac{3}{5}\right)(4) - 0.2(3.20)(3+0.25) - \left[\frac{1}{2}(50)(0.75)^2 - \frac{1}{2}(50)(0.5)^2\right] = 0$$

$v_A = 5.80$ ft/s **Ans**

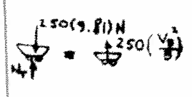
14-39. The "flying car" is a ride at an amusement park which consists of a car having wheels that roll along a track mounted inside a rotating drum. By design the car cannot fall off the track, however motion of the car is developed by applying the car's brake, thereby gripping the car to the track and allowing it to move with a constant speed of the track, $v_t = 3$ m/s. If the rider applies the brake when going from B to A and then releases it at the top of the drum, A, so that the car coasts freely down along the track to B ($\theta = \pi$ rad), determine the speed of the car at B and the normal reaction which the drum exerts on the car at B. Neglect friction during the motion from A to B. The rider and car have a total mass of 250 kg and the center of mass of the car and rider moves along a circular path having a radius of 8 m.



$T_A + \Sigma U_{A-B} = T_B$

$$\frac{1}{2}(250)(3)^2 + 250(9.81)(16) = \frac{1}{2}(250)(v_B)^2$$

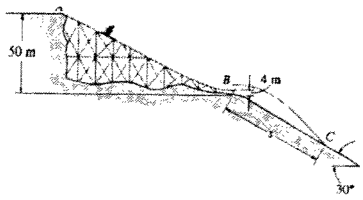
$v_B = 17.97 = 18.0$ m/s **Ans**



$+\uparrow \Sigma F_n = ma_n \qquad N_B - 250(9.81) = 250\left(\frac{(17.97)^2}{8}\right)$

$N_B = 12.5$ kN **Ans**

***14-40.** The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, find the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass of 70 kg.



$T_A + \Sigma U_{A-B} = T_B$

$0 + 70(9.81)(46) = \frac{1}{2}(70)(v_B)^2$

$v_B = 30.04\text{ m/s} = 30.0\text{ m/s}$ **Ans**

$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t$

$s\cos 30° = 0 + 30.04t$

$(+\downarrow)$ $s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

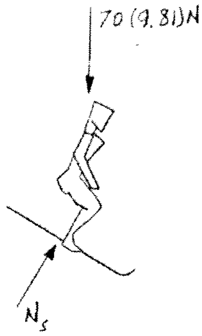
$s\sin 30° + 4 = 0 + 0 + \frac{1}{2}(9.81)t^2$

Eliminating t,

$s^2 - 122.67s - 981.33 = 0$

Solving for the positive root

$s = 130\text{ m}$ **Ans**



14-41 A spring having a stiffness of 5 kN/m is compressed 400 mm. The stored energy in the spring is used to drive a machine which requires 90 W of power. Determine how long the spring can supply energy at the required rate.

$U_{1-2} = \frac{1}{2}(5000)(0.4)^2 = 400\text{ J}$

$P = \frac{U_{1-2}}{t}$; $90 = \frac{400}{t}$

$t = 4.44\text{ s}$ **Ans**

14-42. Determine the power input for a motor necessary to lift 300 lb at a constant rate of 5 ft/s. The efficiency of the motor is $\epsilon = 0.65$.

Power: The power output can be obtained using Eq. 14-10.

$$P = \mathbf{F} \cdot \mathbf{v} = 300(5) = 1500 \text{ ft} \cdot \text{lb/s}$$

Using Eq. 14-11, the required power input for the motor to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\epsilon}$$

$$= \frac{1500}{0.65} = 2307.7 \text{ ft} \cdot \text{lb/s} = 4.20 \text{ hp} \quad \textbf{Ans}$$

14-43. An electrically powered train car draws 30 kW of power. If the car weighs 40 000 lb and starts from rest, determine the maximum speed it attains in 30 s. The mechanical efficiency is $\epsilon = 0.8$.

$$P_i = 30\,000 \text{ W}\left(\frac{1 \text{ hp}}{746 \text{ W}}\right)\left(\frac{550 \text{ ft} \cdot \text{lb/s}}{1 \text{ hp}}\right) = 22.12(10^3) \text{ ft} \cdot \text{lb/s}$$

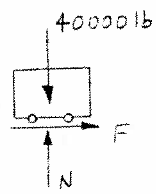
$$P_o = P_i e = 22.12(10^3)(0.8) = 17.694(10^3) \text{ ft} \cdot \text{lb/s}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F = \left(\frac{40\,000}{32.2}\right)\left(\frac{dv}{dt}\right)$$

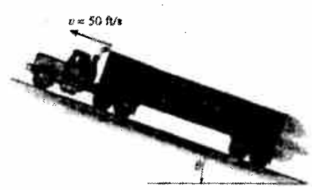
Since $P_o = Fv$, substituting,

$$\int_0^{30}\left(\frac{17\,694}{40\,000}\right)(32.2)\,dt = \int_0^v v\,dv$$

$$v = 29.2 \text{ ft/s} \qquad \textbf{Ans}$$



*__14-44.__ A truck has a weight of 25 000 lb and an engine which transmits a power of 350 hp to *all* the wheels. Assuming that the wheels do not slip on the ground, determine the angle θ of the largest incline the truck can climb at a constant speed of $v = 50$ ft/s.



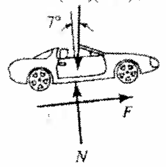
$$+\!\nwarrow \Sigma F_x = ma_x; \qquad F - 25\,000\sin\theta = 0 \qquad F = 25\,000\sin\theta \text{ lb}$$

$$P = \mathbf{F} \cdot \mathbf{v}; \qquad 350(550) = 25\,000\sin\theta(50)$$

$$\theta = \sin^{-1}(0.1540) = 8.86° \qquad \textbf{Ans}$$

14-45. An automobile having a mass of 2 Mg travels up a 7° slope at a constant speed of $v = 100$ km/h. If mechanical friction and wind resistance are neglected, determine the power developed by the engine if the automobile has an efficiency $\epsilon = 0.65$.

Equation of Motion: The force F which is required to maintain the car's constant speed up the slope must be determined first.



$$+\sum F_{x'} = ma_{x'}; \quad F - 2(10^3)(9.81)\sin 7° = 2(10^3)(0)$$

$$F = 2391.08 \text{ N}$$

Power: Here, the speed of the car is $v = \left[\frac{100(10^3)\text{ m}}{\text{h}}\right] \times \left(\frac{1\text{ h}}{3600\text{ s}}\right) = 27.78$ m/s. The power output can be obtained using Eq. 14-10.

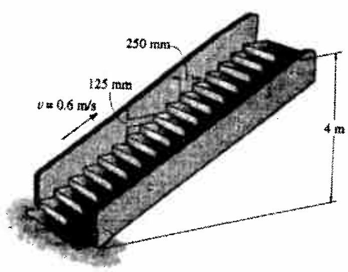
$$P = \mathbf{F} \cdot \mathbf{v} = 2391.08(27.78) = 66.418(10^3)\text{ W} = 66.418\text{ kW}$$

Using Eq. 14-11, the required power input from the engine to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

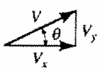
$$= \frac{66.418}{0.65} = 102\text{ kW} \quad \textbf{Ans}$$

14-46. The escalator steps move with a constant speed of 0.6 m/s. If the steps are 125 mm high and 250 mm in length, determine the power of a motor needed to lift an average mass of 150 kg per step. There are 32 steps.



Step height : 0.125 m

The number of steps : $\frac{4}{0.125} = 32$



Total load : $32(150)(9.81) = 47\,088$ N

If load is placed at the center height, $h = \frac{4}{2} = 2$ m, then

$$U = 47\,088\left(\frac{4}{2}\right) = 94.18\text{ kJ}$$

$$v_y = v\sin\theta = 0.6\left(\frac{4}{\sqrt{(32(0.25))^2 + 4^2}}\right) = 0.2683\text{ m/s}$$

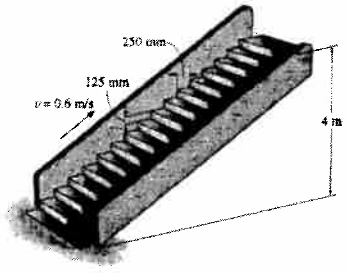
$$t = \frac{h}{v_y} = \frac{2}{0.2683} = 7.454\text{ s}$$

$$P = \frac{U}{t} = \frac{94.18}{7.454} = 12.6\text{ kW} \quad \textbf{Ans}$$

Also,

$$P = \mathbf{F} \cdot \mathbf{v} = 47\,088(0.2683) = 12.6\text{ kW} \quad \textbf{Ans}$$

14-47. If the escalator in Prob. 14-46 is *not moving*, determine the constant speed at which a man having a mass of 80 kg must walk up the steps to generate 100 W of power—the same amount that is needed to power a standard light bulb.



$$P = \frac{U_{1-2}}{t} = \frac{(80)(9.81)(4)}{t} = 100 \qquad t = 31.4\text{ s}$$

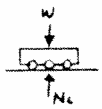
$$v = \frac{s}{t} = \frac{\sqrt{(32(0.25))^2 + 4^2}}{31.4} = 0.285\text{ m/s} \qquad \textbf{Ans}$$

***14-48.** An electric streetcar has a weight of 15 000 lb and accelerates along a horizontal straight road from rest such that the power is always 100 hp. Determine how far it must travel to reach a speed of 40 ft/s.

$$F = ma = \frac{W}{g}\left(\frac{v\,dv}{ds}\right)$$

$$P = Fv = \left[\left(\frac{W}{g}\right)\left(\frac{v\,dv}{ds}\right)\right]v$$

$$\int_0^s P\,ds = \int_0^v \frac{W}{g}v^2\,dv$$

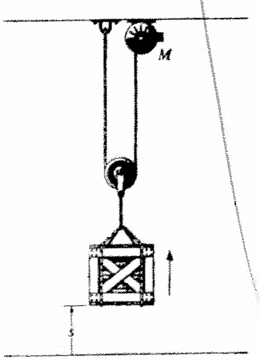


P = constant

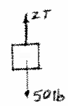
$$Ps = \frac{W}{g}\left(\frac{1}{3}\right)v^3 \qquad s = \frac{W}{3gP}v^3$$

$$s = \frac{(15\,000)(40)^3}{3(32.2)(100)(550)} = 181\text{ ft} \qquad \textbf{Ans}$$

14-49. The 50-lb crate is given a speed of 10 ft/s in t = 4 s starting from rest. If the acceleration is constant, determine the power that must be supplied to the motor when t = 2 s. The motor has an efficiency ϵ = 0.76. Neglect the mass of the pulley and cable.



$$+\uparrow \Sigma F_y = m a_y; \qquad 2T - 50 = \frac{50}{32.2}a$$



$$(+\uparrow)\; v = v_0 + a_c t$$

$$10 = 0 + a(4)$$

$$a = 2.5\text{ ft/s}^2$$

$$T = 26.94\text{ lb}$$

In t = 2 s,

$$(+\uparrow)\quad v = v_0 + a_c t$$

$$v = 0 + 2.5(2) = 5\text{ ft/s}$$

$$s_C + (s_C - s_P) = l$$

$$2v_C = v_P$$

$$2(5) = v_P = 10\text{ ft/s}$$

$$P_O = 26.94(10) = 269.2$$

$$P_{in} = \frac{269.2}{0.76} = 354.49\text{ ft}\cdot\text{lb/s}$$

$$P_{in} = 0.644\text{ hp} \qquad \text{Ans}$$

14-50. A car has a mass m and accelerates along a horizontal straight road from rest such that the power is always a constant amount P. Determine how far it must travel to reach a speed of v.

Power : Since the power ouput is constant, then the traction force F varies with v. Applying Eq. 14 – 10, we have

$$P = \mathbf{F} \cdot \mathbf{v}$$

$$P = Fv \qquad F = \frac{P}{v}$$

Equation of Motion :

$$\xrightarrow{+} \Sigma F_x = ma_x ; \qquad \frac{P}{v} = ma \qquad a = \frac{P}{mv}$$

Kinematics : Applying equation $ds = \frac{v\,dv}{a}$, we have

$$\int_0^s ds = \int_0^v \frac{mv^2}{P} dv \qquad s = \frac{mv^3}{3P} \qquad \textbf{Ans}$$

14-51. To dramatize the loss of energy in an automobile, consider a car having a weight of 5 000 lb that is traveling at 35 mi/h. If the car is brought to a stop, determine how long a 100-W light bulb must burn to expend the same amount of energy. (1 mi = 5280 ft.)

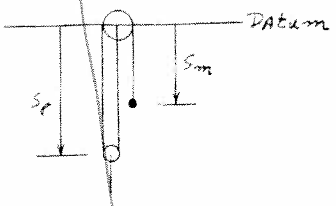
Energy : Here, the speed of the car is $v = \left(\frac{35 \text{ mi}}{\text{h}}\right) \times \left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right) \times \left(\frac{1 \text{ h}}{3600 \text{ s}}\right)$ = 51.33 ft/s. Thus, the kinetic energy of the car is

$$U = \frac{1}{2} mv^2 = \frac{1}{2}\left(\frac{5000}{32.2}\right)\left(51.33^2\right) = 204.59\left(10^3\right) \text{ ft} \cdot \text{lb}$$

The power of the bulb is $P_{bulb} = 100 \text{ W} \times \left(\frac{1 \text{ hp}}{746 \text{ W}}\right) \times \left(\frac{550 \text{ ft} \cdot \text{lb/s}}{1 \text{ hp}}\right)$ = 73.73 ft · lb/s. Thus,

$$t = \frac{U}{P_{bulb}} = \frac{204.59(10^3)}{73.73} = 2774.98 \text{ s} = 46.2 \text{ min} \qquad \textbf{Ans}$$

***14-52.** Determine the power output of the draw-works motor M necessary to lift the 600-lb drill pipe upward with a constant speed of 4 ft/s. The cable is tied to the top of the oil rig, wraps around the lower pulley, then around the top pulley and then to the motor.



$$2s_p + s_M = l$$

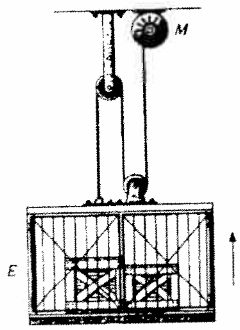
$$2v_p = -v_M$$

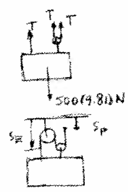
$$2(-4) = -v_M$$

$$v_M = 8 \text{ ft/s}$$

$$P_o = Fv = \left(\frac{600}{2}\right)(8) = 2400 \text{ ft}\cdot\text{lb/s} = 4.36 \text{ hp} \qquad \textbf{Ans}$$

14-53. The 500-kg elevator starts from rest and travels upward with a constant acceleration $a_c = 2 \text{ m/s}^2$. Determine the power output of the motor M when $t = 3$ s. Neglect the mass of the pulleys and cable.





$+\uparrow \Sigma F_y = m\,a_y;\qquad 3T - 500(9.81) = 500(2)$

$$T = 1968.33 \text{ N}$$

$$3s_E - s_P = l$$

$$3v_E = v_P$$

When $t = 3$ s,

$(+\uparrow)\; v = v_0 + a_c t$

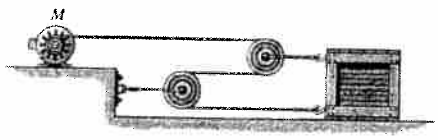
$$v_E = 0 + 2(3) = 6 \text{ m/s}$$

$$v_P = 3(6) = 18 \text{ m/s}$$

$$P_O = 1968.33(18)$$

$$P_O = 35.4 \text{ kW} \qquad \textbf{Ans}$$

14-54. The crate has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively. If the motor M supplies a cable force of $F = (8t^2 + 20)$ N, where t is in seconds, determine the power output developed by the motor when $t = 5$ s.



Time to start motor, $\mu_s = 0.3$

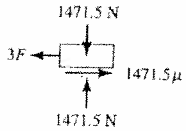
$$\overset{+}{\leftarrow}\sum F_x = 0; \quad 3F - 1471.5(0.3) = 0$$

$$F = 147.15 \text{ N}$$

$$F = 8t^2 + 20$$

$$147.15 = 8t^2 + 20$$

$$t = 3.987 \text{ s}$$



Motion: $\mu_k = 0.2$

$$\overset{+}{\leftarrow}\sum F_x = ma_x; \quad 3F - 1471.5(0.2) = 150a$$

$$3(8t^2 + 20) - 294.3 = 150a$$

$$a = 0.160t^2 - 1.5620$$

$$\int_0^v dv = \int_{3.987}^{5} (0.160t^2 - 1.5620)dt$$

When $t = 5$ s,

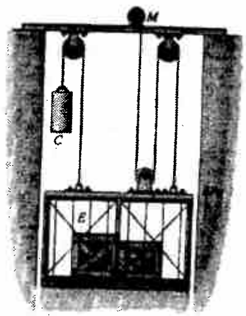
$$v = 0.160\left(\frac{t^3}{3}\right)\Big|_{3.987}^{5} - 1.5620\,t\Big|_{3.987}^{5}$$

$$v = 1.7045 \text{ ft/s}$$

$$P_O = 3[8(5)^2 + 20](1.7045) = 1124.97 \text{ N} \cdot \text{m/s}$$

$$P_O = 1.12 \text{ kW}$$ **Ans**

14-55. The elevator E and its freight have a total mass of 400 kg. Hoisting is provided by the motor M and the 60-kg block C. If the motor has an efficiency of $\epsilon = 0.6$, determine the power that must be supplied to the motor when the elevator is hoisted upward at a constant speed of $v_E = 4$ m/s.

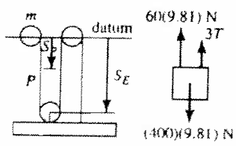


Elevator :

Since $a = 0$,

$$+\uparrow \Sigma F_y = 0; \quad 60(9.81) + 3T - 400(9.81) = 0$$

$$T = 1111.8 \text{ N}$$



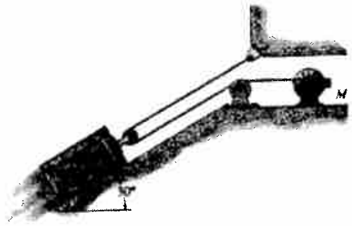
$$2s_E + (s_E - s_P) = l$$

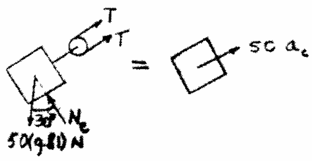
$$3v_E = v_P$$

Since $v_E = -4$ m/s, $v_P = -12$ m/s

$$P_i = \frac{\mathbf{F} \cdot \mathbf{v}_E}{e} = \frac{(1111.8)(12)}{0.6} = 22.2 \text{ kW}$$ **Ans**

•14-56. The 50-kg crate is hoisted up the 30° incline by the pulley system and motor M. If the crate starts from rest and by constant acceleration attains a speed of 4 m/s after traveling 8 m along the plane, determine the power that must be supplied to the motor at this instant. Neglect friction along the plane. The motor has an efficiency of $\epsilon = 0.74$.





$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$(4)^2 = 0 + 2a_c(8 - 0)$$

$$a_c = 1\ \text{m/s}^2$$

$+\nearrow \Sigma F_x = ma_x;\quad 2T - 50(9.81)\sin 30° = (50)(1) \qquad T = 147.6\ \text{N}$

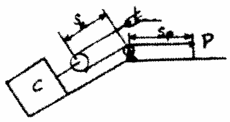
$$2s_C + s_P = l$$

$$2v_C = -v_P$$

$$(2)(-4) = -v_P$$

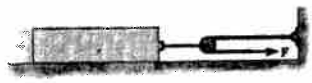
$$v_P = 8\ \text{m/s}$$

$$P_o = \mathbf{T} \cdot \mathbf{v}_P = 147.6(8) = 1181\ \text{W}$$



$P_i = \frac{P_o}{e} = \frac{1181}{0.74} = 1595.9\ \text{W} = 1.60\ \text{kW}$ **Ans**

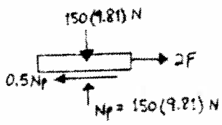
14-57. The block has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. If a force $F = (60t^2)$ N, where t is in seconds, is applied to the cable, determine the power developed by the force when $t = 5$ s. *Hint:* First determine the time needed for the force to cause motion.



$\xrightarrow{+} \Sigma F_x = 0;\quad 2F - 0.5(150)(9.81) = 0$

$$F = 367.875 = 60t^2$$

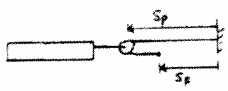
$$t = 2.476\ \text{s}$$



$\xrightarrow{+} \Sigma F_x = ma_x;\quad 2(60t^2) - 0.4(150)(9.81) = 150a_p$

$$a_p = 0.8t^2 - 3.924$$

$$dv = a\,dt$$



$$\int_0^v dv = \int_{2.476}^{5} (0.8t^2 - 3.924)\,dt$$

$$v = \left(\frac{0.8}{3}\right)t^3 - 3.924t\Big|_{2.476}^{5} = 19.38\ \text{m/s}$$

$$s_p + (s_p - s_F) = l$$

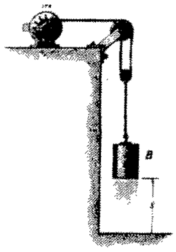
$$2v_p = v_F$$

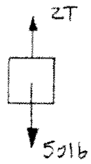
$$v_F = 2(19.38) = 38.76\ \text{m/s}$$

$$F = 60(5)^2 = 1500\ \text{N}$$

$P = \mathbf{F} \cdot \mathbf{v} = 1500(38.76) = 58.1\ \text{kW}$ **Ans**

14-58. The 50-lb load is hoisted by the pulley system and motor *M*. If the crate starts from rest and by constant acceleration attains a speed of 15 ft/s after rising $s = 6$ ft, determine the power that must be supplied to the motor at this instant. The motor has an efficiency of $\epsilon = 0.76$. Neglect the mass of the pulleys and cable.





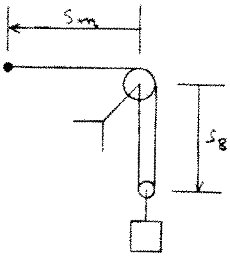
$(+\uparrow) \quad v^2 = v_0^2 + 2\,a_c(s - s_0)$

$(15)^2 = 0 + 2\,a_c(6 - 0)$

$a_c = 18.75 \text{ ft/s}^2$

$+\uparrow \Sigma F_y = m\,a_y; \quad 2T - 50 = \frac{50}{32.2}(18.75)$

$T = 39.56 \text{ lb}$



$2\,s_B + s_M = l$

$2\,v_B = -v_M$

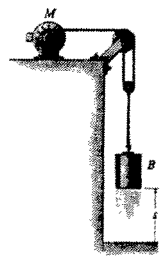
$2(-15) = -v_M$

$v_M = 30 \text{ ft/s}$

$P_o = 30(39.56) = 1186.7 \text{ ft} \cdot \text{lb/s} = 2.16 \text{ hp}$

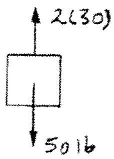
$P_i = \frac{2.16}{0.76} = 2.84 \text{ hp}$ **Ans**

14-59. The 50-lb load is hoisted by the pulley system and motor *M*. If the motor exerts a constant force of 30 lb on the cable, determine the power that must be supplied to the motor if the load has been hoisted $s = 10$ ft starting from rest. The motor has an efficiency of $\epsilon = 0.76$.



$+\uparrow \Sigma F_y = m\,a_y; \quad 2(30) - 50 = \frac{50}{32.2}a_B$

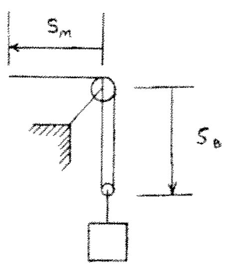
$a_B = 6.44 \text{ ft/s}^2$



$(+\downarrow)\; v^2 = v_0^2 + 2a_c(s - s_0)$

$v_B^2 = 0 + 2(6.44)(10 - 0)$

$v_B = -11.349 \text{ ft/s}$



$2s_B + s_M = l$

$2v_B = -v_M$

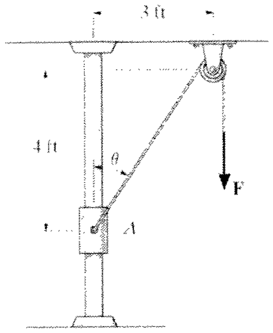
$v_M = -2(-11.349) = 22.698 \text{ ft/s}$

$P_o = \mathbf{F} \cdot \mathbf{v} = 30(22.698) = 680.94 \text{ ft} \cdot \text{lb/s}$

$P_i = \frac{680.94}{0.76} = 895.97 \text{ ft} \cdot \text{lb/s}$

$P_i = 1.63 \text{ hp}$ **Ans**

***14–60.** The 10-lb collar starts from rest at A and is lifted by applying a constant vertical force of $F = 25$ lb to the cord. If the rod is smooth, determine the power developed by the force at the instant $\theta = 60°$.



Work of **F**

$U_{1-2} = 25(5 - 3.464) = 38.40$ lb·ft

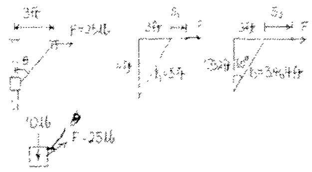
$T_1 + \Sigma U_{1-2} = T_2$

$0 + 38.40 - 10(4 - 1.732) = \frac{1}{2}(\frac{10}{32.2})v^2$

$v = 10.06$ ft/s

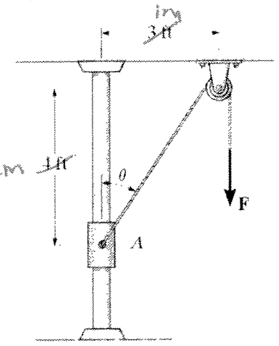
$P = \mathbf{F} \cdot \mathbf{v} = 25\cos 60°(10.06) = 125.76$ ft·lb/s

$P = 0.229$ hp **Ans**



28 100 N 1m/s

~~14–61~~. The ~~10-lb~~ collar starts from rest at A and is lifted with a constant speed of ~~2 ft/s~~ along the smooth rod. Determine the power developed by the force **F** at the instant shown.

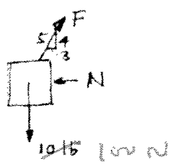


$F\left(\frac{1.2}{1.562}\right) - 100 = 0$

$F = 130$ N

$+\uparrow \Sigma F_y = m\,a_y;\quad F(\frac{4}{5}) - 10 = 0$

~~$F = 12.5$ lb~~



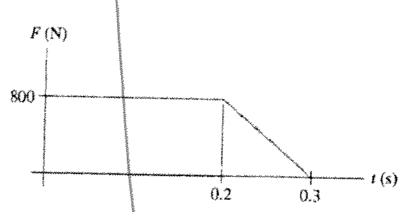
$P = \mathbf{F} \cdot \mathbf{v} = 12.5(\frac{4}{5})(2) = 20$ lb·ft/s

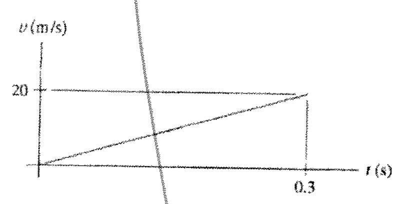
$= 0.0364$ hp **Ans**

$= 130\left(\frac{1.2}{1.562}\right)(1) = 100$ N·m/s

$= 100$ W

14-62. An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the power applied as a function of time and the work done in $t = 0.3$ s.





For $0 \leq t \leq 0.2$

$F = 800$ N

$$v = \frac{20}{0.3}t = 66.67\,t$$

$P = \mathbf{F}\cdot\mathbf{v} = 53.3\,t$ kW **Ans**

For $0.2 \leq t \leq 0.3$

$F = 2400 - 8000\,t$

$v = 66.67\,t$

$P = \mathbf{F}\cdot\mathbf{v} = (160\,t - 533\,t^2)$ kW **Ans**

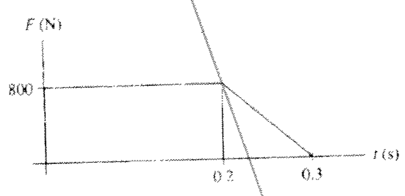
$$u = \int_0^{0.3} P\,dt$$

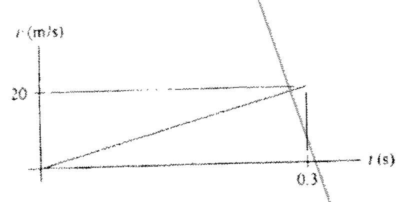
$$u = \int_0^{0.2} 53.3\,t\,dt + \int_{0.2}^{0.3} (160\,t - 533\,t^2)\,dt$$

$$= \frac{53.3}{2}(0.2)^2 + \frac{160}{2}[(0.3)^2 - (0.2)^2] - \frac{533}{3}[(0.3)^3 - (0.2)^3]$$

$= 1.69$ kJ **Ans**

14-63. An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the maximum power developed during the 0.3-second time period.





See solution to Prob. 14–62.

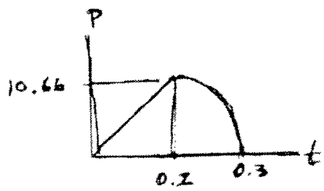
$P = 160\,t - 533\,t^2$

$\frac{dP}{dt} = 160 - 1066.6\,t = 0$

$t = 0.15 \text{ s} < 0.2 \text{ s}$

Thus maximum occurs at $t = 0.2$ s

$P_{max} = 53.3(0.2) = 10.7 \text{ kW}$ **Ans**



•14-64. Solve Prob. 14-8 using the conservation of energy equation.

100 km/h = 27.778 m/s

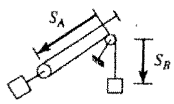
$$T_1 + V_1 = T_2 + V_2$$

$$0 + mgh = \frac{1}{2}mv^2 + 0$$

$$h = \frac{v^2}{2g}$$

$$= \frac{(27.778)^2}{2(9.81)} = 39.3 \quad \textbf{Ans}$$

14-65. Solve Prob. 14-15 using the conservation of energy equation.



$$2s_A + s_B = l$$

$$2\Delta s_A + \Delta s_B = 0$$

$$2v_A + v_B = 0$$

$$T_1 + V_1 = T_2 + V_2$$

$$[0+0] + [0+0] = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)(2v_A)^2 + 10(10) - 60\left(\frac{3}{5}\right)(5)$$

$v_A = 7.18$ ft/s **Ans**

14-66. Solve Prob. 14-18 using the conservation of energy equation.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(100)(0.5)^2 + \frac{1}{2}(50)(0.5)^2 = \frac{1}{2}(20)v^2 + 0$$

$v = 1.37$ m/s **Ans**

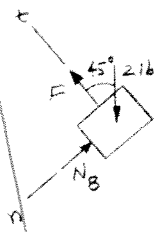
14-67. Solve Prob. 14-31 using the conservation of energy equation.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(20)(2)^2 + 0 = 0 + \frac{1}{2}(50)s^2 + \frac{1}{2}(100)s^2$$

$s = 0.730$ m **Ans**

***14-68.** Solve Prob. 14-36 using the conservation of energy equation.



$\nwarrow \Sigma F_n = ma_n;\quad 2\sin 45° = \left(\frac{2}{32.2}\right)\left(\frac{v_B^2}{1.5}\right)$

$v_B = 5.844$ ft/s

Datum at A:

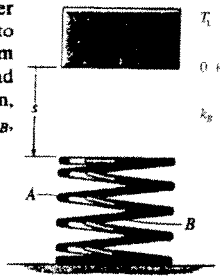
$T_A + V_A = T_B + V_B$

$$0+\frac{1}{2}(2)(\pi(1.5)-l_0)^2 = \frac{1}{2}\left(\frac{2}{32.2}\right)(5.844)^2+\frac{1}{2}(2)\left[\pi(1.5)-\frac{\pi}{4}(1.5)-l_0\right]^2+2(1.5\sin 45°)$$

$l_0 = 2.77$ ft **Ans**

29

14-69. Two equal-length springs are "nested" together in order to form a shock absorber. If it is designed to arrest the motion of a 2-kg mass that is dropped $s = 0.5$ m above the top of the springs from an at-rest position, and the maximum compression of the springs is to be 0.2 m, determine the required stiffness of the inner spring, k_B, if the outer spring has a stiffness $k_A = 400$ N/m.



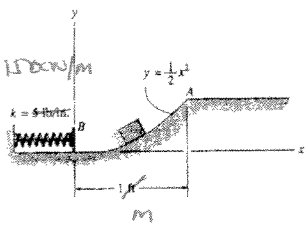
$T_1 + V_1 = T_2 + V_2$

$0+0 = 0-2(9.81)(0.5+0.2)+\frac{1}{2}(400)(0.2)^2+\frac{1}{2}(k_B)(0.2)^2$

$k_B = 287$ N/m **Ans**

30

14-70. Determine the smallest amount the spring at B must be compressed against the 0.5-~~lb~~ kg block so that when it is released from B it slides along the smooth surface and reaches point A.



Datum at B:

$y_A = \frac{1}{2}(1)^2 = 0.5$ ~~ft~~ m

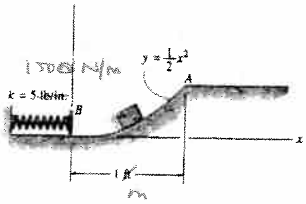
$T_1 + V_1 = T_2 + V_2$

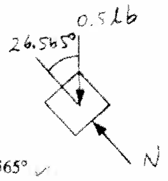
$0+\frac{1}{2}[~~5(12)~~ \; 1500]x^2 = 0+0.5(0.5)$

~~$x = 0.0913$ ft $= 1.10$ in.~~ ~~**Ans**~~

$= 0.0183$ m $= 18.3$ mm

14-71. If the spring is compressed 3 in. against the 0.5-lb block and it is released from rest, determine the normal force of the smooth surface on the block when it reaches point $x = 0.5$ ft.





$$y = \frac{1}{2}x^2\Big|_{x=0.5} = \frac{1}{2}(0.5)^2 = 0.125$$

$$\frac{dy}{dx} = x\Big|_{x=0.5} = 0.5, \qquad \theta = \tan^{-1}(0.5) = 26.565^\circ$$

$$\frac{d^2y}{dx^2} = 1$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+(0.5)^2\right]^{\frac{3}{2}}}{|1|} = 1.398$$

Datum at B:

$$T_1 + V_1 = T_2 + V_2$$

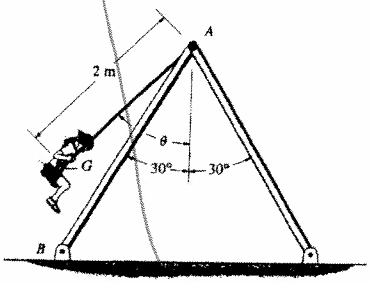
$$0 + \frac{1}{2}[5(12)]\left(\frac{3}{12}\right)^2 = \frac{1}{2}\left(\frac{0.5}{32.2}\right)(v)^2 + 0.5(0.125)$$

$v = 15.279$ ft/s

$$+\nwarrow \Sigma F_n = ma_n; \qquad N - 0.5\cos 26.565^\circ = \frac{0.5}{32.2}\left(\frac{(15.279)^2}{1.398}\right)$$

$N = 3.04$ lb **Ans**

***14-72.** The girl has a mass of 40 kg and center of mass at G. If she is swinging to a maximum height defined by $\theta = 60^\circ$, determine the force developed along each of the four supporting posts such as AB at the instant $\theta = 0^\circ$. The swing is centrally located between the posts.



The maximum tension in the cable occurs when $\theta = 0^\circ$.

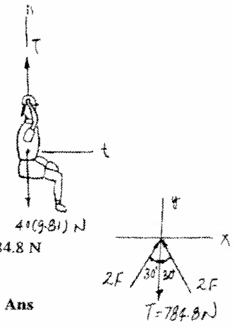
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 40(9.81)(-2\cos 60^\circ) = \tfrac{1}{2}(40)v^2 + 40(9.81)(-2)$$

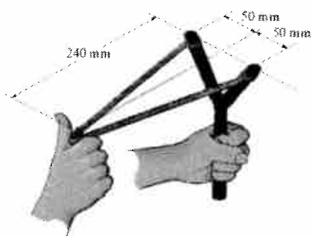
$v = 4.429$ m/s

$$+\uparrow \Sigma F_n = ma_n; \qquad T - 40(9.81) = (40)\left(\frac{4.429^2}{2}\right) \qquad T = 784.8\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad 2(2F)\cos 30^\circ - 784.8 = 0 \qquad F = 227\text{ N}$$ **Ans**



14-73. Each of the two elastic rubber bands of the slingshot has an unstretched length of 200 mm. If they are pulled back to the position shown and released from rest, determine the speed of the 25-g pellet just after the rubber bands become unstretched. Neglect the mass of the rubber bands and the change in elevation of the pellet while it is constrained by the rubber bands. Each rubber band has a stiffness of $k = 50$ N/m.



$$T_1 + V_1 = T_2 + V_2$$

$$0 + (2)\left(\frac{1}{2}\right)(50)\left[\sqrt{(0.05)^2 + (0.240)^2} - 0.2\right]^2 = \frac{1}{2}(0.025)v^2$$

$v = 2.86$ m/s **Ans**

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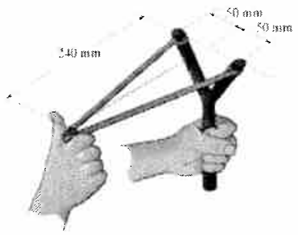
14–74. Each of the two elastic rubber bands of the slingshot has an unstretched length of 200 mm. If they are pulled back to the position shown and released from rest, determine the maximum height the 25-g pellet will reach if it is fired vertically upward. Neglect the mass of the rubber bands and the change in elevation of the pellet while it is constrained by the rubber bands. Each rubber band has a stiffness $k = 50$ N/m.



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left(\frac{1}{2}\right)(50)\left[\sqrt{(0.05)^2 + (0.240)^2} - 0.2\right]^2 = 0 + 0.025(9.81)h$$

$h = 0.416$ m $= 416$ mm **Ans**

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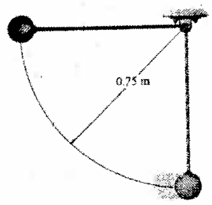
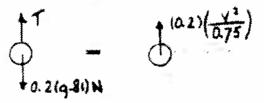
14–75. The bob of the pendulum has a mass of 0.2 kg and is released from rest when it is in the horizontal position shown. Determine its speed and the tension in the cord at the instant the bob passes through its lowest position.





Datum at initial position.

$$T_1 + V_1 = T_2 + V_2$$

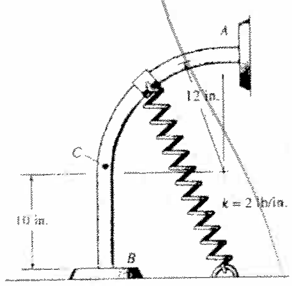
$$0 + 0 = \frac{1}{2}(0.2)(v_2)^2 - (0.2)(9.81)(0.75)$$

$v_2 = 3.836$ m/s $= 3.84$ m/s **Ans**

$+\uparrow \Sigma F_n = ma_n;\quad T - 0.2(9.81) = 0.2\left(\frac{(3.836)^2}{0.75}\right)$

$T = 5.89$ N **Ans**

***14–76.** The 5-lb collar is released from rest at A and travels along the smooth guide. Determine the speed of the collar just before it strikes the stop at B. The spring has an unstretched length of 12 in.



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 5\left(\frac{22}{12}\right) + \frac{1}{2}(24)\left(\frac{10}{12}\right)^2 = \frac{1}{2}\left(\frac{5}{32.2}\right)v_B^2 + 0 + 0$$

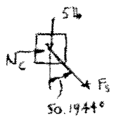
$v_B = 15.0$ ft/s **Ans**

14-77. The 5-lb collar is released from rest at A and travels along the smooth guide. Determine its speed when its center reaches point C and the normal force it exerts on the rod at this point. The spring has an unstretched length of 12 in., and point C is located just before the end of the curved portion of the rod.

$$T_A + V_A = T_C + V_C$$

$$0 + 0 + \frac{1}{2}[2(12)](\frac{10}{12})^2 + 5(\frac{12}{12}) = \frac{1}{2}(\frac{5}{32.2})v^2 + \frac{1}{2}[2(12)](\sqrt{(\frac{12}{12})^2 + (\frac{10}{12})^2} - \frac{12}{12}]^2$$

$v = 12.556\ \text{ft/s} = 12.6\ \text{ft/s}$ **Ans**



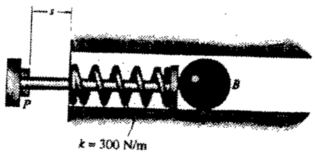
$\overset{+}{\searrow} \Sigma F_n = m\,a_n;\quad N_C + F_s \sin 50.1944° = \frac{5}{32.2}(\frac{(12.556)^2}{1})$

$F_s = ks;\quad F_s = 2(12)[\sqrt{(\frac{12}{12})^2 + (\frac{10}{12})^2} - \frac{12}{12}] = 7.2410\ \text{lb}$

Thus,

$N_C = 18.9\ \text{lb}$ **Ans**

14-78. The firing mechanism of a pinball machine consists of a plunger P having a mass of 0.25 kg and a spring of stiffness $k = 300$ N/m. When $s = 0$, the spring is compressed 50 mm. If the arm is pulled back such that $s = 100$ mm and released, determine the speed of the 0.3-kg pinball B *just before* the plunger strikes the stop, i.e., $s = 0$. Assume all surfaces of contact to be smooth. The ball moves in the horizontal plane. Neglect friction, the mass of the spring, and the rolling motion of the ball.

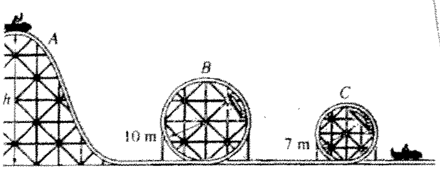


$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(300)(0.1 + 0.05)^2 = \frac{1}{2}(0.25)(v_2)^2 + \frac{1}{2}(0.3)(v_2)^2 + \frac{1}{2}(300)(0.05)^2$$

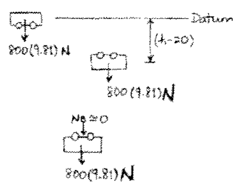
$v_2 = 3.30\ \text{m/s}$ **Ans**

14-79. The roller-coaster car has a mass of 800 kg, including its passenger, and starts from the top of the hill A with a speed $v_A = 3$ m/s. Determine the minimum height h of the hill crest so that the car travels around both inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C?



$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(800)(3)^2 + 0 = \frac{1}{2}(800)(v_B^2) - 800(9.81)(h - 20)$$



$+\downarrow \Sigma F_n = m\,a_n;\quad 800(9.81) = 800(\frac{v_B^2}{10})$

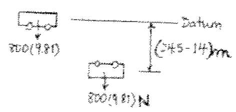
Thus, $v_B = 9.90\ \text{m/s}$

$h = 24.5\ \text{m}$ **Ans**

At B: $N_B = 0$ **Ans** (For h to be minimum)

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}(800)(3)^2 + 0 = \frac{1}{2}(800)(v_C)^2 - 800(9.81)(24.5 - 14)$$



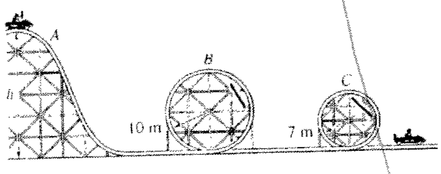
$v_C = 14.69\ \text{m/s}$

$a_n = \frac{14.69^2}{7}$

$+\downarrow \Sigma F_n = m\,a_n;\quad N_C + 800(9.81) = 800(\frac{14.69^2}{7})$

$N_C = 16.8\ \text{kN}$ **Ans**

***14-80.** The roller-coaster car has a mass of 800 kg, including its passenger. If it is released from rest at the top of the hill A, determine the minimum height h of the hill crest so that the car travels around both inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C?

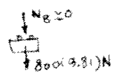


Since friction is neglected, the car will travel around the 7 m - loop provided it first travels around the 10 m - loop.

$T_A + V_A = T_B + V_B$

$0 + 0 = \frac{1}{2}(800)(v_B^2) - 800(9.81)(h-20)$

$+\downarrow \Sigma F_n = m a_n;\qquad 800(9.81) = 800(\frac{v_B^2}{10})$



Thus, $\qquad v_B = 9.90$ m/s

$h = 25.0$ m $\qquad$ **Ans**

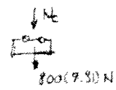
At B: $\qquad N_B = 0 \qquad$ **Ans** (For h to be minimum.)

$T_A + V_A = T_C + V_C$

$0 + 0 = \frac{1}{2}(800)(v_C)^2 - 800(9.81)(25-14)$

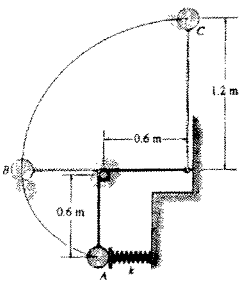
$v_C = 14.69$ m/s

$+\downarrow \Sigma F_n = m a_n;\qquad N_C + 800(9.81) = 800(\frac{(14.69)^2}{7})$

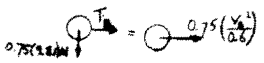


$N_C = 16.8$ kN $\qquad$ **Ans**

14-81. The 0.75-kg bob of a pendulum is fired from rest at position A by a spring which has a stiffness of $k = 6$ kN/m and is compressed 125 mm. Determine the speed of the bob and the tension in the cord when the bob is at positions B and C. Point B is located on the path where the radius of curvature is still 0.6 m, i.e., just before the cord becomes horizontal.

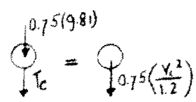


Datum at A:



$T_A + V_A = T_B + V_B$

$0 + \frac{1}{2}(6000)(0.125)^2 = \frac{1}{2}(0.75)(v_B)^2 + 0.75(9.81)(0.6)$



$v_B = 10.64 = 10.6$ m/s $\qquad$ **Ans**

$\overset{+}{\rightarrow}\Sigma F_n = ma_n;\qquad T_B = 0.75\left(\frac{(10.64)^2}{0.6}\right) = 142$ N $\qquad$ **Ans**

$T_A + V_A = T_C + V_C$

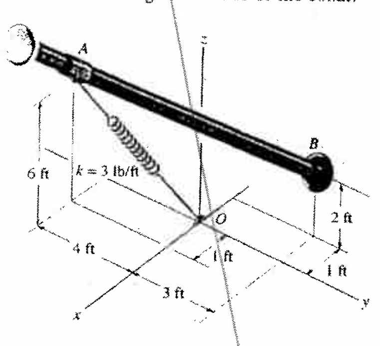
$0 + \frac{1}{2}(6000)(0.125)^2 = \frac{1}{2}(0.75)(v_C)^2 + 0.75(9.81)(1.8)$

$v_C = 9.470 = 9.47$ m/s $\qquad$ **Ans**

$+\downarrow \Sigma F_n = ma_n;\qquad T_C + 0.75(9.81) = 0.75\left(\frac{(9.470)^2}{1.2}\right)$

$T_C = 48.7$ N $\qquad$ **Ans**

14-82. The spring has a stiffness $k = 3$ lb/ft and an unstretched length of 2 ft. If it is attached to the 5-lb smooth collar and the collar is released from rest at A, determine the speed of the collar just before it strikes the end of the rod at B. Neglect the size of the collar.



Datum at B.

$|r_{OA}| = \sqrt{(1)^2 + (4)^2 + (6)^2} = 7.28$ ft

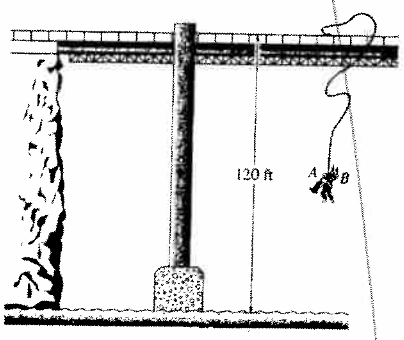
$|r_{OB}| = \sqrt{(1)^2 + (3)^2 + (2)^2} = 3.74$ ft

$T_1 + V_1 = T_2 + V_2$

$0 + (5)(6-2) + \frac{1}{2}(3)(7.28-2)^2 = \frac{1}{2}(\frac{5}{32.2})v_B^2 + \frac{1}{2}(3)(3.74-2)^2$

$v_B = 27.2$ ft/s **Ans**

14-83. Just for fun, two 150-lb engineering students A and B intend to jump off the bridge from rest using an elastic cord (bungee cord) having a stiffness $k = 80$ lb/ft. They wish to just reach the surface of the river, when A, attached to the cord, lets go of B at the instant they touch the water. Determine the proper unstretched length of the cord to do the stunt, and calculate the maximum acceleration of student A and the maximum height he reaches above the water after the rebound. From your results, comment on the feasibility of doing this stunt.



$T_1 + V_1 = T_2 + V_2$

$0 + 2(150)(120) = 0 + \frac{1}{2}(80)(x)^2$

$x = 30$ ft

Unstretched length of cord.

$120 = l + 30$

$l = 90$ ft **Ans**

When A lets go of B.

$T_2 + V_2 = T_3 + V_3$

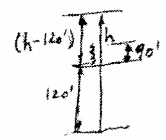
$0 + \frac{1}{2}(80)(30)^2 = 0 + (150)h$

$h = 240$ ft

This is not possible since 90 ft cord would have to stretch again, i.e., $h_{max} = 120 + 90 = 210$ ft

Thus $h > 120 + 90 = 210$ ft

$T_2 + V_2 = T_3 + V_3$



$0 + \frac{1}{2}(80)(30)^2 = 0 + 150h + \frac{1}{2}(80)[(h-120) - 90]^2$

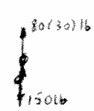
$36\,000 = 150h + 40(h^2 - 420h + 44\,100)$

$h^2 - 416.25h + 43\,200 = 0$

Choosing the root > 210 ft

$h = 219$ ft **Ans**

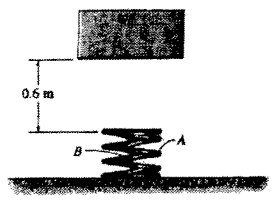
$+\uparrow \Sigma F_y = ma_y;\quad 80(30) - 150 = \frac{150}{32.2}a$



$a = 483$ ft/s^2 **Ans**

It would not be a good idea to perform the stunt since $a = 15$ g which is excessive and A rises $219' - 120' = 99$ ft above the bridge!

•14-84. Two equal-length springs having a stiffness $k_A = 300$ N/m and $k_B = 200$ N/m are "nested" together in order to form a shock absorber. If a 2-kg block is dropped from an at-rest position 0.6 m above the top of the springs, determine their deformation when the block momentarily stops.



Datum at initial position :

$T_1 + V_1 = T_2 + V_2$

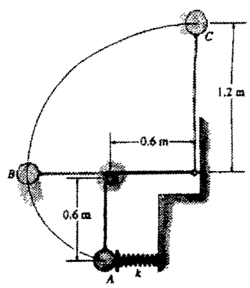
$0 + 0 = 0 - 2(9.81)(0.6 + x) + \frac{1}{2}(300 + 200)(x)^2$

$250x^2 - 19.62x - 11.772 = 0$

Solving for the positive root,

$x = 0.260$ m **Ans**

14-85. The 0.75-kg bob of a pendulum is fired from rest at position A. If the spring is compressed 50 mm and released, determine (a) its stiffness k so that the speed of the bob is zero when it reaches point B, where the radius of curvature is still 0.6 m, and (b) the stiffness k so that when the bob reaches point C the tension in the cord is zero.



a)

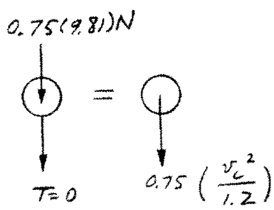
Datum at A :

$T_A + V_A = T_B + V_B$

$0 + \frac{1}{2}(k)(0.05)^2 = 0 + (0.75)(9.81)(0.6)$

$k = 3532$ N/m $= 3.53$ kN/m **Ans**

b)



$+\downarrow \Sigma F_n = ma_n; \quad 0.75(9.81) = 0.75\left(\frac{v_C^2}{1.2}\right)$

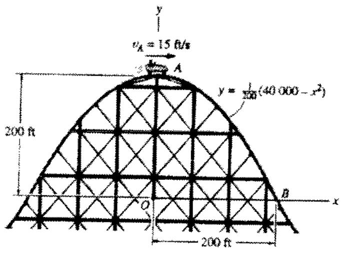
$v_C = 3.431$ m/s

$T_A + V_A = T_C + V_C$

$0 + \frac{1}{2}(k)(0.05)^2 = \frac{1}{2}(0.75)(3.431)^2 + (0.75)(9.81)(1.8)$

$k = 14\,126 = 14.1$ kN/m **Ans**

14-86. The roller-coaster car has a speed of 15 ft/s when it is at the crest of a vertical parabolic track. Determine the car's velocity and the normal force it exerts on the track when it reaches point B. Neglect friction and the mass of the wheels. The total weight of the car and the passengers is 350 lb.



$$y = \frac{1}{200}(40\,000 - x^2)$$

$$\frac{dy}{dx} = -\frac{1}{100}x\Big|_{x=200} = -2, \qquad \theta = \tan^{-1}(-2) = -63.43°$$

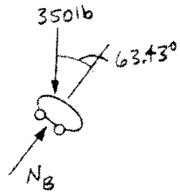
$$\frac{d^2y}{dx^2} = -\frac{1}{100}$$

Datum at A :

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\left(\frac{350}{32.2}\right)(15)^2 + 0 = \frac{1}{2}\left(\frac{350}{32.2}\right)(v_B)^2 - 350(200)$$

$v_B = 114.48 = 114$ ft/s **Ans**

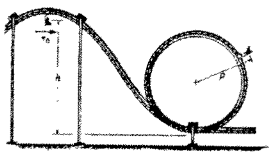


$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (-2)^2\right]^{\frac{3}{2}}}{\left|-\frac{1}{100}\right|} = 1118.0 \text{ ft}$$

$$+\!\swarrow \Sigma F_n = ma_n; \quad 350\cos 63.43° - N_B = \left(\frac{350}{32.2}\right)\frac{(114.48)^2}{1118.0}$$

$N_B = 29.1$ lb **Ans**

14-87. The Raptor is an outside loop roller coaster in which riders are belted into seats resembling ski-lift chairs. Determine the minimum speed v_0 at which the cars should coast down from the top of the hill, so that passengers can just make the loop without leaving contact with their seats. Neglect friction, the size of the car and passenger, and assume each passenger and car has a mass m.



Datum at ground :

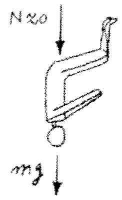
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_0^2 + mgh = \frac{1}{2}mv_1^2 + mg2\rho$$

$$v_1 = \sqrt{v_0^2 + 2g(h - 2\rho)}$$

$$+\downarrow \Sigma F_n = ma_n; \quad mg = m\left(\frac{v_1^2}{\rho}\right)$$

$$v_1 = \sqrt{g\rho}$$

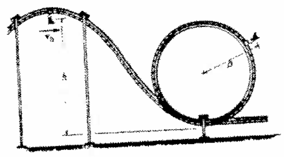


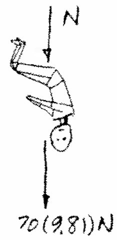
Thus,

$$g\rho = v_0^2 + 2gh - 4g\rho$$

$v_0 = \sqrt{g(5\rho - 2h)}$ **Ans**

***14-88.** The Raptor is an outside loop roller coaster in which riders are belted into seats resembling ski-lift chairs. If the cars travel at $v_0 = 4$ m/s when they are at the top of the hill, determine their speed when they are at the top of the loop and the reaction of the 70-kg passenger on his seat at this instant. The car has a mass of 50 kg. Take $h = 12$ m, $\rho = 5$ m. Neglect friction and the size of the car and passenger.





Datum at ground :

$$T_1 + V_1 = T_2 + V_2$$

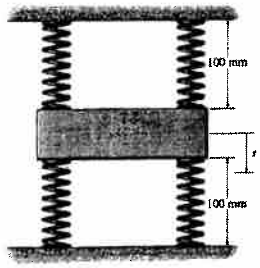
$$\frac{1}{2}(120)(4)^2 + 120(9.81)(12) = \frac{1}{2}(120)(v_1)^2 + 120(9.81)(10)$$

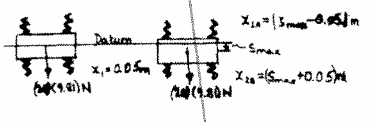
$v_1 = 7.432$ m/s **Ans**

$$+\downarrow \Sigma F_n = ma_n; \quad 70(9.81) + N = 70\left(\frac{(7.432)^2}{5}\right)$$

$N = 86.7$ N **Ans**

14-89. A block having a mass of 20 kg is attached to four springs. If each spring has a stiffness of $k = 2$ kN/m and an unstretched length of 150 mm, determine the *maximum* downward vertical displacement s_{max} of the block if it is released from rest when $s = 0$.





Place the datum at the initial elevation of the block.

$$T_1 + V_1 = T_2 + V_2$$

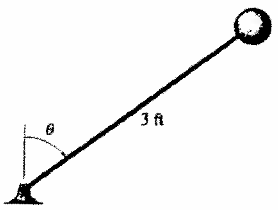
$$0 + 4\left[\frac{1}{2}(2000)(0.05)^2\right] = 0 + 2\left[\frac{1}{2}(2000)(s_{max} - 0.05)^2\right] + 2\left[\frac{1}{2}(2000)(s_{max} + 0.05)^2\right] - (20)(9.81)s_{max}$$

$$4000s_{max}^2 - 196.2s_{max} = 0$$

Solving,

$s_{max} = 0.0490$ m $= 49.0$ mm **Ans**

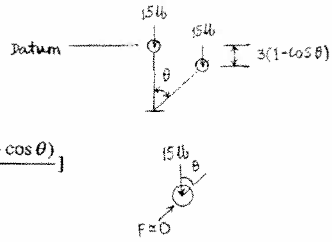
14-90. The ball has a weight of 15 lb and is fixed to a rod having a negligible mass. If it is released from rest when $\theta = 0°$, determine the angle θ at which the compressive force in the rod becomes zero.



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}\left(\frac{15}{32.2}\right)v^2 - 15(3)(1-\cos\theta)$$

$$v^2 = 193.2(1-\cos\theta)$$



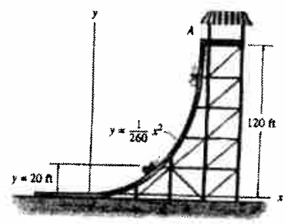
$+\swarrow \Sigma F_n = m\,a_n;$ $\quad 15\cos\theta = \frac{15}{32.2}\left[\frac{193.2(1-\cos\theta)}{3}\right]$

$$\cos\theta = 2 - 2\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

$\theta = 48.2°$ **Ans**

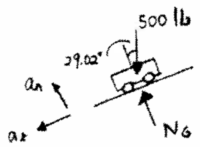
14-91. The ride at an amusement park consists of a gondola which is lifted to a height of 120 ft at A. If it is released from rest and falls along the parabolic track, determine the speed at the instant $y = 20$ ft. Also determine the normal reaction of the tracks on the gondola at this instant. The gondola and passenger have a total weight of 500 lb. Neglect the effects of friction.



$$y = \frac{1}{260}x^2$$

$$\frac{dy}{dx} = \frac{1}{130}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{130}$$



At $y = 20$ ft

$x = 72.11$ ft

$\tan\theta = \frac{dy}{dx} = 0.555, \quad \theta = 29.02°$

$$\rho = \frac{[1+(0.555)^2]^{3/2}}{\frac{1}{130}} = 194.40 \text{ ft}$$

$\nwarrow + \Sigma F_n = m\,a_n;\quad N_G - 500\cos 29.02° = \frac{500}{32.2}\left(\frac{v^2}{194.40}\right) \qquad (1)$

Datum at A:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}\left(\frac{500}{32.2}\right)v^2 - 500(100)$$

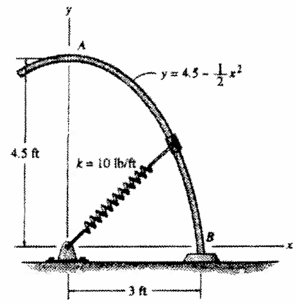
$$v^2 = 6440$$

$v = 80.2$ ft/s **Ans**

Substituting into Eq. (1) yields

$N_G = 952$ lb **Ans**

***14-92.** The 2-lb collar has a speed of 5 ft/s at A. The attached spring has an unstretched length of 2 ft and a stiffness of $k = 10$ lb/ft. If the collar moves over the smooth rod, determine its speed when it reaches point B, the normal force of the rod on the collar, and the rate of decrease in its speed.



Datum at B :

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + \frac{1}{2}(10)(4.5-2)^2 + 2(4.5) = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_B)^2 + \frac{1}{2}(10)(3-2)^2 + 0$$

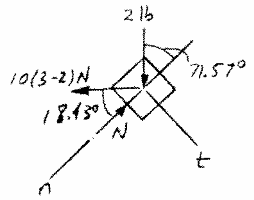
$v_B = 34.060$ ft/s $= 34.1$ ft/s **Ans**

$$y = 4.5 - \frac{1}{2}x^2$$

$$\frac{dy}{dx} = \tan\theta = -x\Big|_{x=3} = -3$$

$$\theta = -71.57^\circ \qquad \frac{d^2y}{dx^2} = -1$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+(-3)^2\right]^{\frac{3}{2}}}{|-1|} = 31.623 \text{ ft}$$



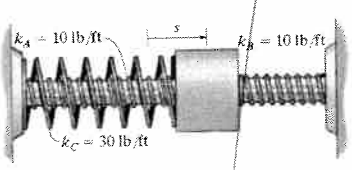
$$+\swarrow \Sigma F_n = ma_n; \qquad -N + 10\cos 18.43^\circ + 2\cos 71.57^\circ = \left(\frac{2}{32.2}\right)\left(\frac{(34.060)^2}{31.623}\right)$$

$N = 7.84$ lb **Ans**

$$+\searrow \Sigma F_t = ma_t; \qquad 2\sin 71.57^\circ - 10\sin 18.43^\circ = \left(\frac{2}{32.2}\right)a_t$$

$a_t = -20.4$ ft/s^2 **Ans**

14–93. The 20-lb collar is constrained to move on the smooth rod. It is attached to the three springs which are unstretched when $s = 0$. If the collar is displaced $s = 0.5$ ft and released from rest, determine its speed when $s = 0$.

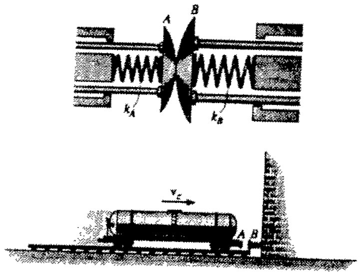


$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(10)(0.5)^2 + \frac{1}{2}(10)(0.5)^2 + \frac{1}{2}(30)(0.5)^2 = \frac{1}{2}\left(\frac{20}{32.2}\right)v^2 + 0$$

$v = 4.49$ ft/s **Ans**

14-94. A tank car is stopped by two spring bumpers A and B, having a stiffness of $k_A = 15(10^3)$ lb/ft and $k_B = 20(10^3)$ lb/ft, respectively. Bumper A is attached to the car, whereas bumper B is attached to the wall. If the car has a weight of $25(10^3)$ lb and is freely coasting at 3 ft/s, determine the maximum deflection of each spring at the instant the bumpers stop the car.



$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{25\,000}{32.2}\right)(3)^2 + 0 = 0 + \frac{1}{2}(15\,000)(x_A)^2 + \frac{1}{2}(20\,000)(x_B)^2$$

Since the force in the springs is the same at any instant,

$$F = k_A x_A = k_B x_B \qquad 15\,000 x_A = 20\,000 x_B$$

$$x_A = 1.333 x_B$$

Solving,

$x_A = 0.516$ ft **Ans**

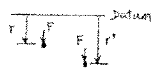
$x_B = 0.387$ ft **Ans**

14-95. If the mass of the earth is M_e, show that the gravitational potential energy of a body of mass m located a distance r from the center of the earth is $V_g = -GM_e m/r$. Recall that the gravitational force acting between the earth and the body is $F = G(M_e m/r^2)$, Eq. 13–1. For the calculation, locate the datum at $r \to \infty$. Also, prove that F is a conservative force.

The work is computed by moving F from position r to a greater position r'.

$$V = -U = -\int F\,dr$$

$$= -G M_e m \int_r^{r'} \frac{dr}{r^2}$$



$$= -G M_e m\left(\frac{1}{r} - \frac{1}{r'}\right)$$

As $r' \to \infty$,

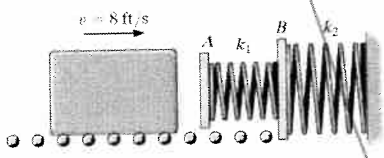
$$V = \frac{-G M_e m}{r} \qquad \textbf{Q.E.D.}$$

To be conservative, require

$$F = -\nabla V = -\frac{\partial}{\partial r}\left(-\frac{G M_e m}{r}\right)$$

$$= \frac{-G M_e m}{r^2} \qquad \textbf{Q.E.D.}$$

***14–96.** The double-spring bumper is used to stop the 1500-lb steel billet in the rolling mill. Determine the maximum deflection of the plate A caused by the billet if it strikes the plate with a speed of 8 ft/s. Neglect the mass of the springs, rollers and the plates A and B. Take $k_1 = 3000$ lb/ft, $k_2 = 45000$ lb/ft.



$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{1500}{32.2}\right)(8)^2 + 0 = 0 + \tfrac{1}{2}(3000)s_1^2 + \tfrac{1}{2}(4500)s_2^2 \qquad [1]$$

$$F_s = 3000s_1 = 4500s_2; \qquad [2]$$

$$s_1 = 1.5s_2$$

Solving Eqs.[1] and [2] yields :

$s_2 = 0.5148$ ft $\quad s_1 = 0.7722$ ft

$s_A = s_1 + s_2 = 0.7722 + 0.5148 = 1.29$ ft **Ans**