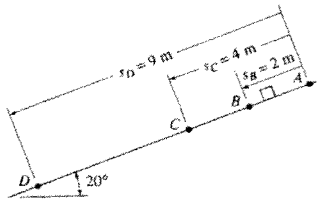
13-1. Determine the gravitational attraction between two spheres which are just touching each other. Each sphere has a mass of 10 kg and a radius of 200 mm.

The distance between the centers of the spheres is $r = 400$ mm $= 0.4$ m.

$$F = G\frac{m_1 \cdot m_2}{r^2} = 66.73\left(10^{-12}\right)\left(\frac{(10)(10)}{(0.4)^2}\right) = 41.7\left(10^{-9}\right)\text{N} = 41.7\text{ nN} \qquad \textbf{Ans}$$

13-2. By using an inclined plane to retard the motion of a falling object, and thus make the observations more accurate, Galileo was able to determine experimentally that the distance through which an object moves in free fall is proportional to the square of the time for travel. Show that this is the case, i.e., $s \propto t^2$, by determining the time t_B, t_C, and t_D needed for a block of mass m to slide from rest at A to points B, C, and D, respectively. Neglect the effects of friction.



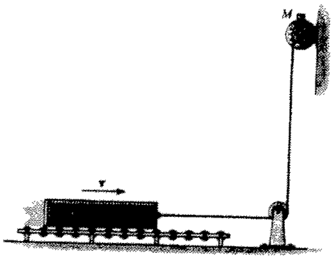
$$W \sin 20° = \frac{W}{g}a$$

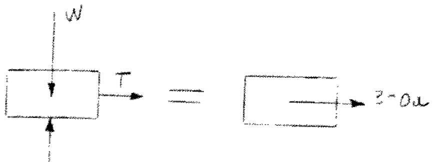
$$a = 9.81(\sin 20°) = 3.355 \text{ m/s}^2$$

$$s = \frac{1}{2}at^2$$

s	t	
2 m	1.09 s	**Ans**
4 m	1.54 s	**Ans**
9 m	2.32 s	**Ans**

13-3. The 300-kg bar B, originally at rest, is being towed over a series of small rollers. Determine the force in the cable when $t = 5$ s, if the motor M is drawing in the cable for a short time at a rate of $v = (0.4t^2)$ m/s, where t is in seconds $(0 \leq t \leq 6 \text{ s})$. How far does the bar move in 5 s? Neglect the mass of the cable, pulley, and the rollers.





$\xrightarrow{+} \Sigma F_x = ma_x;\qquad T = 300a$

$v = 0.4t^2$

$$a = \frac{dv}{dt} = 0.8t$$

When $t = 5$ s, $a = 4 \text{ m/s}^2$

$T = 300(4) = 1200$ N $= 1.20$ kN **Ans**

$ds = v\,dt$

$$\int_0^s ds = \int_0^5 0.4t^2\,dt$$

$$s = \left(\frac{0.4}{3}\right)(5)^3 = 16.7\text{ m} \qquad \textbf{Ans}$$

***13-4.** A crate having a mass of 60 kg falls horizontally off the back of a truck which is traveling at 80 km/h. Determine the coefficient of kinetic friction between the road and the crate if the crate slides 45 m on the ground with no tumbling along the road before coming to rest. Assume that the initial speed of the crate along the road is 80 km/h.

$$80\text{ km/h} = \frac{80(10^3)}{3600} = 22.22\text{ m/s}$$

$$+\uparrow \Sigma F_y = ma_y; \quad N_C - 60(9.81) = 0 \quad N_C = 588.6\text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \quad \mu_k(588.6) = 60a \quad a = 9.81\mu_k$$

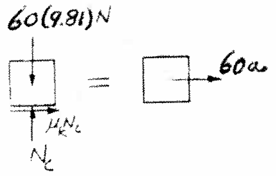
$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = (22.22)^2 - 2a(45 - 0)$$

$$a = 5.487\text{ m/s}^2$$

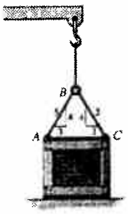
Thus,

$$\mu_k = \frac{5.487}{9.81} = 0.559 \quad \textbf{Ans}$$



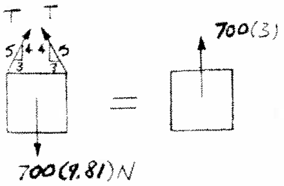
4

13-5. The crane lifts the 700-kg bin with an initial acceleration of 3 m/s². Determine the force in each of the supporting cables due to this motion.



$$+\uparrow \Sigma F_y = ma_y; \quad 2T\left(\frac{4}{5}\right) - 700(9.81) = 700(3)$$

$$T = 5.60\text{ kN} \quad \textbf{Ans}$$

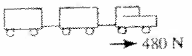


5

13-6. The baggage truck A has a mass of 800 kg and is used to pull the two cars, each with mass 300 kg. If the tractive force F on the truck is $F = 480$ N, determine the initial acceleration of the truck. What is the acceleration of the truck if the coupling at C suddenly fails? The car wheels are free to roll. Neglect the mass of the wheels.

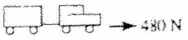
$$\overset{+}{\rightarrow} \sum F_x = ma_x; \quad 480 = [800 + 2(300)]a$$

$$a = 0.3429 = 0.343\text{ m/s}^2 \quad \textbf{Ans}$$

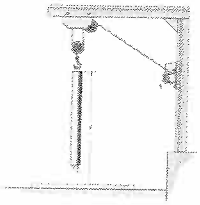


$$\overset{+}{\rightarrow} \sum F_x = ma_x; \quad 480 = (800 + 300)a$$

$$a = 0.436\text{ m/s}^2 \quad \textbf{Ans}$$

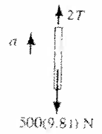


13-~~7~~6. The 500-kg fuel assembly for a nuclear reactor is being lifted out from the core of the nuclear reactor using the pulley system shown. It is hoisted upward with a constant acceleration such that $s = 0$ and $v = 0$ when $t = 0$, and $s = 2.5$ m when $t = 1.5$ s. Determine the tension in the cable at A during the motion.



$$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$2.5 = 0 + 0 + \frac{1}{2}(a)(1.5)^2$$

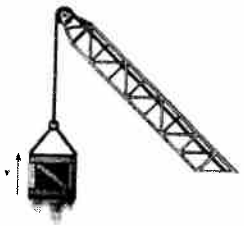


$a = 2.222\ \text{m/s}^2$

$+\uparrow \Sigma F_y = m\,a_y;\qquad 2T - 500(9.81) = 500(2.222)$

$T = 3008\ \text{N} = 3.01\ \text{kN}$ **Ans**

***13-8.** The 200-kg crate is suspended from the cable of a crane. Determine the force in the cable when $t = 2$ s if the crate is moving upward with (a) a constant velocity of 2 m/s, and (b) a speed of $v = (0.2t^2 + 2)$ m/s, where t is in seconds.



a) $+\uparrow \Sigma F_y = ma_y;\qquad T - 200(9.81) = 0$

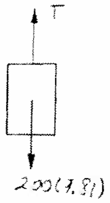
$T = 1.96\ \text{kN}$ **Ans**

b) $v = 0.2t^2 + 2$

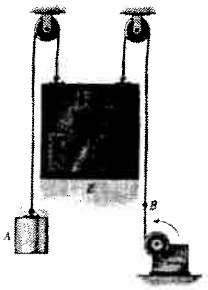
$$a = \frac{dv}{dt} = 0.4t\Big|_{t=2\,s} = 0.8\ \text{m/s}^2$$

$+\uparrow \Sigma F_y = ma_y;\qquad T - 200(9.81) = 200(0.8)$

$T = 2.12\ \text{kN}$ **Ans**



13-~~9~~7. The elevator E has a mass of 500 kg, and the counterweight at A has a mass of 150 kg. If the motor supplies a constant force of 5 kN on the cable at B, determine the speed of the elevator when $t = 3$ s, starting from rest. Neglect the mass of the pulleys and cable.

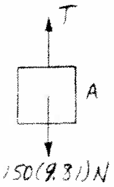


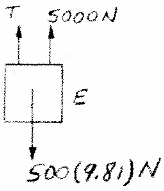
For A:

$+\downarrow \Sigma F_y = ma_y;\qquad 150(9.81) - T = 150a_A \qquad (1)$

For E:

$+\downarrow \Sigma F_y = ma_y;\qquad 500(9.81) - 5000 - T = 500a_E \qquad (2)$





$s_A + s_E = l$

$a_A = -a_E \qquad (3)$

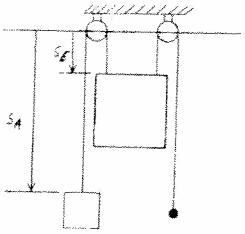
Solving:

$T = 1110\ \text{N}$

$a_E = -2.410\ \text{m/s}^2 = 2.410\ \text{m/s}^2 \uparrow$

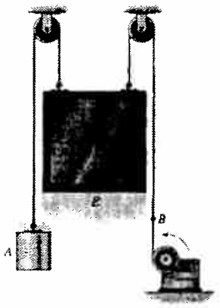
$(+\uparrow)\qquad v = v_0 + a_c t$

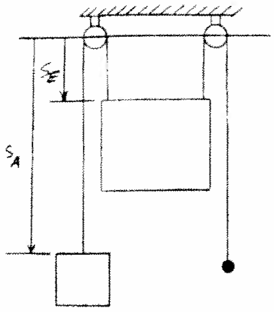
$v_E = 0 + 2.410(3) = 7.23\ \text{m/s} \uparrow$ **Ans**



8

13-10. The elevator E has a mass of 500 kg and the counterweight at A has a mass of 150 kg. If the elevator attains a speed of 10 m/s after it rises 40 m, determine the constant force developed in the cable at B. Neglect the mass of the pulleys and cable.





$(+\downarrow) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$(-10)^2 = (0)^2 + 2a_E(-40-0)$

$a_E = -1.25 \text{ m/s}^2 = 1.25 \text{ m/s}^2 \uparrow$

$s_A + s_E = l$

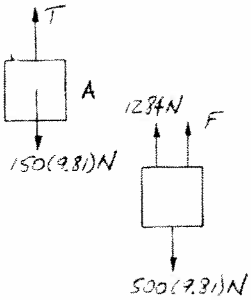
$a_A = -a_E$

For A:

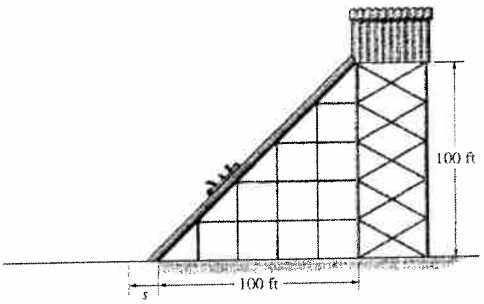
$+\downarrow \Sigma F_y = ma_y; \qquad 150(9.81) - T = 150(1.25) \qquad T = 1.284 \text{ kN}$

For E:

$+\downarrow \Sigma F_y = ma_y; \qquad 500(9.81) - 1284 - F = 500(-1.25) \qquad F = 4.25 \text{ kN}$ **Ans**



13-11. The water-park ride consists of an 800-lb sled which slides from rest down the incline and then into the pool. If the frictional resistance on the incline is $F_r = 30$ lb, and in the pool for a short distance $F_r = 80$ lb, determine how fast the sled is traveling when $s = 5$ ft.



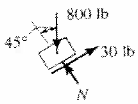
$+\swarrow \sum F_x = ma_x; \qquad 800 \sin 45^\circ - 30 = \frac{800}{32.2}a$

$a = 21.561 \text{ ft/s}^2$

$v_1^2 = v_0^2 + 2a_c(s - s_0)$

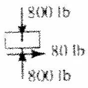
$v_1^2 = 0 + 2(21.561)(100\sqrt{(2-0)})$

$v_1 = 78.093 \text{ ft/s}$



$\overset{+}{\leftarrow} \sum F_x = ma_x; \qquad -80 = \frac{800}{32.2}a$

$a = -3.22 \text{ ft/s}^2$

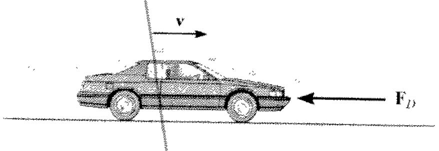


$v_2^2 = v_1^2 + 2a_c(s_2 - s_1)$

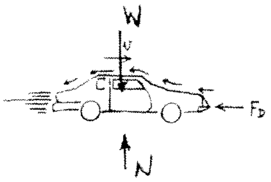
$v_2^2 = (78.093)^2 + 2(-3.22)(5 - 0)$

$v_2 = 77.9 \text{ ft/s}$ **Ans**

13–12. A car of mass m is traveling at a slow velocity v_0. If it is subjected to the drag resistance of the wind, which is proportional to its velocity, i.e., $F_D = kv$, determine the distance and the time the car will travel before its velocity becomes $0.5v_0$. Assume no other frictional forces act on the car.



$\xrightarrow{+} \Sigma F_x = m a_x;$ $\quad -F_D = m\frac{dv}{dt} \qquad (1)$



$$-kv = m\frac{dv}{dt}$$

$$-\frac{k}{m}\int_0^t dt = \int_{v_0}^{v}\frac{dv}{v}$$

$$-\frac{k}{m}t = \ln\left(\frac{v}{v_0}\right)$$

$$t = \frac{m}{k}\ln\left(\frac{v_0}{v}\right)$$

Set $v = 0.5\,v_0$, $\quad t = \frac{m}{k}\ln(2)$

$t = 0.693\frac{m}{k}$ **Ans**

From Eq.(1),

$$-kv = m\frac{v\,dv}{dx}$$

$$-kv\,dx = mv\,dv$$

$$-\int_0^x k\,dx = \int_{v_0}^{v} m\,dv$$

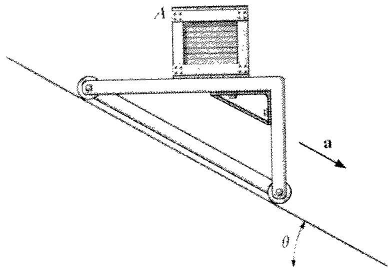
$$-kx = m(v - v_0)$$

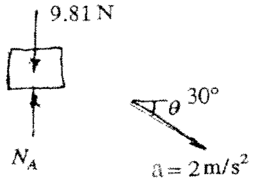
At $v = 0.5\,v_0$, $\quad x = \frac{m}{k}(0.5\,v_0)$

$x = 0.5\frac{mv_0}{k}$ **Ans**

9

13–1~~3~~. Determine the normal force the 10-kg crate A exerts on the smooth cart if the cart is given an acceleration of $a = 2\ \text{m/s}^2$ down the plane. Also, what is the acceleration of the crate? Set $\theta = 30°$.





$\overset{+}{\leftarrow} \Sigma F_x = ma_x;\qquad 0 = 10a_x;\ a_x = 0$

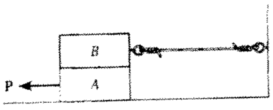
$+\downarrow \Sigma F_y = ma_y;\qquad 9.81(10) - N_A = 10(2\cos 60°)$

$N_A = 88.1\ \text{N}$ **Ans**

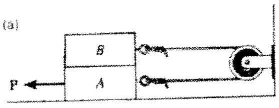
$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(0)^2 + (2\sin 30°)^2} = 1\ \text{m/s}^2$$ **Ans**

10

13–1~~4~~. Each of the two blocks has a mass m. The coefficient of kinetic friction at all surfaces of contact is μ. If a horizontal force **P** moves the bottom block, determine the acceleration of the bottom block in each case.



(a)

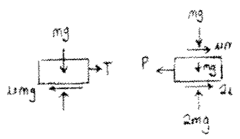


(b)

(a) Block A :

$\overset{+}{\leftarrow} \Sigma F_x = m\,a_x;\qquad P - 3\mu mg = m\,a_A$

$$a_A = \frac{P}{m} - 3\mu g$$ **Ans**



(b) $s_B + s_A = l$

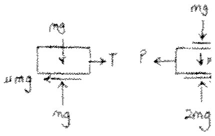
$a_A = -a_B$ (1)

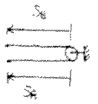
Block A :

$\overset{+}{\leftarrow} \Sigma F_x = m\,a_x;\qquad P - T - 3\mu mg = m\,a_A$ (2)

Block B :

$\overset{+}{\leftarrow} \Sigma F_x = m\,a_x;\qquad \mu mg - T = m\,a_B$ (3)



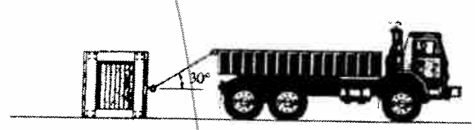


Subtract Eq.(3) from Eq.(2) :

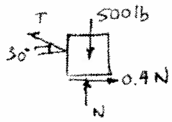
$$P - 4\mu mg = m(a_A - a_B)$$

Use Eq.(1); $\quad a_A = \dfrac{P}{2m} - 2\mu g$ **Ans**

13-15. The driver attempts to tow the crate using a rope that has a tensile strength of 200 lb. If the crate is originally at rest and has a weight of 500 lb, determine the greatest acceleration it can have if the coefficient of static friction between the crate and the road is $\mu_s = 0.4$, and the coefficient of kinetic friction is $\mu_k = 0.3$.



Equilibrium: In order to slide the crate, the towing force must overcome static friction.



$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -T\cos 30° + 0.4N = 0$ [1]

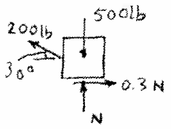
$+\uparrow \Sigma F_y = 0; \quad N + T\sin 30° - 500 = 0$ [2]

Solving Eqs.[1] and [2] yields :

$T = 187.6$ lb $\quad N = 406.2$ lb

Since $T < 200$ lb, the cord will not break at the moment the crate slides.

After the crate begins to slide, the kinetic friction is used for the calculation.

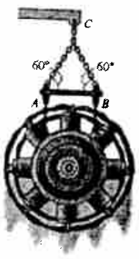


$+\uparrow \Sigma F_y = ma_y; \quad N + 200\sin 30° - 500 = 0 \quad N = 400$ lb

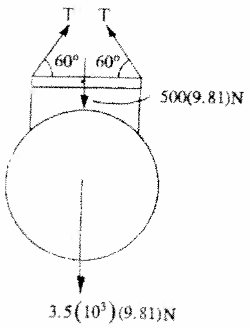
$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \quad 200\cos 30° - 0.3(400) = \frac{500}{32.2}a$

$a = 3.43$ ft/s² **Ans**

***13-16.** The 3.5-Mg engine is suspended from a 500-kg spreader beam and hoisted by a crane which gives it an acceleration of 4 m/s² when it has a velocity of 2 m/s. Determine the force in chains AC and AD during the lift.



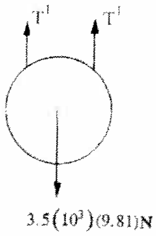
System :



$+\uparrow \Sigma F_y = ma_y; \quad 2T\sin 60° - 500(9.81) - 3.5(10^3)(9.81) = [3.5(10^3) + 500]4$

$T_{AC} = T = 31.9$ kN **Ans**

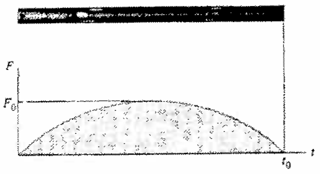
Engine :



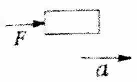
$+\uparrow \Sigma F_y = ma_y; \quad 2T' - 3.5(10^3)(9.81) = 3.5(10^3)(4)$

$T_{AD} = T' = 24.2$ kN **Ans**

13-17. The bullet of mass m is given a velocity due to gas pressure caused by the burning of powder within the barrel of the gun. Assuming this pressure creates a force of $F = F_0 \sin(\pi t/t_0)$ on the bullet, determine the velocity of the bullet at any instant it is in the barrel. What is the bullet's maximum velocity? Also, determine the position of the bullet in the barrel as a function of time.



$\xrightarrow{+} \Sigma F_x = ma_x;\qquad F_0 \sin\left(\frac{\pi t}{t_0}\right) = ma$



$a = \frac{dv}{dt} = \left(\frac{F_0}{m}\right)\sin\left(\frac{\pi t}{t_0}\right)$

$\int_0^v dv = \int_0^t \left(\frac{F_0}{m}\right)\sin\left(\frac{\pi t}{t_0}\right) dt \qquad v = -\left(\frac{F_0 t_0}{\pi m}\right)\cos\left(\frac{\pi t}{t_0}\right)\Big]_0^t$

$v = \left(\frac{F_0 t_0}{\pi m}\right)\left(1 - \cos\left(\frac{\pi t}{t_0}\right)\right)$ **Ans**

v_{max} occurs when $\cos\left(\frac{\pi t}{t_0}\right) = -1$, or $t = t_0$.

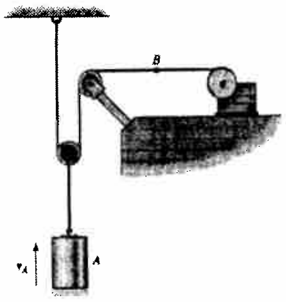
$v_{max} = \frac{2F_0 t_0}{\pi m}$ **Ans**

$\int_0^s ds = \int_0^t \left(\frac{F_0 t_0}{\pi m}\right)\left(1 - \cos\left(\frac{\pi t}{t_0}\right)\right)dt$

$s = \left(\frac{F_0 t_0}{\pi m}\right)\left[t - \frac{t_0}{\pi}\sin\left(\frac{\pi t}{t_0}\right)\right]_0^t$

$s = \left(\frac{F_0 t_0}{\pi m}\right)\left(t - \frac{t_0}{\pi}\sin\left(\frac{\pi t}{t_0}\right)\right)$ **Ans**

13-18. The 400-lb cylinder at A is hoisted using the motor and the pulley system shown. If the speed of point B on the cable is increased at a constant rate from zero to $v_B = 10$ ft/s in $t = 5$ s, determine the tension in the cable at B to cause the motion.



$2s_A + s_B = l$

$2a_A = -a_B$

$\left(\xrightarrow{+}\right) \quad v = v_0 + a_c t$

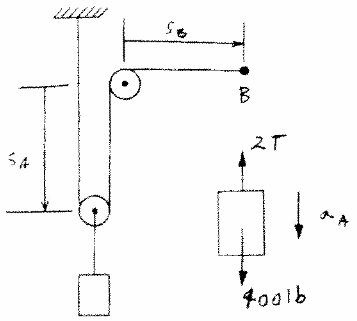
$10 = 0 + a_B(5)$

$a_B = 2 \text{ ft/s}^2$

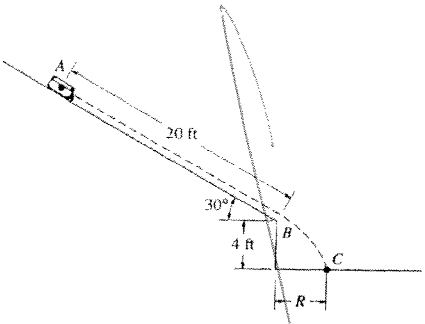
$a_A = -1 \text{ ft/s}^2$

$+\downarrow \Sigma F_y = ma_y; \qquad 400 - 2T = \left(\frac{400}{32.2}\right)(-1)$

Thus, $T = 206$ lb **Ans**

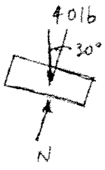


13-19. A 40-lb suitcase slides from rest 20 ft down the smooth ramp. Determine the point where it strikes the ground at C. How long does it take to go from A to C?



$+\searrow \Sigma F_x = m\,a_x; \quad 40\sin 30° = \frac{40}{32.2}a$

$a = 16.1\ \text{ft/s}^2$



$(+\searrow)\ v^2 = v_0^2 + 2\,a_c(s - s_0);$

$v_B^2 = 0 + 2(16.1)(20)$

$v_B = 25.38\ \text{ft/s}$

$(+\searrow)\ v = v_0 + a_c\,t;$

$25.38 = 0 + 16.1\,t_{AB}$

$t_{AB} = 1.576\ \text{s}$

$(\overset{+}{\rightarrow})\ s_x = (s_x)_0 + (v_x)_0\,t$

$R = 0 + 25.38\cos 30°\,(t_{BC})$

$(+\downarrow)\ s_y = (s_y)_0 + (v_y)_0\,t + \frac{1}{2}a_c t^2$

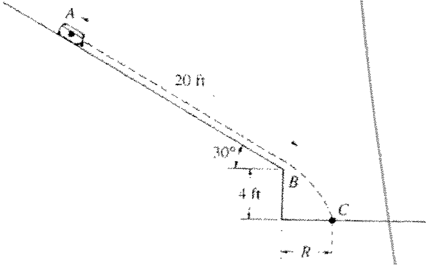
$4 = 0 + 25.38\sin 30°\,t_{BC} + \frac{1}{2}(32.2)(t_{BC})^2$

$t_{BC} = 0.2413\ \text{s}$

$R = 5.30\ \text{ft}$ **Ans**

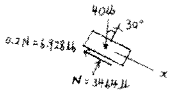
Total time $= t_{AB} + t_{BC} = 1.82\ \text{s}$ **Ans**

***13-20.** Solve Prob. 13–19 if the suitcase has an initial velocity down the ramp of $v_A = 10\ \text{ft/s}$ and the coefficient of kinetic friction along AB is $\mu_k = 0.2$.



$+\searrow \Sigma F_x = m\,a_x; \quad 40\sin 30° - 6.928 = \frac{40}{32.2}a$

$a = 10.52\ \text{ft/s}^2$



$(+\searrow)\ v^2 = v_0^2 + 2\,a_c(s - s_0);$

$v_B^2 = (10)^2 + 2(10.52)(20)$

$v_B = 22.82\ \text{ft/s}$

$(+\searrow)\ v = v_0 + a_c\,t;$

$22.82 = 10 + 10.52\,t_{AB}$

$t_{AB} = 1.219\ \text{s}$

$(\overset{+}{\rightarrow})\ s_x = (s_x)_0 + (v_x)_0\,t$

$R = 0 + 22.82\cos 30°\,(t_{BC})$

$(+\downarrow)\ s_y = (s_y)_0 + (v_y)_0\,t + \frac{1}{2}a_c t^2$

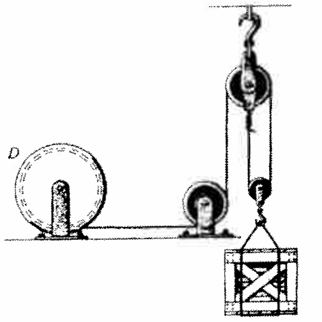
$4 = 0 + 22.82\sin 30°\,t_{BC} + \frac{1}{2}(32.2)(t_{BC})^2$

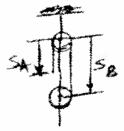
$t_{BC} = 0.2572\ \text{s}$

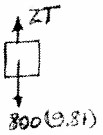
$R = 5.08\ \text{ft}$ **Ans**

Total time $= t_{AB} + t_{BC} = 1.48\ \text{s}$ **Ans**

13-21. The winding drum D is drawing in the cable at an accelerated rate of 5 m/s². Determine the cable tension if the suspended crate has a mass of 800 kg.







$$s_A + 2s_B = l$$

$$a_A = -2a_B$$

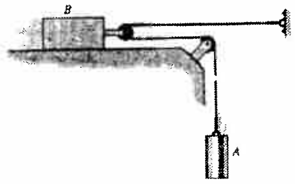
$$5 = -2a_B$$

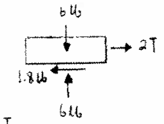
$$a_B = -2.5\ \text{m/s}^2 = 2.5\ \text{m/s}^2 \uparrow$$

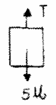
$+\uparrow \Sigma F_y = ma_y;$ $\quad 2T - 800(9.81) = 800(2.5)$

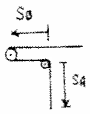
$T = 4924\ \text{N} = 4.92\ \text{kN}$ **Ans**

13-22. At a given instant the 5-lb weight A is moving downward with a speed of 4 ft/s. Determine its speed 2 s later. Block B has a weight of 6 lb, and the coefficient of kinetic friction between it and the horizontal plane is $\mu_k = 0.3$. Neglect the mass of the pulleys and cord.









$\overset{+}{\leftarrow} \Sigma F_x = ma_x;$ $\quad 1.8 - 2T = \left(\frac{6}{32.2}\right)a_B$

$+\downarrow \Sigma F_y = ma_y;$ $\quad 5 - T = \left(\frac{5}{32.2}\right)a_A$

$$s_A + 2s_B = l$$

$$a_A = -2a_B$$

Solving,

$$T = 1.85\ \text{lb}$$

$$a_A = 20.31\ \text{ft/s}^2$$

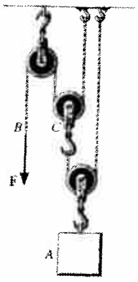
$$a_B = -10.16\ \text{ft/s}^2$$

$(+\downarrow)$ $\quad v = v_0 + a_c t$

$v_A = 4 + 20.31(2) = 44.6\ \text{ft/s}$ **Ans**

13 150N 300N

13-~~23~~. A force F = ~~15 lb~~ is applied to the cord. Determine how high the ~~30-lb~~ block A rises in 2 s starting from rest. Neglect the weight of the pulleys and cord.



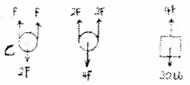
-300 300

$+\uparrow \Sigma F_y = ma_y;$ $\quad \cancel{-30} + 4F = \frac{\cancel{30}}{\cancel{32.2}} a_A$

9.81

$F =$ ~~15 lb~~ 150 N

$a_A =$ ~~32.2 ft/s²~~ 9.81 m/s²



$(+\uparrow)\; s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

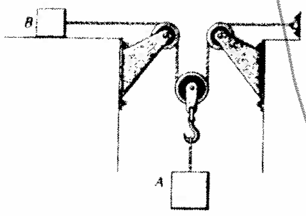
9.81

$s = 0 + 0 + \frac{1}{2}(\cancel{32.2})(2)^2$

$s =$ ~~64.4 ft~~ **Ans**

19.62m

***13-24.** At a given instant the 10-lb block A is moving downward with a speed of 6 ft/s. Determine its speed 2 s later. Block B has a weight of 4 lb, and the coefficient of kinetic friction between it and the horizontal plane is $\mu_k = 0.2$. Neglect the mass of the pulleys and cord.



Block A :

$+\downarrow \Sigma F_y = ma_y;$ $\quad 10 - 2T = \frac{10}{32.2} a_A$

Block B :

$\overset{+}{\leftarrow} \Sigma F_x = ma_x;$ $\quad -T + 0.2(4) = \frac{4}{32.2} a_B$

$2s_A + s_B = l$

$2a_A = -a_B$

Solving;

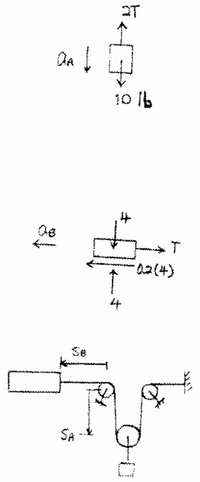
$T = 3.38$ lb

$a_A = 10.403$ ft/s^2

$a_B = -20.81$ ft/s^2

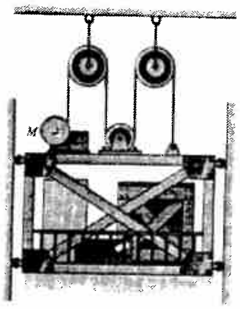
$(+\downarrow)\; v_A = (v_A)_0 + a_A t$

$v_A = 6 + 10.403(2) = 26.8$ ft/s **Ans**



14

13-~~25~~. A freight elevator, including its load, has a mass of 500 kg. It is prevented from rotating due to the track and wheels mounted along its sides. If the motor M develops a constant tension $T = 1.50$ kN in its attached cable, determine the velocity of the elevator when it has moved upward 3 m starting from rest. Neglect the mass of the pulleys and cables.



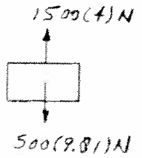
$+\uparrow \Sigma F_y = ma_y;$ $\quad 1500(4) - 500(9.81) = 500a$

$a = 2.19$ m/s^2

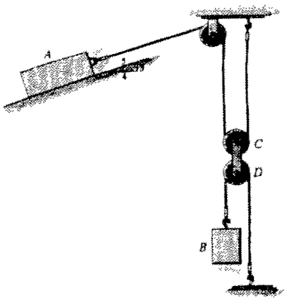
$(+\uparrow)\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$v^2 = 0 + 2(2.19)(3)$

$v = 3.62$ m/s **Ans**



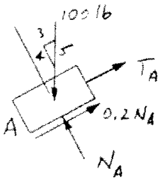
13-26. At the instant shown the 100-lb block A is moving down the plane at 5 ft/s while being attached to the 50-lb block B. If the coefficient of kinetic friction is $\mu_k = 0.2$, determine the acceleration of A and the distance A slides before it stops. Neglect the mass of the pulleys and cables.



Block A :

$\overset{+}{\searrow}\ \Sigma F_x = ma_x;\qquad -T_A - 0.2N_A + 100\left(\frac{3}{5}\right) = \left(\frac{100}{32.2}\right)a_A$

$+\nwarrow \Sigma F_y = ma_y;\qquad N_A - 100\left(\frac{4}{5}\right) = 0$



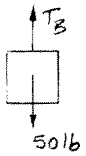
Thus,

$T_A - 44 = -3.1056a_A \qquad (1)$

Block B :

$+\uparrow \Sigma F_y = ma_y;\qquad T_B - 50 = \left(\frac{50}{32.2}\right)a_B$

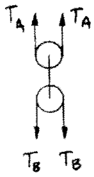
$T_B - 50 = 1.553a_B \qquad (2)$



Pulleys at C and D :

$+\uparrow \Sigma F_y = 0;\qquad 2T_A - 2T_B = 0$

$T_A = T_B \qquad (3)$



Kinematics :

$s_A + 2s_C = l$

$s_D + (s_D - s_B) = l'$

$s_C + d + s_D = d'$

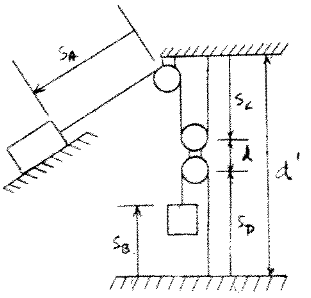
Thus,

$a_A = -2a_C$

$2a_D = a_B$

$a_C = -a_D$,

so that $\quad a_A = a_B \qquad (4)$



Solving Eqs. (1) – (4) :

$a_A = a_B = -1.288\ \text{ft/s}^2$

$T_A = T_B = 48.0\ \text{lb}$

Thus,

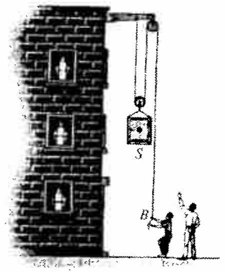
$a_A = 1.29\ \text{ft/s}^2$ **Ans**

$(+\swarrow)\qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$0 = (5)^2 + 2(-1.288)(s - 0)$

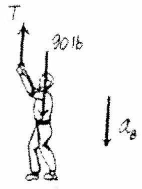
$s = 9.70\ \text{ft}$ **Ans**

13-27. The safe S has a weight of 200 lb and is supported by the rope and pulley arrangement shown. If the end of the rope is given to a boy B of weight 90 lb, determine his acceleration if in the confusion he doesn't let go of the rope. Neglect the mass of the pulleys and rope.



Equation of Motion : The tension T developed in the cord is the same throughout the entire cord since the cord passes over the smooth pulleys. From FBD(a),

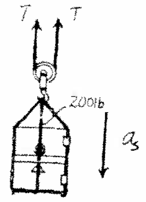
$$+\uparrow \Sigma F_y = ma_y ; \qquad T-90=-\left(\frac{90}{32.2}\right)a_B \qquad [1]$$



(a)

From FBD(b),

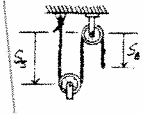
$$+\uparrow \Sigma F_y = ma_y ; \qquad 2T-200=-\left(\frac{200}{32.2}\right)a_S \qquad [2]$$



(b)

Kinematic : Establish the position - coordinate equation, we have

$$2s_S + s_B = l$$



Taking time derivative twice yields

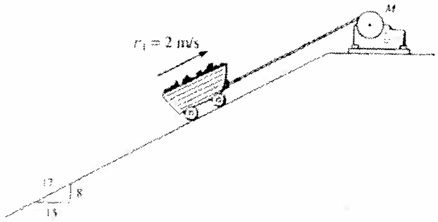
$$(+\downarrow) \qquad 2a_S + a_B = 0 \qquad [3]$$

Solving Eqs. [1], [2] and [3] yields

$$a_B = -2.30 \text{ ft/s}^2 = 2.30 \text{ ft/s}^2 \uparrow \qquad \textbf{Ans}$$

$$a_S = 1.15 \text{ ft/s}^2 \downarrow \qquad T = 96.43 \text{ lb}$$

***13-28.** The 400-kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200t^2)$ N, where t is in seconds. If the car has an initial velocity $v_1 = 2$ m/s when $t = 0$, determine its velocity when $t = 2$ s.

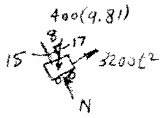


$$\nearrow + \Sigma F_{x'} = ma_{x'} ; \qquad 3200t^2 - 400(9.81)\left(\tfrac{8}{17}\right) = 400a \qquad a = 8t^2 - 4.616$$

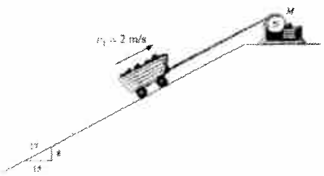
$$dv = a\,dt$$

$$\int_2^v dv = \int_0^2 \left(8t^2 - 4.616\right) dt$$

$$v = 14.1 \text{ m/s} \qquad \textbf{Ans}$$



13-29. The 400-kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200t^2)$ N, where t is in seconds. If the car has an initial velocity $v_1 = 2$ m/s at $s = 0$ and $t = 0$, determine the distance it moves up the plane when $t = 2$ s.



$$\nearrow + \Sigma F_{x'} = ma_{x'} ; \qquad 3200t^2 - 400(9.81)\left(\tfrac{8}{17}\right) = 400a \qquad a = 8t^2 - 4.616$$

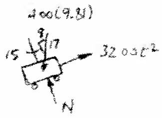
$$dv = a\,dt$$

$$\int_2^v dv = \int_0^t \left(8t^2 - 4.616\right) dt$$

$$v = \frac{ds}{dt} = 2.667t^3 - 4.616t + 2$$

$$\int_0^s ds = \int_0^2 \left(2.667t^3 - 4.616t + 2\right) dt$$

$$s = 5.43 \text{ m} \qquad \textbf{Ans}$$



■13-30. The tanker has a weight of $800(10^6)$ lb and is traveling forward at $v_0 = 3$ ft/s in still water when the engines are shut off. If the drag resistance of the water is proportional to the speed of the tanker at any instant and can be approximated by $F_D = (400(10^3)v)$ lb, where v is in ft/s, determine the time needed for the tanker's speed to become 1.5 ft/s. Given the initial velocity of $v_0 = 3$ ft/s, through what distance must the tanker travel before it stops?

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -\left(400\left(10^3\right) v\right) = \left(\frac{800(10^6)}{32.2}\right)a$$

$$a = \frac{dv}{dt}$$

$$\int_0^t -0.0161\, dt = \int_3^v \frac{dv}{v}$$

$$-0.0161t = \ln\left(\frac{v}{3}\right) \qquad (1)$$

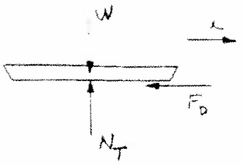
When $v = 1.5$ ft/s,

$t = 43.1$ s **Ans**

$$v = \frac{ds}{dt} = 3e^{-0.0161t}$$

$$\int_0^{s_{max}} ds = \int_0^{\infty} 3e^{-0.0161t} dt$$

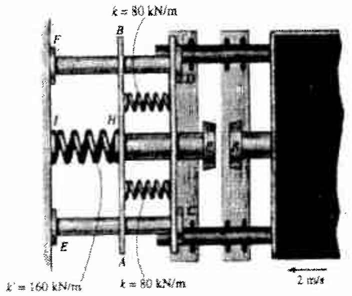
$$s_{max} = \left.\frac{-3e^{-0.0161t}}{0.0161}\right|_0^{\infty} = 186 \text{ ft}$$ **Ans**



Note that from Eq. (1) it is seen that as $v \rightarrow 0$, $t \rightarrow \infty$. Hence it takes an infinite amount of time to stop the tanker. In reality, however, the drag equation $F_D = \left(400\left(10^3\right) v\right)$ lb changes as the tanker slows down, and hence the dependence of v on t also changes.

15

13-31. The spring mechanism is used as a shock absorber for railroad cars. Determine the maximum compression of spring HI if the fixed bumper R of a 5-Mg railroad car, rolling freely at 2 m/s, strikes the plate P. Bar AB slides along the guide paths CE and DF. The ends of all springs are attached to their respective members and are originally unstretched.



The springs stretch or compress an equal amount x. Thus,

$$F_s = (2k + k')x = \left[2(80)\left(10^3\right) + 160\left(10^3\right)\right]x = 320\,000x$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 320\,000x = 5000a$$

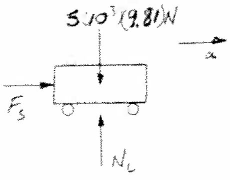
$$a = 64x$$

$$\left(\xleftarrow{+}\right) \qquad a\, dx = v\, dv$$

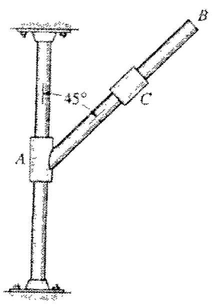
$$-\int_0^x 64x\, dx = \int_2^0 v\, dv$$

$$32x^2 = 2$$

$x = 0.25$ m $= 250$ mm **Ans**



***13-32.** The 2-kg collar C is free to slide along the smooth shaft AB. Determine the acceleration of collar C if (a) the shaft is fixed from moving, (b) collar A, which is fixed to shaft AB, moves downward at constant velocity along the vertical rod, and (c) collar A is subjected to a downward acceleration of 2 m/s^2. In all cases, the collar moves in the plane.

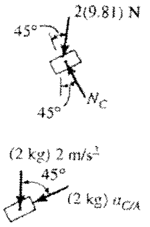


$+\swarrow \Sigma F_{x'} = ma_{x'};\quad 2(9.81)\sin 45° = 2a_C \qquad a_C = 6.94 \text{ m/s}^2$

(b) From part (a) $\mathbf{a}_{C/A} = 6.94 \text{ m/s}^2$

$\mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{C/A}$ Where $\mathbf{a}_A = \mathbf{0}$

$= 6.94 \text{ m/s}^2$



(c)

$\mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{C/A}$

$= \underset{\downarrow}{2} + \underset{\swarrow}{a_{C/A}}$ [1]

$+\swarrow \Sigma F_{x'} = ma_{x'};\quad 2(9.81)\sin 45° = 2(2\cos 45° + a_{C/A})$

$a_{C/A} = 5.5225 \text{ m/s}^2 \swarrow$

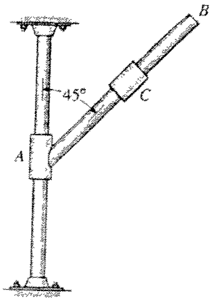
From Eq.[1]

$\mathbf{a}_C = \underset{\downarrow}{2} + \underset{\swarrow}{5.5225} = \underset{\leftarrow}{3.905} + \underset{\downarrow}{5.905}$

$a_C = \sqrt{3.905^2 + 5.905^2} = 7.08 \text{ m/s}^2$ **Ans**

$\theta = \tan^{-1}\dfrac{5.905}{3.905} = 56.5°$ **Ans**

13-33. The 2-kg collar C is free to slide along the smooth shaft AB. Determine the acceleration of collar C if collar A is subjected to an upward acceleration of 4 m/s^2.



$\xleftarrow{+} \Sigma F_x = ma_x;\quad N\sin 45° = 2a_{C/AB}\sin 45°$

$N = 2a_{C/AB}$

$+\uparrow \Sigma F_y = ma_y;\quad N\cos 45° - 19.62 = 2(4) - 2a_{C/AB}\cos 45°$

$a_{C/AB} = 9.76514$

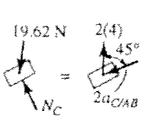
$\mathbf{a}_C = \mathbf{a}_{AB} + \mathbf{a}_{C/AB}$

$(a_C)_x = 0 + 9.76514\sin 45° = 6.905 \leftarrow$

$(a_C)_y = 4 - 9.76514\cos 45° = 2.905 \downarrow$

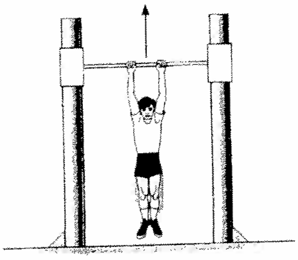
$a_C = \sqrt{(6.905)^2 + (2.905)^2} = 7.49 \text{ m/s}^2$ **Ans**

$\theta = \tan^{-1}\left(\dfrac{2.905}{6.905}\right) = 22.8°$ **Ans**



16

13-34. The boy having a weight of ~~80 lb~~ 400N hangs uniformly from the bar. Determine the force in each of his arms in $t = 2$ s if the bar is moving upward with (a) a constant velocity of ~~3 ft/s,~~ 1 m/s and (b) a speed of $v = (\cancel{4}t^2)$ ~~ft~~ m/s, where t is in seconds.

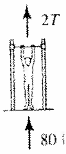


a) $T = \cancel{40\text{ lb}}$ 400 N **Ans**

b) $v = \cancel{4t^2}$ t^2

$a = \cancel{8t}$ $2t$

$+\uparrow \sum F_y = ma_y;\quad 2T - \cancel{80}\,400 = \frac{\cancel{80}\,400}{\cancel{32.2}\,9.81}(\cancel{8t})\,2t$



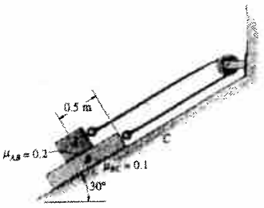
At $t = 2$ s,

$T = \cancel{59.9}$ lb **Ans**

281.55 N

17

13-35. The 10-kg block A rests on the 50-kg plate B in the position shown. Neglecting the mass of the rope and pulley, and using the coefficients of kinetic friction indicated, determine the time needed for block A to slide 0.5 m *on the plate* when the system is released from rest.

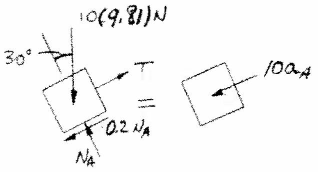


Block A :

$+\nwarrow \Sigma F_y = ma_y;\quad N_A - 10(9.81)\cos 30° = 0 \quad N_A = 84.96\text{ N}$

$+\swarrow \Sigma F_x = ma_x;\quad -T + 0.2(84.96) + 10(9.81)\sin 30° = 10a_A$

$T - 66.04 = -10a_A \quad (1)$



Block B :

$+\nwarrow \Sigma F_y = ma_y;\quad N_B - 84.96 - 50(9.81)\cos 30° = 0$

$N_B = 509.7\text{ N}$

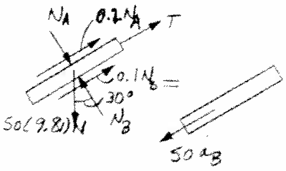
$+\swarrow \Sigma F_x = ma_x;\quad -0.2(84.96) - 0.1(509.7) - T + 50(9.81\sin 30°) = 50a_B$

$177.28 - T = 50a_B \quad (2)$

$s_A + s_B = l$

$\Delta s_A = -\Delta s_B$

$a_A = -a_B \quad (3)$



Solving Eqs. (1) – (3) :

$a_B = 1.854\text{ m/s}^2$

$a_A = -1.854\text{ m/s}^2 \quad T = 84.58\text{ N}$

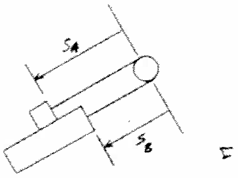
In order to slide 0.5 m along the plate the block must move 0.25 m. Thus,

$(+\swarrow)\quad s_B = s_A + s_{B/A}$

$-\Delta s_A = \Delta s_A + 0.5$

$\Delta s_A = -0.25\text{ m}$

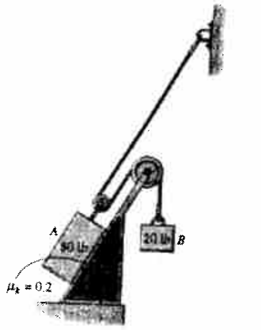
$(+\swarrow)\quad s_A = s_0 + v_0 t + \frac{1}{2}a_A t^2$



$-0.25 = 0 + 0 + \frac{1}{2}(-1.854)t^2$

$t = 0.519$ s **Ans**

***13-36.** Determine the acceleration of block A when the system is released. The coefficient of kinetic friction and the weight of each block are indicated. Neglect the mass of the pulleys and cord.



Block A :

$\overset{+}{\nwarrow}\Sigma F_y = ma_y$;　$N_A - 80\cos 60° = 0$

$\overset{+}{\swarrow}\Sigma F_x = ma_x$;　$80\sin 60° - 0.2N_A - 2T = \left(\frac{80}{32.2}\right)a_A$

Block B :

$+\downarrow \Sigma F_y = ma_y$;　$-T + 20 = \left(\frac{20}{32.2}\right)a_B$

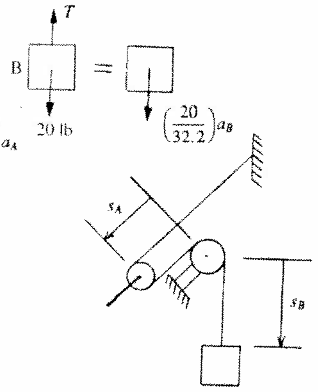
$2s_A + s_B = l$

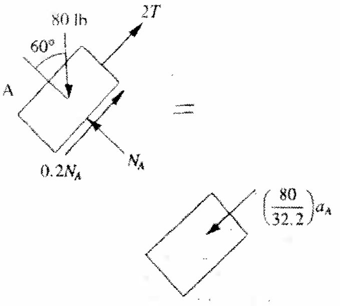
$2a_A = -a_B$

Solving,

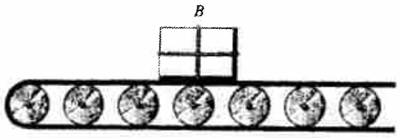
$N_A = 40\text{ lb}$　$T = 25.32\text{ lb}$　$a_B = -8.57\text{ ft/s}^2$

$a_A = 4.28\text{ ft/s}^2$　**Ans**





13-37. The conveyor belt is moving at 4 m/s. If the coefficient of static friction between the conveyor and the 10-kg package B is $\mu_s = 0.2$, determine the shortest time the belt can stop so that the package does not slide on the belt.



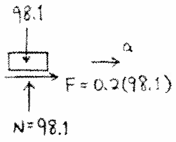
$\overset{+}{\rightarrow}\Sigma F_x = m\,a_x$;　$0.2(98.1) = 10\,a$

$a = 1.962\text{ m/s}^2$

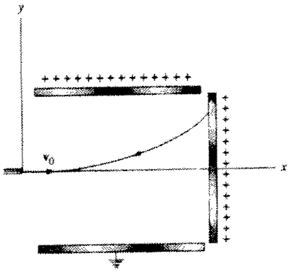
$(\overset{+}{\rightarrow})\ v = v_0 + a_c t$

$4 = 0 + 1.962\,t$

$t = 2.04\text{ s}$　**Ans**



13-38. An electron of mass m is discharged with an initial horizontal velocity of v_0. If it is subjected to two fields of force for which $F_x = F_0$ and $F_y = 0.3F_0$, where F_0 is constant, determine the equation of the path, and the speed of the electron at any time t.



$$\xrightarrow{+} \Sigma F_x = ma_x, \qquad F_0 = ma_x$$

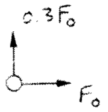
$$+\uparrow \Sigma F_y = ma_y: \qquad 0.3F_0 = ma_y$$

Thus,

$$\int_{v_0}^{v_x} dv_x = \int_0^t \frac{F_0}{m}\,dt$$

$$v_x = \frac{F_0}{m}t + v_0$$

$$\int_0^{v_y} dv_y = \int_0^t \frac{0.3F_0}{m}\,dt \qquad v_y = \frac{0.3F_0}{m}t$$



$$v = \sqrt{\left(\frac{F_0}{m}t + v_0\right)^2 + \left(\frac{0.3F_0}{m}t\right)^2}$$

$$= \frac{1}{m}\sqrt{1.09F_0^2t^2 + 2F_0 tmv_0 + m^2v_0^2} \qquad \textbf{Ans}$$

$$\int_0^x dx = \int_0^t \left(\frac{F_0}{m}t + v_0\right)dt$$

$$x = \frac{F_0 t^2}{2m} + v_0 t$$

$$\int_0^y dy = \int_0^t \frac{0.3F_0}{m}t\,dt$$

$$y = \frac{0.3F_0t^2}{2m}$$

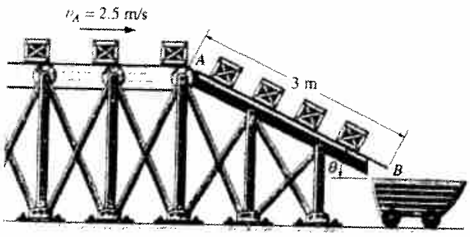
$$t = \left(\sqrt{\frac{2m}{0.3F_0}}\right) y^{\frac{1}{2}}$$

$$x = \frac{F_0}{2m}\left(\frac{2m}{0.3F_O}\right)y + v_0\left(\sqrt{\frac{2m}{0.3F_0}}\right) y^{\frac{1}{2}}$$

$$x = \frac{y}{0.3} + v_0\left(\sqrt{\frac{2m}{0.3F_0}}\right) y^{\frac{1}{2}} \qquad \textbf{Ans}$$

19

13-39. The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's speed is $v_A = 2.5$ m/s, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is $\mu_k = 0.3$, determine the speed at which each crate slides off the ramp at B. Assume that no tipping occurs. Take $\theta = 30°$.



$$\nearrow + \Sigma F_y = ma_y; \quad N_C - 12(9.81)\cos 30° = 0$$

$$N_C = 101.95 \text{ N}$$

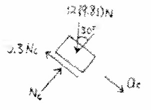
$$+\searrow \Sigma F_x = ma_x; \quad 12(9.81)\sin 30° - 0.3(101.95) = 12\,a_C$$

$$a_C = 2.356 \text{ m/s}^2$$

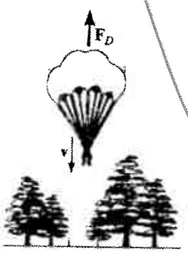
$$(+\searrow) \quad v_B^2 = v_A^2 + 2\,a_c(s_B - s_A)$$

$$v_B^2 = (2.5)^2 + 2(2.356)(3-0)$$

$$v_B = 4.5152 = 4.52 \text{ m/s} \qquad \textbf{Ans}$$



***13-40.** A parachutist having a mass m opens his parachute from an at-rest position at a very high altitude. If the atmospheric drag resistance is $F_D = kv^2$, where k is a constant, determine his velocity when he has fallen for a time t. What is his velocity when he lands on the ground? This velocity is referred to as the *terminal velocity*, which is found by letting the time of fall $t \to \infty$.



$$+\downarrow \Sigma F_z = m\,a_z; \quad mg - kv^2 = m\frac{dv}{dt}$$

$$m\int_0^v \frac{m\,dv}{(mg - kv^2)} = \int_0^t dt$$

$$\frac{m}{k}\int_0^v \frac{dv}{\frac{mg}{k} - v^2} = t$$

$$\frac{m}{k}\left(\frac{1}{2\sqrt{\frac{mg}{k}}}\right)\ln\left[\frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}\right]_0^v = t$$

$$\frac{k}{m}t\left(2\sqrt{\frac{mg}{k}}\right) = \ln\frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}$$

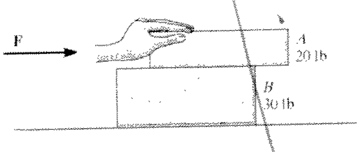
$$e^{2t\sqrt{\frac{mg}{k}}} = \frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}$$

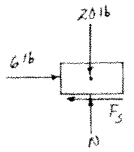
$$\sqrt{\frac{mg}{k}}\,e^{2t\sqrt{\frac{mg}{k}}} - v\,e^{2t\sqrt{\frac{mg}{k}}} = \sqrt{\frac{mg}{k}} + v$$

$$v = \sqrt{\frac{mg}{k}}\left[\frac{e^{2t\sqrt{\frac{mg}{k}}} - 1}{e^{2t\sqrt{\frac{mg}{k}}} + 1}\right] \qquad \textbf{Ans}$$

$$\text{When } t \to \infty \qquad v_t = \sqrt{\frac{mg}{k}} \qquad \textbf{Ans}$$

13–41. Block B rests on a smooth surface. If the coefficients of static and kinetic friction between A and B are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively, determine the acceleration of each block if someone pushes horizontally on block A with a force of (*a*) $F = 6$ lb, and (*b*) $F = 50$ lb.





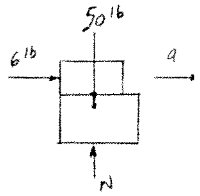
(a)

Block A

$$\xrightarrow{+} \Sigma F_x = 0; \quad 6 - F_s = 0 \quad F_s = 6 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad N - 20 = 0 \quad N = 20 \text{ lb}$$

Since $F_s = 6 \text{ lb} < \mu_s N = 0.4(20) = 8 \text{ lb}$, block A will not slide. As a result, blocks A and B move together with the same acceleration.

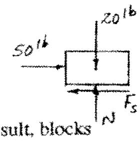


$$\xrightarrow{+} \Sigma F_x = ma_x; \quad 6 = \frac{50}{32.2}a \quad a = 3.86 \text{ ft/s}^2$$

$$a_A = a_B = a = 3.86 \text{ ft/s}^2 \qquad \textbf{Ans}$$

(b)

Block A

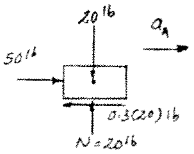


$$\xrightarrow{+} \Sigma F_x = 0; \quad 50 - F_s = 0 \quad F_s = 50 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad N - 20 = 0 \quad N = 20 \text{ lb}$$

Since $F_s = 50 \text{ lb} > \mu_s N = 0.4(20) = 8 \text{ lb}$, block A will slide. As a result, blocks A and B move with different accelerations.

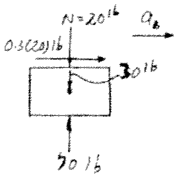
Since block A is sliding, μ_k will be used in the calculation.



$$\xrightarrow{+} \Sigma F_x = ma_x; \quad 50 - 0.3(20) = \frac{20}{32.2}a_A$$

$$a_A = 70.8 \text{ ft/s}^2 \qquad \textbf{Ans}$$

For Block B

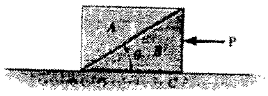


$$\xrightarrow{+} \Sigma F_x = ma_x; \quad 0.3(20) = \frac{30}{32.2}a_B$$

$$a_B = 6.44 \text{ ft/s}^2 \qquad \textbf{Ans}$$

20

13-42. Blocks A and B each have a mass m. Determine the largest horizontal force P which can be applied to B so that A will not move relative to B. All surfaces are smooth.



Require

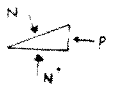
$a_A = a_B = a$

Block A :

$+\uparrow \Sigma F_y = 0; \quad N\cos\theta - mg = 0$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \quad N\sin\theta = ma$

$a = g\tan\theta$



Block B :

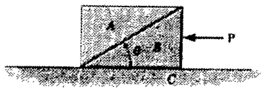
$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \quad P - N\sin\theta = ma$

$P - mg\tan\theta = mg\tan\theta$

$P = 2mg\tan\theta$ **Ans**

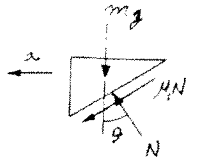
21

13-43. Blocks A and B each have a mass m. Determine the largest horizontal force P which can be applied to B so that A will not slip up B. The coefficient of static friction between A and B is μ_s. Neglect any friction between B and C.



Require

$a_A = a_B = a$



Block A :

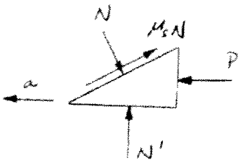
$+\uparrow \Sigma F_y = 0; \quad N\cos\theta - \mu_s N\sin\theta - mg = 0$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \quad N\sin\theta + \mu_s N\cos\theta = ma$

$$N = \frac{mg}{\cos\theta - \mu_s\sin\theta}$$

$$a = g\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

Block B :



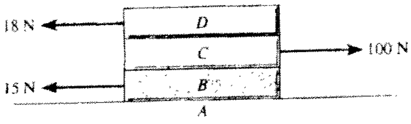
$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \quad P - \mu_s N\cos\theta - N\sin\theta = ma$

$$P - mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) = mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

$$P = 2mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) \quad \textbf{Ans}$$

22

***13-44.** Each of the three plates has a mass of 10 kg. If the coefficients of static and kinetic friction at each surface of contact are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively, determine the acceleration of each plate when the three horizontal forces are applied.



Plates B, C and D

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \quad 100 - 15 - 18 - F = 0$$

$$F = 67 \text{ N}$$

$$F_{max} = 0.3(294.3) = 88.3 \text{ N} > 67 \text{ N}$$

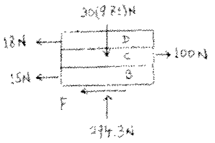


Plate B will not slip

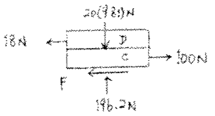
$$a_B = 0 \qquad \textbf{Ans}$$

Plates D and C

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \quad 100 - 18 - F = 0$$

$$F = 82 \text{ N}$$

$$F_{max} = 0.3(196.2) = 58.86 \text{ N} \nless 82 \text{ N}$$

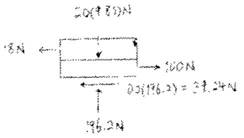


Slipping between B and C.

Assume no slipping between D and C,

$$\stackrel{+}{\rightarrow} \Sigma F_x = m\,a_x; \quad 100 - 39.24 - 18 = 20\,a_x$$

$$a_x = 2.138 \text{ m/s}^2 \rightarrow$$

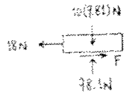


Check slipping between D and C.

$$\stackrel{+}{\rightarrow} \Sigma F_x = m\,a_x; \quad F - 18 = 10(2.138)$$

$$F = 39.38 \text{ N}$$

$$F_{max} = 0.3(98.1) = 29.43 \text{ N} < 39.38 \text{ N}$$



Slipping between D and C.

Plate C:

$$\stackrel{+}{\rightarrow} \Sigma F_x = m\,a_x; \quad 100 - 39.24 - 19.62 = 10\,a_C$$

$$a_C = 4.11 \text{ m/s}^2 \rightarrow \qquad \textbf{Ans}$$

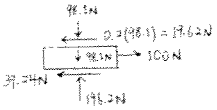
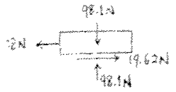


Plate D:

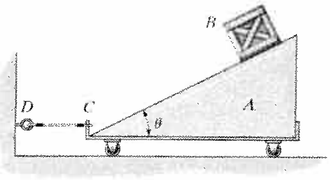
$$\stackrel{+}{\rightarrow} \Sigma F_x = m\,a_x; \quad 19.62 - 18 = 10\,a_D$$

$$a_D = 0.162 \text{ m/s}^2 \rightarrow \qquad \textbf{Ans}$$



2)

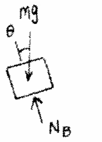
13-45. Crate B has a mass m and is released from rest when it is on top of cart A, which has a mass $3m$. Determine the tension in cord CD needed to hold the cart from moving while B is sliding down A. Neglect friction.



Crate B :

$$\nwarrow + \Sigma F_y = m\,a_y; \qquad N_B - mg\cos\theta = 0$$

$$N_B = mg\cos\theta$$

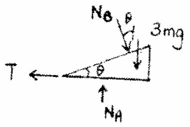


Cart :

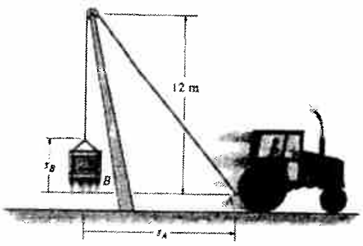
$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -T + N_B\sin\theta = 0$$

$$T = mg\sin\theta\cos\theta$$

$$T = \frac{mg}{2}\sin 2\theta \qquad \textbf{Ans}$$



■13-46. The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor is traveling to the right at a constant speed of 4 m/s, determine the tension in the rope when $s_A = 5$ m. When $s_A = 0$, $s_B = 0$.



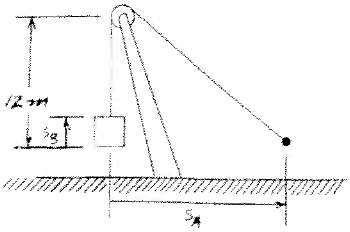
$$12 - s_B + \sqrt{s_A^2 + (12)^2} = 24$$

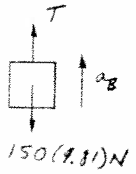
$$-\dot{s}_B + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A\dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A\dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A\ddot{s}_A\right) = 0$$

$$\ddot{s}_B = -\left[\frac{s_A^2\dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A\ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{((5)^2 + 144)^{\frac{3}{2}}} - \frac{(4)^2 + 0}{((5)^2 + 144)^{\frac{1}{2}}}\right] = 1.0487 \text{ m/s}^2$$

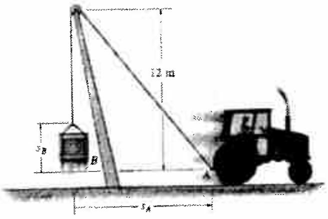




$$+\uparrow \Sigma F_y = ma_y; \qquad T - 150(9.81) = 150(1.0487)$$

$$T = 1.63 \text{ kN} \qquad \textbf{Ans}$$

13-47. The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor is traveling to the right with an acceleration of 3 m/s^2 and has a velocity of 4 m/s at the instant $s_A = 5$ m, determine the tension in the rope at this instant. When $s_A = 0$, $s_B = 0$.



$$12 - s_B + \sqrt{s_A^2 + (12)^2} = 24$$

$$-\dot{s}_B + \frac{1}{2}\left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(2 s_A \dot{s}_A\right) = 0$$

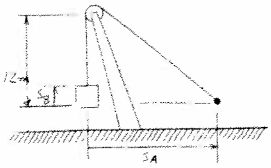
$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A \dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \ddot{s}_A\right) = 0$$

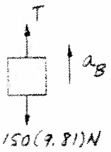
$$\ddot{s}_B = -\left[\frac{s_A^2 \dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A \ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{((5)^2 + 144)^{\frac{3}{2}}} - \frac{(4)^2 + (5)(3)}{((5)^2 + 144)^{\frac{1}{2}}}\right] = 2.2025 \text{ m/s}^2$$

$+\uparrow \Sigma F_y = ma_y;$ $\quad T - 150(9.81) = 150(2.2025)$

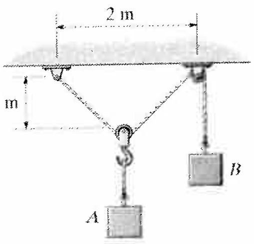
$T = 1.80$ kN **Ans**





24

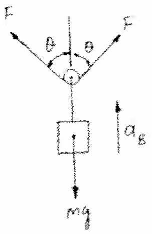
***13-48.** Block B has a mass m and is hoisted using the cord and pulley system shown. Determine the magnitude of force **F** as a function of the block's vertical position y so that when **F** is applied the block rises with a constant acceleration $\mathbf{a}_B$. Neglect the mass of the cord and pulleys.

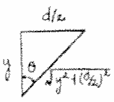


$+\uparrow \Sigma F_y = ma_y;$ $\quad 2F\cos\theta - mg = ma_B$ $\quad$ where $\cos\theta = \dfrac{y}{\sqrt{y^2 + \left(\frac{d}{2}\right)^2}}$

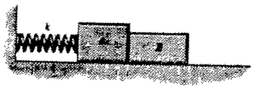
$$2F\left(\frac{y}{\sqrt{y^2 + \left(\frac{d}{2}\right)^2}}\right) - mg = ma_B$$

$$F = \frac{m(a_B + g)\sqrt{4y^2 + d^2}}{4y}$$ **Ans**





13-49. Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, determine the distance both blocks slide on the smooth surface before they begin to separate. What is their velocity at this instant?



Block A :

$$\xrightarrow{+} \Sigma F_x = ma_x ; \qquad -k(x-d) - N = m_A a_A$$

Block B :

$$\xrightarrow{+} \Sigma F_x = ma_x ; \qquad N = m_B a_B$$

Since $a_A = a_B = a$,

$$-k(x-d) - m_B a = m_A a$$

$$a = \frac{k(d-x)}{(m_A + m_B)} \qquad N = \frac{km_B (d-x)}{(m_A + m_B)}$$

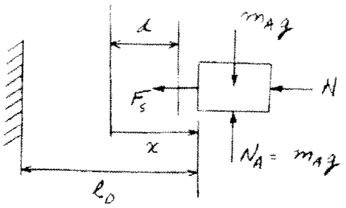
$N = 0$ when $d - x = 0$, or $\quad x = d \quad$ **Ans**

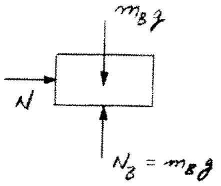
$$v\,dv = a\,dx$$

$$\int_0^v v\,dv = \int_0^d \frac{k(d-x)}{(m_A + m_B)}\,dx$$

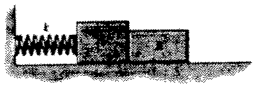
$$\frac{1}{2}v^2 = \frac{k}{(m_A + m_B)}\left[(d)x - \frac{1}{2}x^2\right]_0^d = \frac{1}{2}\frac{kd^2}{(m_A + m_B)}$$

$$v = \sqrt{\frac{kd^2}{(m_A + m_B)}} \quad \textbf{Ans}$$





13-50. Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, show that for separation to occur it is necessary that $d > 2\mu_k g(m_A + m_B)/k$, where μ_k is the coefficient of kinetic friction between the blocks and the ground. Also, what is the distance the blocks slide on the surface before they separate?



Block A :

$\xrightarrow{+} \Sigma F_x = ma_x;$ $\quad -k(x-d)-N-\mu_k m_A g = m_A a_A$

Block B :

$\xrightarrow{+} \Sigma F_x = ma_x;$ $\quad N-\mu_k m_B g = m_B a_B$

Since $a_A = a_B = a$,

$$a = \frac{k(d-x)-\mu_k g(m_A+m_B)}{(m_A+m_B)} = \frac{k(d-x)}{(m_A+m_B)} - \mu_k g$$

$$N = \frac{km_B(d-x)}{(m_A+m_B)}$$

$N = 0$, then $x = d$ for separation. **Ans**

At the moment of separation :

$v\,dv = a\,dx$

$$\int_0^v v\,dv = \int_0^d\left[\frac{k(d-x)}{(m_A+m_B)}\,dx - \mu_k g\right]dx$$

$$\frac{1}{2}v^2 = \frac{k}{(m_A+m_B)}\left[(d)x - \frac{1}{2}x^2 - \mu_k g\,x\right]_0^d$$

$$v = \sqrt{\frac{kd^2 - 2\mu_k g(m_A+m_B)d}{(m_A+m_B)}}$$

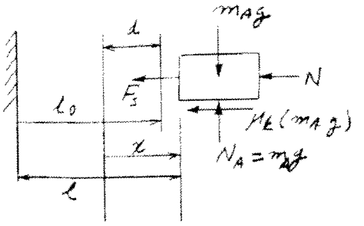
Require $v > 0$, so that

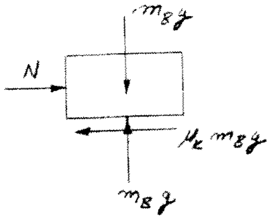
$kd^2 - 2\mu_k g(m_A+m_B)d > 0$

Thus,

$kd > 2\mu_k g(m_A+m_B)$

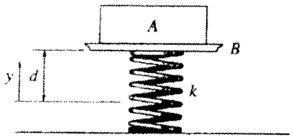
$d > \frac{2\mu_k g}{k}(m_A+m_B)$ Q.E.D.



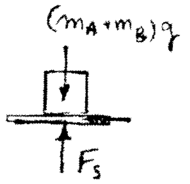


25

13-51. The block A has a mass m_A and rests on the pan B, which has a mass m_B. Both are supported by a spring having a stiffness k that is attached to the bottom of the pan and to the ground. Determine the distance d the pan should be pushed down from the equilibrium position and then released from rest so that separation of the block will take place from the surface of the pan at the instant the spring becomes unstretched.



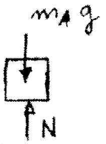
For Equilibrium



$$+\uparrow \Sigma F_y = ma_y; \qquad F_s = (m_A + m_B)g$$

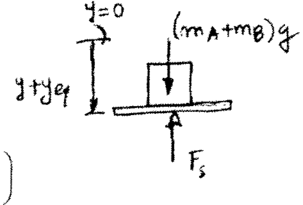
$$y_{eq} = \frac{F_s}{k} = \frac{(m_A + m_B)g}{k}$$

Block:



$$+\uparrow \Sigma F_y = ma_y; \qquad -m_A g + N = m_A a$$

Block and pan



$$+\uparrow \Sigma F_y = ma_y; \qquad -(m_A + m_B)g + k(y_{eq} + y) = (m_A + m_B)a$$

Thus,

$$-(m_A + m_B)g + k\left[\left(\frac{m_A + m_B}{k}\right)g + y\right] = (m_A + m_B)\left(\frac{-m_A g + N}{m_A}\right)$$

Require $y = d$, $N = 0$

$$kd = -(m_A + m_B)g$$

Since d is downward,

$$d = \frac{(m_A + m_B)g}{k} \qquad \textbf{Ans}$$

26

***13-52.** Determine the mass of the sun, knowing that the distance from the earth to the sun is 149.6(10^6) km. *Hint:* Use Eq. 13–1 to represent the force of gravity acting on the earth.

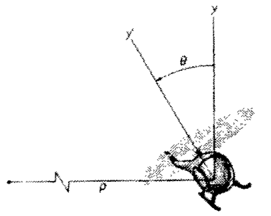
$\Sigma F_n = ma_n;\quad G\frac{M_e M_s}{R^2} = M_e\frac{v^2}{R}\quad M_s = \frac{v^2 R}{G}$

$v = \frac{s}{t} = \frac{2\pi(149.6)(10^9)}{365(24)(3600)} = 29\ 81\left(10^3\right)\ \text{m/s}$

$M_s = \frac{\left[(29.81)(10^3)\right]^2(149.6)(10^9)}{66.73(10^{-12})} = 1.99\left(10^{30}\right)$ **kg** **Ans**

27

13-53. The 1.40-Mg helicopter is traveling at a constant speed of 40 m/s along the horizontal curved path while banking at $\theta = 30°$. Determine the force acting normal to the blade, i.e., in the y' direction, and the radius of curvature of the path.

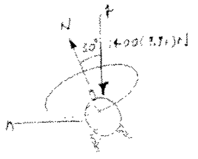


$+\uparrow\Sigma F_y = ma_y;\quad N\cos 30° - 1400(9.81) = 0$

$N = 15\ 859\ \text{N} = 15.9\ \text{kN}$ **Ans**

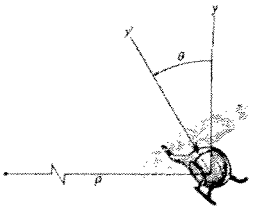
$\overset{+}{\leftarrow}\Sigma F_n = ma_n;\quad 15\ 859\sin 30° = 1400\left(\frac{(40)^2}{\rho}\right)$

$\rho = 282\ \text{m}$ **Ans**



28

13-54. The 1.40-Mg helicopter is traveling at a constant speed of 33 m/s along the horizontal curved path having a radius of curvature of $\rho = 300$ m. Determine the force the blade exerts on the frame and the bank angle θ.



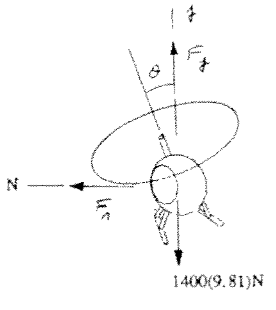
$+\uparrow\Sigma F_y = ma_y;\quad F_y - 1400(9.81) = 0$

$F_y = 13\ 734\ \text{N}$

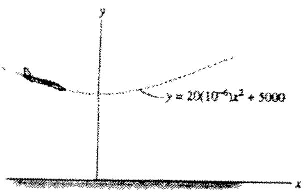
$\overset{+}{\leftarrow}\Sigma F_n = ma_n;\quad F_n = 1400\left(\frac{(33)^2}{300}\right) = 5082\ \text{N}$

$F_R = \sqrt{(13\ 734)^2 + (5082)^2} = 14.6\ \text{kN}$ **Ans**

$\theta = \tan^{-1}\left(\frac{F_n}{F_y}\right) = \tan^{-1}\left(\frac{5082}{13\ 734}\right) = 20.3°$ **Ans**



13-55. The plane is traveling at a constant speed of 800 ft/s along the curve $y = 20(10^{-6})x^2 + 5000$, where x and y are in feet. If the pilot has a weight of 180 lb, determine the normal and tangential components of the force the seat exerts on the pilot when the plane is at its lowest point.

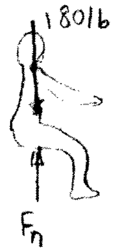


$$y = 20\left(10^{-6}\right)x^2 + 5000$$

$$\frac{dy}{dx} = 40\left(10^{-6}\right)x\Big|_{x=0} = 0$$

$$\frac{d^2y}{dx^2} = 40\left(10^{-6}\right)$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1+0]^{\frac{3}{2}}}{|40(10)^{-6}|} = 25\left(10^3\right)$$

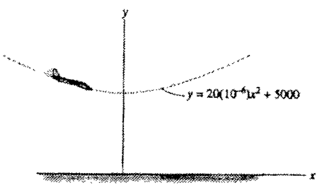


$+\uparrow\Sigma F_n = ma_n;\qquad F_n - 180 = \left(\frac{180}{32.2}\right)\left(\frac{(800)^2}{25(10^3)}\right)$

$F_n = 323$ lb **Ans**

$\overset{+}{\rightarrow}\Sigma F_t = ma_t;\qquad F_t = 0$ **Ans**

***13-56.** The jet plane is traveling at a constant speed of 1000 ft/s along the curve $y = 20(10^{-6})x^2 + 5000$, where x and y are in feet. If the pilot has a weight of 180 lb, determine the normal and tangential components of the force the seat exerts on the pilot when $y = 10\,000$ ft.



$$y = 20\left(10^{-6}\right)x^2 + 5000$$

$$10\,000 = 20\left(10^{-6}\right)x^2 + 5000$$

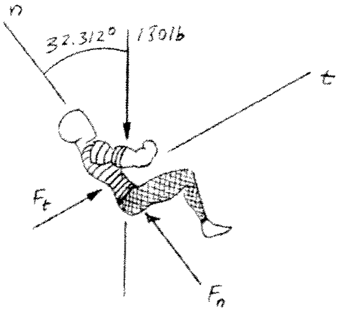
$$x = 15\,811 \text{ ft}$$

$$\frac{dy}{dx} = \tan\theta = 40\left(10^{-6}\right)x\Big|_{x=15\,811} = 0.63246$$

$$\theta = 32.31^\circ$$

$$\frac{d^2y}{dx^2} = 40\left(10^{-6}\right)$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+(0.63246)^2\right]^{\frac{3}{2}}}{|40(10^{-6})|} = 41.413\left(10^3\right)\text{ ft}$$



$+\nwarrow\Sigma F_n = ma_n;\qquad F_n - 180\cos 32.31^\circ = \left(\frac{180}{32.2}\right)\left(\frac{(1000)^2}{41.413(10)^3}\right)$

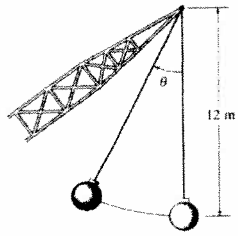
$F_n = 287$ lb **Ans**

$+\nearrow\Sigma F_t = ma_t;\qquad F_t - 180\sin 32.31^\circ = 0$

$F_t = 96.2$ lb **Ans**

29

13-57. The 600-kg wrecking ball is suspended from the crane by a cable having a negligible mass. If the ball has a speed $v = 8$ m/s at the instant it is at its lowest point, $\theta = 0°$, determine the tension in the cable at this instant. Also, determine the angle θ to which the ball swings before it stops.



$$+\uparrow \Sigma F_n = m\,a_n; \qquad T - 600(9.81) = 600(\frac{8^2}{12})$$

$$T = 9086 \text{ N} = 9.09 \text{ kN} \qquad \textbf{Ans}$$

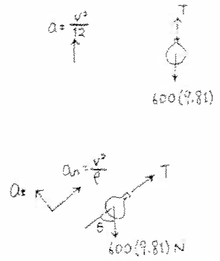
$$\nwarrow \Sigma F_t = m\,a_t; \qquad -600(9.81)\sin\theta = 600\,a_t$$

Set $a_t\,(12\,d\theta) = v\,dv$

$$-9.81(12)\int_0^\theta \sin\theta\,d\theta = \int_8^0 v\,dv$$

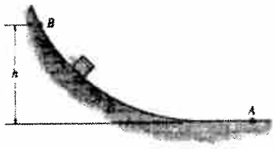
$$-9.81(12)(-\cos\theta + 1) = -\frac{1}{2}(8)^2$$

$\theta = 43.3°$ **Ans**



30

13-58. Prove that if the block is released from rest at point B of a smooth path of *arbitrary shape*, the speed it attains when it reaches point A is equal to the speed it attains when it falls freely through a distance h; i.e., $v = \sqrt{2gh}$.



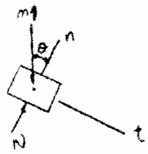
$$+\searrow \Sigma F_t = ma_t; \qquad mg\sin\theta = ma_t \qquad a_t = g\sin\theta$$

$$v\,dv = a_t\,ds = g\sin\theta\,ds \qquad \text{However} \quad dy = ds\sin\theta$$

$$\int_0^v v\,dv = \int_0^h g\,dy$$

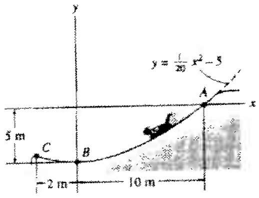
$$\frac{v^2}{2} = gh$$

$v = \sqrt{2gh}$ **Q.E.D.**



31

13-59. The sled and rider have a total mass of 80 kg and start from rest at $A(10 \text{ m}, 0)$. If the sled descends the smooth slope, which may be approximated by a parabola, determine the normal force that the ground exerts on the sled at the instant it arrives at point B. Neglect the size of the sled and rider. *Hint:* Use the result of Prob. 13-58.



Velocity of the sled : $x = 0$, $h = -5$ m.

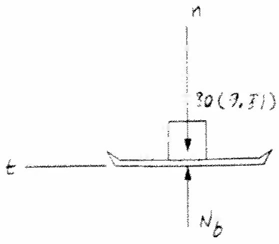
$$v = \sqrt{2gh} = \sqrt{2(9.81)(5)} = 9.9045 \text{ m/s}$$

$$\frac{dy}{dx} = \frac{1}{10}x\Big|_{x=0} = 0 \qquad \frac{d^2y}{dx^2} = \frac{1}{10}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\Bigg|_{x=0} = \frac{(1+0^2)^{3/2}}{\left|\frac{1}{10}\right|} = 10 \text{ m}$$

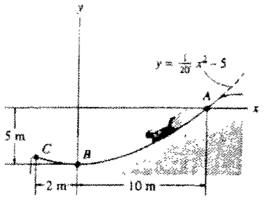
$$+\uparrow \Sigma F_n = ma_n; \qquad N_b - 80(9.81) = 80\left(\frac{(9.9045)^2}{10}\right)$$

$N_b = 1.57$ kN **Ans**



32

***13-60.** The sled and rider have a total mass of 80 kg and start from rest at $A(10\text{ m}, 0)$. If the sled descends the smooth slope which may be approximated by a parabola, determine the normal force that the ground exerts on the sled at the instant it arrives at point C. Neglect the size of the sled and rider. *Hint:* Use the result of Prob. 13-58.



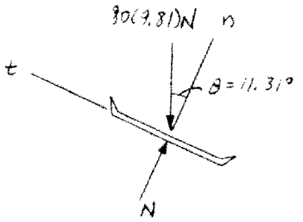
Velocity of the sled : $x = -2$ m, $h = -4.80$ m.

$$v = \sqrt{2gh} = \sqrt{2(9.81)(4.80)} = 9.704 \text{ m/s}$$

$$\frac{dy}{dx} = \frac{1}{10}x\bigg|_{x=-2\text{ m}} = -0.2 \qquad \frac{d^2y}{dx^2} = \frac{1}{10}$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\Bigg|_{x=-2\text{ m}} = \frac{\left[1+(-0.2)^2\right]^{3/2}}{\left|\frac{1}{10}\right|} = 10.606 \text{ m}$$

$$\tan\theta = \frac{dy}{dx}\bigg|_{x=-2\text{ m}} = -0.2 \qquad \theta = |11.31°|$$



$$\nearrow + \Sigma F_n = ma_n; \qquad N - 80(9.81)\cos 11.31° = 80\left(\frac{(9.704)^2}{10.606}\right)$$

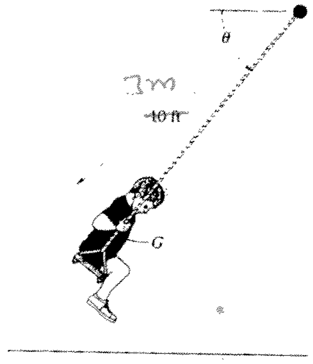
$N = 1.48$ kN **Ans**

33

5m/s

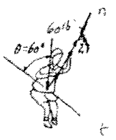
13-61. At the instant $\theta = 60°$, the boy's center of mass G has a downward speed v_G = 15 ft/s. Determine the rate of increase in his speed and the tension in each of the two supporting cords of the swing at this instant. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.

250N



$\swarrow + \Sigma F_t = ma_t; \quad \overset{250}{\cancel{60}}\cos 60° = \frac{\overset{250}{\cancel{60}}}{\underset{9.81}{\cancel{32.2}}}a_t \qquad a_t = \overset{4.905\text{ m/s}^2}{\cancel{16.1\text{ ft/s}^2}}$ **Ans**

$\nwarrow + \Sigma F_n = ma_n;$ ~~$2T - 60\sin 60° = \frac{60}{32.2}\left(\frac{15^2}{10}\right) \qquad T = 46.9$ lb **Ans**~~

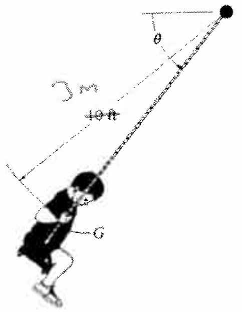


$$2T - 250\sin 60° = \frac{250}{9.81}\left(\frac{5^2}{10}\right)$$

$T = 140$ N

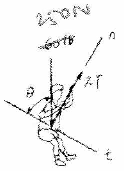
34

13-62. At the instant $\theta = 60°$, the boy's center of mass G is momentarily at rest. Determine his speed and the tension in each of the two supporting cords of the swing when $\theta = 90°$. The boy has a weight of ~~60 lb~~. Neglect his size and the mass of the seat and cords. 250N



$+\searrow \Sigma F_t = ma_t;$ $\quad \overset{250}{\cancel{60}}\cos\theta = \frac{\overset{250}{\cancel{60}}}{\underset{9.81}{\cancel{32.2}}}a_t \quad a_t = \cancel{32.2}\cos\theta \quad 9.81\cos\theta$

$\nearrow+\Sigma F_n = ma_n;$ $\quad 2T - \overset{250}{\cancel{60}}\sin\theta = \frac{\overset{250}{\cancel{60}}}{\underset{9.81}{\cancel{32.2}}}\left(\frac{v^2}{\underset{3}{\cancel{40}}}\right) \quad [1]$



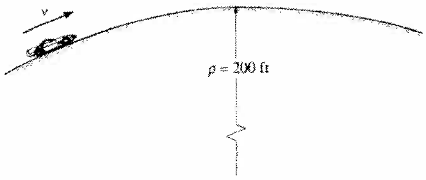
$v\,dv = a\,ds \qquad$ however $ds = \underset{3}{\cancel{40}}d\theta$

$\int_0^v v\,dv = \int_{60°}^{90°} \overset{9.81}{\cancel{32.2}}\cos\theta\,d\theta$

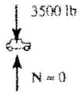
$v = \overset{2.81\ m/s}{\cancel{9.289\ ft/s}}$ **Ans**

From Eq.[1] $\quad 2T - \overset{250}{\cancel{60}}\sin 90° = \frac{\overset{250}{\cancel{60}}}{\underset{9.81}{\cancel{32.2}}}\left(\frac{\overset{2.81^2}{\cancel{9.289^2}}}{\underset{3}{\cancel{40}}}\right) \quad$ ~~$T = 38.0$ lb~~ **Ans** T = 158.5 N

13-63. If the crest of the hill has a radius of curvature $\rho = 200$ ft, determine the maximum constant speed at which the car can travel over it without leaving the surface of the road. Neglect the size of the car in the calculation. The car has a weight of 3500 lb.

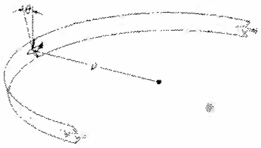


$\downarrow \sum F_n = ma_n; \quad 3500 = \frac{3500}{32.2}\left(\frac{v^2}{200}\right)$



$v = 80.2$ ft/s **Ans**

***13-64.** The airplane, traveling at a constant speed of 50 m/s, is executing a horizontal turn. If the plane is banked at $\theta = 15°$, when the pilot experiences only a normal force on the seat of the plane, determine the radius of curvature ρ of the turn. Also, what is the normal force of the seat on the pilot if he has a mass of 70 kg?



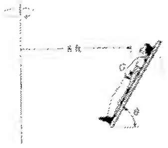
$+\uparrow \sum F_b = ma_b; \quad N_P \sin 15° - 70(9.81) = 0$

$N_P = 2.65$ kN **Ans**

$\overset{+}{\leftarrow} \sum F_n = ma_n; \quad N_P \cos 15° = 70\left(\frac{50^2}{\rho}\right)$

$\rho = 68.3$ m **Ans**

13-65. The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. Determine the resultant normal and frictional forces the cushion exerts on him if, due to rotation about the z axis, he has a constant speed $v = 20$ ft/s. Neglect the size of the man. Take $\theta = 60°$.



$+\nwarrow \sum F_y = m(a_n)_y; \quad N - 150\cos 60° = \frac{150}{32.2}\left(\frac{20^2}{8}\right)\sin 60°$

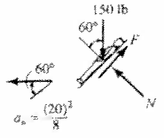
$N = 277$ lb **Ans**

$+\swarrow \sum F_x = m(a_n)_x; \quad -F + 150\sin 60° = \frac{150}{32.2}\left(\frac{20^2}{8}\right)\cos 60°$

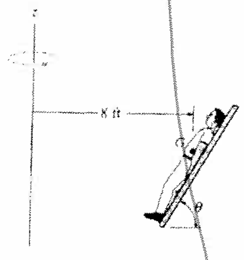
$F = 13.4$ lb **Ans**

Note: No slipping occurs

Since $\mu_s N = 138.4$ lb > 13.4 lb



13-66. The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. If he rotates about the z axis with a constant speed $v = 30$ ft/s, determine the smallest angle θ of the cushion at which he will begin to slip off.

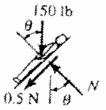


$$\overset{+}{\leftarrow}\sum F_n = ma_n; \quad 0.5N\cos\theta + N\sin\theta = \frac{150}{32.2}\left(\frac{(30)^2}{8}\right)$$

$$+\uparrow\sum F_b = 0; \quad -150 + N\cos\theta - 0.5N\sin\theta = 0$$

$$N = \frac{150}{\cos\theta - 0.5\sin\theta}$$

$$\frac{(0.5\cos\theta + \sin\theta)150}{(\cos\theta - 0.5\sin\theta)} = \frac{150}{32.2}\left(\frac{(30)^2}{8}\right)$$

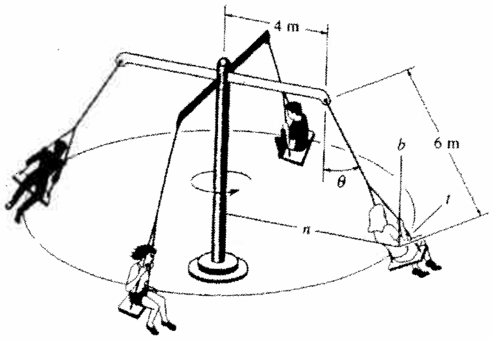


$$0.5\cos\theta + \sin\theta = 3.49378\cos\theta - 1.74689\sin\theta$$

$\theta = 47.5°$ **Ans**

35

13-67. Determine the constant speed of the passengers on the amusement-park ride if it is observed that the supporting cables are directed at $\theta = 30°$ from the vertical. Each chair including its passenger has a mass of 80 kg. Also, what are the components of force in the n, t, and b directions which the chair exerts on a 50-kg passenger during the motion?

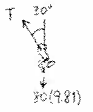


$$\overset{+}{\leftarrow}\Sigma F_n = m\,a_n; \quad T\sin30° = 80(\frac{v^2}{4 + 6\sin30°})$$

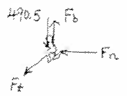
$$+\uparrow\Sigma F_b = 0; \quad T\cos30° - 80(9.81) = 0$$

$$T = 906.2 \text{ N}$$

$v = 6.30$ m/s **Ans**



$\leftarrow a_n = \frac{v^2}{4+6\sin30°}$



$\Sigma F_n = m\,a_n; \quad F_n = 50(\frac{(6.30)^2}{7}) = 283$ N **Ans**

$\Sigma F_t = m\,a_t; \quad F_t = 0$ **Ans**

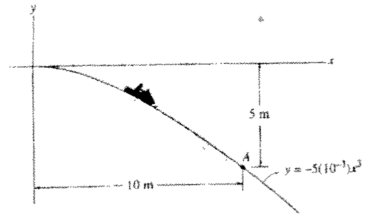
$\leftarrow a_n = \frac{(6.30)^2}{7}$

$\Sigma F_b = m\,a_b; \quad F_b - 490.5 = 0$

$F_b = 490$ N **Ans**

26

***13-68.** The 200-kg snowmobile with passenger is traveling down the hill at a constant speed of 6 m/s. Determine the resultant normal force and the resultant frictional force exerted on the tracks at the instant it reaches point *A*. Neglect the size of the snowmobile.



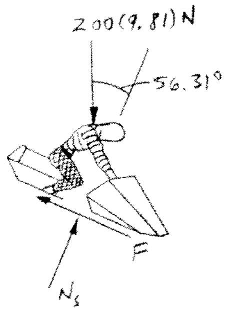
$$y = -5(10^{-3})x^3\Big|_{x=10\text{ m}} = -5\text{ m}$$

$$\frac{dy}{dx} = -15(10^{-3})x^2\Big|_{x=10\text{ m}} = -1.5$$

$$\frac{d^2y}{dx^2} = -30(10^{-3})x\Big|_{x=10\text{ m}} = -0.3$$

$$\theta = \tan^{-1}(-1.5) = -56.31^\circ$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+(-1.5)^2\right]^{\frac{3}{2}}}{|-0.3|} = 19.53\text{ m}$$



$+\searrow \Sigma F_t = ma_t;\quad -F + 200(9.81)\sin 56.31^\circ = 0$

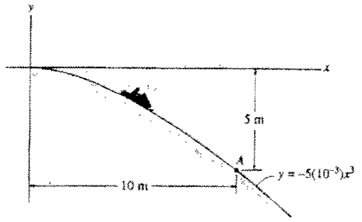
$F = 1632\text{ N} = 1.63\text{ kN}$ **Ans**

$+\swarrow \Sigma F_n = ma_n;\quad -N_s + 200(9.81)\cos 56.31^\circ = 200\left(\frac{(6)^2}{19.53}\right)$

$N_s = 720\text{ N}$ **Ans**

27

13-69. The 200-kg snowmobile with passenger is traveling down the hill such that when it is at point *A*, it is traveling at 4 m/s and increasing its speed at 2 m/s². Determine the resultant normal force and the resultant frictional force exerted on the tracks at this instant. Neglect the size of the snowmobile.



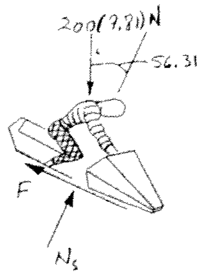
$$y = -5(10^{-3})x^3\Big|_{x=10\text{ m}} = -5\text{ m}$$

$$\frac{dy}{dx} = -15(10^{-3})x^2\Big|_{x=10\text{ m}} = -1.5$$

$$\frac{d^2y}{dx^2} = -30(10^{-3})x\Big|_{x=10\text{ m}} = -0.3$$

$$\theta = \tan^{-1}(-1.5) = -56.31^\circ$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+(-1.5)^2\right]^{\frac{3}{2}}}{|-0.3|} = 19.53\text{ m}$$



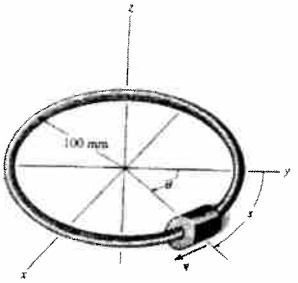
$+\searrow \Sigma F_t = ma_t;\quad -F + 200(9.81)\sin 56.31^\circ = 200(2)$

$F = 1.23\text{ kN}$ **Ans**

$+\swarrow \Sigma F_n = ma_n;\quad -N_s + 200(9.81)\cos 56.31^\circ = 200\left(\frac{(4)^2}{19.53}\right)$

$N_s = 924\text{ N}$ **Ans**

13-70. A collar having a mass of 0.75 kg and negligible size slides over the surface of a horizontal circular rod for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the collar is given a speed of 4 m/s and then released at $\theta = 0°$, determine how far, s, it slides on the rod before coming to rest.



$\Sigma F_t = ma_t; \qquad -0.3N_C = 0.75a_t \qquad (1)$

$\Sigma F_n = ma_n; \qquad (N_C)_n = 0.75\left(\frac{v^2}{0.1}\right)$

$\Sigma F_z = 0; \qquad (N_C)_z - 0.75(9.81) = 0 \qquad (N_C)_z = 7.3575$

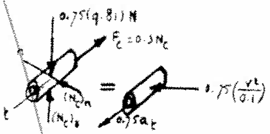
Hence, $N_C = \sqrt{(7.3575)^2 + (7.5v^2)^2}$

Since $v\,dv = a_t\,ds$, then from Eq. (1),

$$v\,dv = -\frac{0.3}{0.75}\left(\sqrt{(7.3575)^2 + (7.5v^2)^2}\right)ds$$

$$\int_4^v \frac{v\,dv}{\left(\sqrt{(7.3575)^2 + (7.5v^2)^2}\right)} = \int_0^s -0.4\,ds$$

$$\int_4^v \frac{v\,dv}{\sqrt{(0.981)^2 + v^4}} = -3s$$



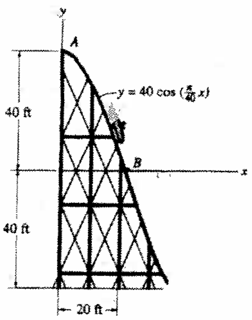
$$s = -\frac{1}{6}\ln\left(\frac{v^2 + \sqrt{(0.981)^2 + v^4}}{32.03}\right)$$

When $v = 0$,

$$s = -\frac{1}{6}\ln\left(\frac{0.981}{32.03}\right)$$

$= 0.581$ m **Ans**

13-71. The roller coaster car and passenger have a total weight of 600 lb and starting from rest at A travel down a track that has the shape shown. Determine the normal force of the tracks on the car when the car is at point B, where it has a velocity of 15 ft/s. Neglect friction and the size of the car and passenger.



At point B :

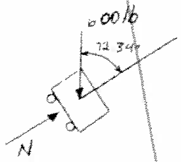
$$y = 40\cos\left(\frac{\pi}{40}x\right)$$

$$\frac{dy}{dx} = -\pi\sin\left(\frac{\pi}{40}x\right)\Big|_{x=20} = -\pi$$

$$\theta = \tan^{-1}(-\pi) = -72.34°$$

$$\frac{d^2y}{dx^2} = -\frac{\pi^2}{40}\cos\left(\frac{\pi}{40}x\right)\Big|_{x=20} = 0$$

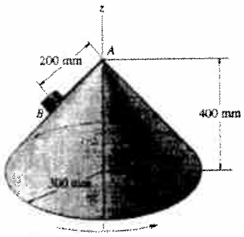
$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{1}{0} \to \infty$$



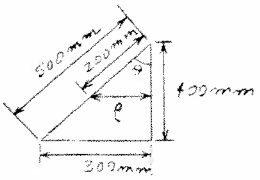
$\swarrow\Sigma F_n = ma_n; \qquad N_t - 600\cos 72.34° = \left(\frac{600}{32.2}\right)\left(\frac{(v_B)^2}{\infty}\right)$

$N_t = 182$ lb **Ans**

***13-72.** The smooth block B, having a mass of 0.2 kg, is attached to the vertex A of the right circular cone using a light cord. The cone is rotating at a constant angular rate about the z axis such that the block attains a speed of 0.5 m/s. At this speed, determine the tension in the cord and the reaction which the cone exerts on the block. Neglect the size of the block.



$$\frac{\rho}{200}=\frac{300}{500};\qquad \rho = 120\text{ mm} = 0.120\text{ m}$$



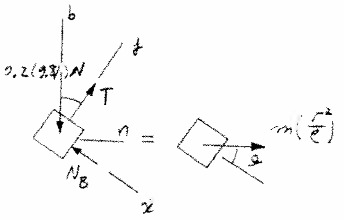
$$+\nearrow\Sigma F_y = ma_y;\qquad T-0.2(9.81)\left(\frac{4}{5}\right)=\left[0.2\left(\frac{(0.5)^2}{0.120}\right)\right]\left(\frac{3}{5}\right)$$

$$T = 1.82\text{ N}\qquad \textbf{Ans}$$

$$+\searrow\Sigma F_x = ma_x;\qquad N_B-0.2(9.81)\left(\frac{3}{5}\right)=-\left[0.2\left(\frac{(0.5)^2}{0.120}\right)\right]\left(\frac{4}{5}\right)$$

$$N_B = 0.844\text{ N}\qquad \textbf{Ans}$$

Also,



$$\xrightarrow{+}\ \Sigma F_n = ma_n;\qquad T\left(\frac{3}{5}\right)-N_B\left(\frac{4}{5}\right)=0.2\left(\frac{(0.5)^2}{0.120}\right)$$

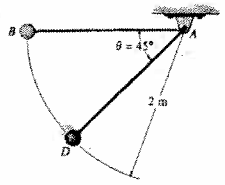
$$+\uparrow\Sigma F_b = 0;\qquad T\left(\frac{4}{5}\right)+N_B\left(\frac{3}{5}\right)-0.2(9.81)=0$$

$$T = 1.82\text{ N}\qquad \textbf{Ans}$$

$$N_B = 0.844\text{ N}\qquad \textbf{Ans}$$

58

13-73. The 5-kg pendulum bob B is released from rest when $\theta = 0°$. Determine the initial tension in the cord and also at the instant the bob reaches point D, $\theta = 45°$. Neglect the size of the bob.



Initially, $v = 0$ so $a_n = 0$.

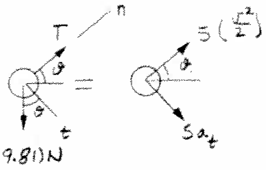
$$\xrightarrow{+}\Sigma F_n = ma_n;\qquad T = 0\qquad \textbf{Ans}$$

In the general position

$$+\searrow\Sigma F_t = ma_t;\qquad 5(9.81)\cos\theta = 5a_t$$

$$a_t = 9.81\cos\theta$$

$$+\nearrow\Sigma F_n = ma_n;\qquad T-5(9.81)\sin\theta = 5\left(\frac{v^2}{2}\right)\qquad (1)$$



Kinematics :

$v\,dv = a_t\,ds$, where $ds = 2\,d\theta$

$$\int_0^v v\,dv = 19.62\int_{0°}^{45°}\cos\theta\,d\theta$$

$$\frac{1}{2}v^2 = 19.62\big[\sin\theta\big]_{0°}^{45°} = 13.87$$

$$v = 5.268\text{ m/s}$$

Substituting into Eq. (1), with $\theta = 45°$ yields

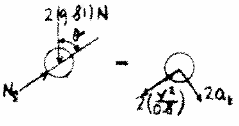
$$T = 104\text{ N}\qquad \textbf{Ans}$$

13-74. A ball having a mass of 2 kg and negligible size moves within a smooth vertical circular slot. If it is released from rest when $\theta = 10°$, determine the force of the slot on the ball when the ball arrives at points A and B.

$+\searrow \Sigma F_t = ma_t;\quad 2(9.81)\sin\theta = 2a_t$

$a_t = 9.81\sin\theta \quad (1)$

$+\swarrow \Sigma F_n = ma_n;\quad -N_s + 2(9.81)\cos\theta = 2\left(\frac{v^2}{0.8}\right) \quad (2)$



$v\,dv = a_t\,ds = a_t(0.8\,d\theta)$

$v\,dv = 9.81\sin\theta(0.8\,d\theta)$

At A: $\int_0^{v_A} v\,dv = 7.848\int_{10°}^{90°} \sin\theta\,d\theta$

$\frac{1}{2}v_A^2 = 7.848\left[-\cos 90° + \cos 10°\right]$

$v_A = 3.932 \text{ m/s}$

From Eq (2):

$N_s = 2(9.81)\cos 90° - 2\left(\frac{(3.932)^2}{0.8}\right) = -38.6 \text{ N}$

$N_s = 38.6 \text{ N}$ **Ans**

At B: $\int_0^{v_B} v\,dv = 7.848\int_{10°}^{170°} \sin\theta\,d\theta$

$\frac{1}{2}v_B^2 = 7.848\left[-\cos 170° + \cos 10°\right]$

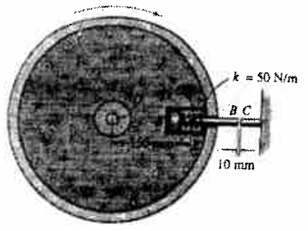
$v_B = 5.560 \text{ m/s}$

$N_s = 2(9.81)\cos 170° - 2\left(\frac{(5.560)^2}{0.8}\right)$

$N_s = -96.6 \text{ N}$

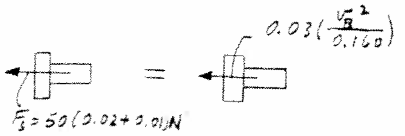
So that $N_s = 96.6 \text{ N}$ **Ans**

13-75. The rotational speed of the disk is controlled by a 30-g smooth contact arm AB which is spring-mounted on the disk. When the disk is *at rest*, the center of mass G of the arm is located 150 mm from the center O, and the preset compression in the spring is 20 mm. If the initial gap between B and the contact at C is 10 mm, determine the (controlling) speed v_G of the arm's mass center, G, which will close the gap. The disk rotates in the horizontal plane. The spring has a stiffness of $k = 50$ N/m, and its ends are attached to the contact arm at D and to the disk at E.

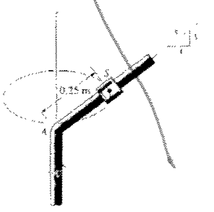


$\overset{+}{\leftarrow} \Sigma F_n = ma_n;\quad 50(0.03) = 0.03\left(\frac{v_G^2}{0.160}\right)$

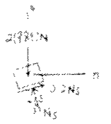
$v_G = 2.83 \text{ m/s}$ **Ans**



•13-76. The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the minimum constant speed the spool can have so that it does not slip down the rod.



$\rho = 0.25(\frac{4}{5}) = 0.2$ m



$\overset{+}{\leftarrow} \Sigma F_n = m\,a_n;\quad N_s(\frac{3}{5}) - 0.2N_s(\frac{4}{5}) = 2(\frac{v^2}{0.2})$

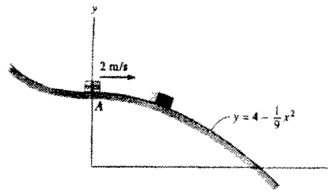
$+\uparrow \Sigma F_b = m\,a_b;\quad N_s(\frac{4}{5}) + 0.2N_s(\frac{3}{5}) - 2(9.81) = 0$

$N_s = 21.3$ N

$v = 0.969$ m/s **Ans**

39

13-77. The 35-kg box has a speed of 2 m/s when it is at A on the smooth ramp. If the surface is in the shape of a parabola, determine the normal force on the box at the instant $x = 3$ m. Also, what is the rate of increase in its speed at this instant?

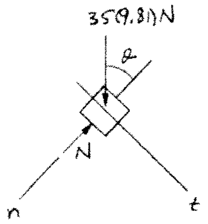


$y = 4 - \frac{1}{9}x^2$

$\frac{dy}{dx} = \tan\theta = -\frac{2}{9}x\Big|_{x=3} = -0.6667 \qquad \theta = -33.69°$

$\frac{d^2y}{dx^2} = -\frac{2}{9}$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+\left(-\frac{2}{9}x\right)^2\right]^{\frac{3}{2}}}{\left|-\frac{2}{9}\right|} = 4.5\left(1+0.04938x^2\right)^{\frac{3}{2}}\Big|_{x=3} = 7.812\text{ m}$$



$+\swarrow \Sigma F_n = ma_n;\quad 35(9.81)\cos\theta - N = 35\left(\frac{v^2}{7.812}\right) \quad (1)$

$+\searrow \Sigma F_t = ma_t;\quad 35(9.81)\sin\theta = 35a_t$

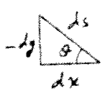
$a_t = 9.81\sin\theta \quad (2)$

$v\,dv = a_t\,ds$

$v\,dv = 9.81\sin\theta\,ds$

$v\,dv = 9.81\left(-\frac{dy}{ds}\right)ds = -9.81\,dy$

When $x = 0$, $y = 4$. When $x = 3$, $y = 4 - \frac{1}{9}(3)^2 = 3$. Thus,



$\int_2^v v\,dv = -\int_4^3 9.81\,dy$

$\frac{1}{2}v^2 - \frac{1}{2}(2)^2 = -9.81(3-4)$

$v = 4.86$ m/s

From Eqs. (1) and (2) for $\theta = 33.69°$

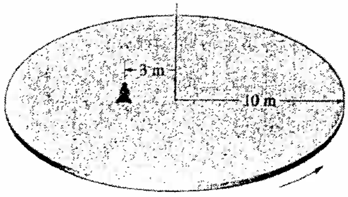
$35(9.81)\cos 33.69° - N = 35\left(\frac{(4.86)^2}{7.812}\right)$

$N = 180$ N **Ans**

$a_t = 9.81\sin 33.69° = 5.44$ m/s^2 **Ans**

40

13-78. The man has a mass of 80 kg and sits 3 m from the center of the rotating platform. Due to the rotation his speed is increased from rest by $\dot{v} = 0.4\ \text{m/s}^2$. If the coefficient of static friction between his clothes and the platform is $\mu_s = 0.3$, determine the time required to cause him to slip.



$\Sigma F_t = m\,a_t;\quad F_t = 80(0.4)$

$F_t = 32\ \text{N}$

$\Sigma F_n = m\,a_n;\quad F_n = (80)\dfrac{v^2}{3}$

$F = \mu_s N_m = \sqrt{(F_t)^2 + (F_n)^2}$

$0.3(80)(9.81) = \sqrt{(32)^2 + ((80)\frac{v^2}{3})^2}$

$55\,432 = 1024 + (6400)(\frac{v^4}{9})$

$v = 2.9575\ \text{m/s}$

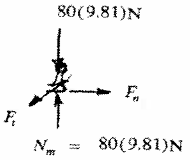
$a_t = \dfrac{dv}{dt} = 0.4$

$\int_0^v dv = \int_0^t 0.4\,dt$

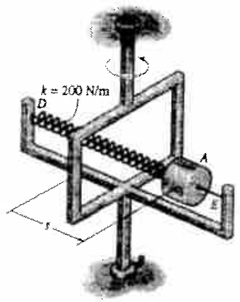
$v = 0.4\,t$

$2.9575 = 0.4\,t$

$t = 7.39\ \text{s}$ **Ans**



13-79. The collar A, having a mass of 0.75 kg, is attached to a spring having a stiffness of $k = 200$ N/m. When rod BC rotates about the vertical axis, the collar slides outward along the smooth rod DE. If the spring is unstretched when $s = 0$, determine the constant speed of the collar in order that $s = 100$ mm. Also, what is the normal force of the rod on the collar? Neglect the size of the collar.



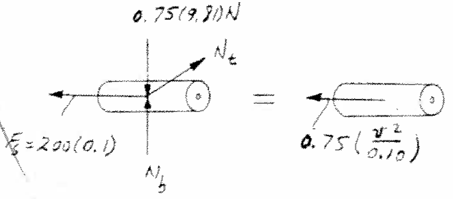
$\Sigma F_b = 0;\quad N_b - 0.75(9.81) = 0\quad N_b = 7.36$

$\Sigma F_n = ma_n;\quad 200(0.1) = 0.75\left(\dfrac{v^2}{0.10}\right)$

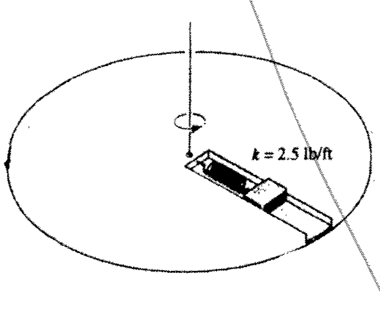
$\Sigma F_t = ma_t;\quad N_t = 0$

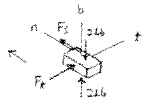
$v = 1.63\ \text{m/s}$ **Ans**

$N = \sqrt{(7.36)^2 + (0)^2} = 7.36\ \text{N}$ **Ans**



***13-80.** The block has a weight of 2 lb and it is free to move along the smooth slot in the rotating disk. The spring has a stiffness of 2.5 lb/ft and an unstretched length of 1.25 ft. Determine the force of the spring on the block and the tangential component of force which the slot exerts on the side of the block, when the block is at rest with respect to the disk and is traveling with a constant speed of 12 ft/s.





$\Sigma F_n = ma_n;\quad F_s = \frac{2}{32.2}(\frac{12^2}{\rho})$

$\Sigma F_t = ma_t;\quad F_t = 0$ **Ans**

$F_s = ks;\quad F_s = 2.5(\rho - 1.25)$

$2.5(32.2)(\rho^2 - 1.25\rho) = 288$

$\rho^2 - 1.25\rho - 3.58 = 0$

Choosing the positive root,

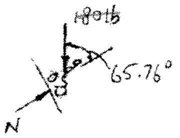
$\rho = 2.62$ ft

$F_s = 2.5(2.62 - 1.25) = 3.42$ lb **Ans**

13-~~81~~ 41. If the bicycle and rider have a total weight of ~~180~~ lb, determine the resultant normal force acting on the bicycle when it is at point A while it is freely coasting at v_A = ~~6 ft~~/s. Also, compute the increase in the bicyclist's speed at this point. Neglect the resistance due to the wind and the size of the bicycle and rider.

800N

2m/s



$y = \cancel{20 \cos(\frac{\pi}{20}x)}$ $8\cos(\frac{\pi}{8}x)$

$\frac{dy}{dx} = -\pi \sin(\frac{\pi}{\cancel{20}_{8}}x)\Big|_{x = \cancel{5\text{ ft}}\, 2} = -2.221$

$\theta = \tan^{-1}(-2.221) = -65.76°$ ✓

$\frac{d^2y}{dx^2} = -\frac{\pi^2}{\cancel{20}_{8}}\cos(\frac{\pi}{\cancel{20}_{8}}x)\Big|_{x = \cancel{5\text{ ft}}\, 2m} = \cancel{-0.3489}$ -0.872

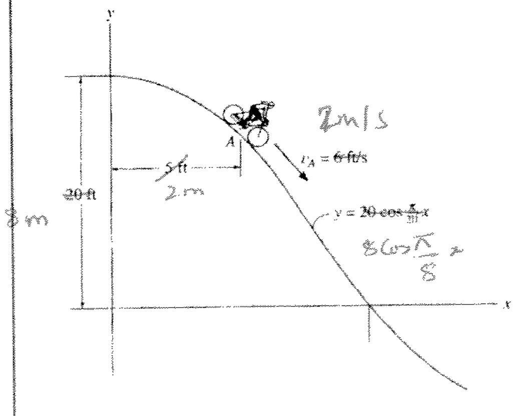
$\rho = \left|\frac{[1 + (-2.221)^2]^{3/2}}{\cancel{-0.3489}_{-0.872}}\right| = \cancel{41.43\text{ ft}}$ 16.57 m

$\searrow + \Sigma F_t = m\,a_t;\quad \cancel{180}_{800} \sin 65.76° = \frac{\cancel{180}^{800}}{\cancel{32.2}_{9.81}} a_t$

$a_t = \cancel{29.4\text{ ft/s}^2}$ 8.945 m/s^2 **Ans**

$\swarrow + \Sigma F_n = m\,a_n;\quad 180\cos 65.76° - N = \frac{180}{32.2}(\frac{6^2}{41.43})$

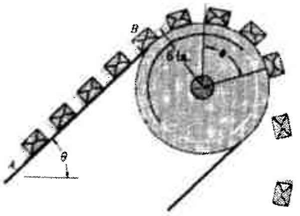
$N = 69.0$ lb **Ans**



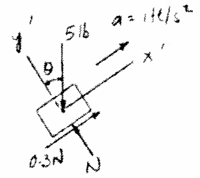
$800 \cos 65.76 - N = \frac{800}{9.81}\left(\frac{2^2}{16.57}\right)$

$N = 308$ N ✓

13-82. The 5-lb packages ride on the surface of the conveyor belt. If the belt starts from rest and increases to a constant speed of 2 ft/s in 2 s, determine the maximum angle θ so that none of the packages slip on the inclined surface AB of the belt. The coefficient of static friction between the belt and a package is $\mu_s = 0.3$. At what angle ϕ do the packages first begin to slip off the surface of the belt after the belt is moving at its constant speed of 2 ft/s? Neglect the size of the packages.



$v = v_1 + a_c t; \quad 2 = 0 + a_c(2); \quad a_c = 1 \text{ ft/s}^2$



$+\nwarrow \Sigma F_{y'} = ma_{y'}; \quad N - 5\cos\theta = 0 \qquad (1)$

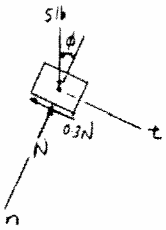
$+\nearrow \Sigma F_{x'} = ma_{x'}; \quad 0.3N - 5\sin\theta = \frac{5}{32.2}(1) \qquad (2)$

Solving Eqs. (1) and (2) yields :

$\theta = 15.0°$ **Ans**

$N = 4.83$ lb

For circular motion



$+\swarrow \Sigma F_n = ma_n; \quad 5\cos\phi - N = \frac{5}{32.2}\left(\frac{2^2}{0.5}\right) \qquad (3)$

$+\searrow \Sigma F_t = ma_t; \quad 5\sin\phi - 0.3N = 0 \qquad (4)$

Solving Eqs. (3) and (4) yields :

$\phi = 12.6°$ **Ans**

$N = 3.64$ lb

13-83. A particle, having a mass of 1.5 kg, moves along a path defined by the equations $r = (4 + 3t)$ m, $\theta = (t^2 + 2)$ rad, and $z = (6 - t^3)$ m, where t is in seconds. Determine the r, θ, and z components of force which the path exerts on the particle when $t = 2$ s.

$r = 4 + 3t|_{t=2\text{ s}} = 10$ m $\qquad \dot{r} = 3$ m/s $\qquad \ddot{r} = 0$

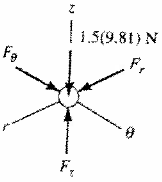
$\theta = t^2 + 2 \qquad \dot{\theta} = 2t|_{t=2\text{ s}} = 4$ rad/s $\qquad \ddot{\theta} = 2$ rad/s^2

$z = 6 - t^3 \qquad \dot{z} = -3t^2 \qquad \ddot{z} = -6t|_{t=2\text{ s}} = -12$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 10(4)^2 = -160$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 10(2) + 2(3)(4) = 44$ m/s^2

$a_z = \ddot{z} = -12$ m/s^2

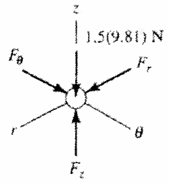


$\Sigma F_r = ma_r; \quad F_r = 1.5(-160) = -240$ N **Ans**

$\Sigma F_\theta = ma_\theta; \quad F_\theta = 1.5(44) = 66$ N **Ans**

$\Sigma F_z = ma_z; \quad F_z - 1.5(9.81) = 1.5(-12) \quad F_z = -3.28$ N **Ans**

***13-84.** The path of motion of a 5-lb particle in the horizontal plane is described in terms of polar coordinates as $r = (2t + 1)$ ft and $\theta = (0.5t^2 - t)$ rad, where t is in seconds. Determine the magnitude of the unbalanced force acting on the particle when $t = 2$ s.



$r = 2t + 1|_{t=2\,s} = 5$ ft $\qquad \dot r = 2$ ft/s $\qquad \ddot r = 0$

$\theta = 0.5t^2 - t|_{t=2\,s} = 0$ rad $\qquad \dot\theta = t - 1|_{t=2\,s} = 1$ rad/s $\qquad \ddot\theta = 1$ rad/s^2

$a_r = \ddot r - r\dot\theta^2 = 0 - 5(1)^2 = -5$ ft/s^2

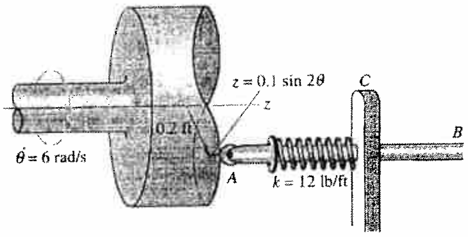
$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 5(1) + 2(2)(1) = 9$ ft/s^2

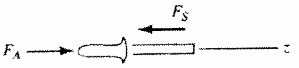
$\Sigma F_r = ma_r;\qquad F_r = \dfrac{5}{32.2}(-5) = -0.7764$ lb

$\Sigma F_\theta = ma_\theta;\qquad F_\theta = \dfrac{5}{32.2}(9) = 1.398$ lb

$F = \sqrt{F_r^2 + F_\theta^2} = \sqrt{(-0.7764)^2 + (1.398)^2} = 1.60$ lb **Ans**

13-85. The spring-held follower AB has a weight of 0.75 lb and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.2$ ft and $z = (0.1 \sin 2\theta)$ ft. If the cam is rotating at a constant rate of 6 rad/s, determine the force at the end A of the follower when $\theta = 45°$. In this position the spring is compressed 0.4 ft. Neglect friction at the bearing C.





$z = 0.1 \sin 2\theta$

$\dot z = 0.2 \cos 2\theta\dot\theta$

$\ddot z = -0.4 \sin 2\theta\dot\theta^2 + 0.2 \cos 2\theta\ddot\theta$

$\dot\theta = 6$ rad/s

$\ddot\theta = 0$

$\ddot z = -14.4 \sin 2\theta$

$\sum F_z = ma_z;\qquad F_A - 12(z + 0.3) = m\ddot z$

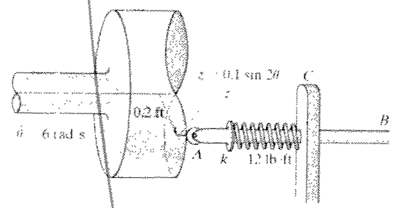
$F_A - 12(0.1 \sin 2\theta + 0.3) = \dfrac{0.75}{32.2}(-14.4 \sin 2\theta)$

For $\theta = 45°$,

$F_A - 12(0.4) = \dfrac{0.75}{32.2}(-14.4)$

$F_A = 4.46$ lb **Ans**

13-86. The spring-held follower AB has a weight of 0.75 lb and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.2$ ft and $z = (0.1 \sin 2(\theta))$ ft. If the cam is rotating at a constant rate of 6 rad/s, determine the maximum and minimum force the follower exerts on the cam if the spring is compressed 0.2 ft when $\theta = 45°$



$z = 0.1 \sin 2\theta$

$\dot{z} = 0.2 \cos 2\theta\dot{\theta}$

$\ddot{z} = -0.4 \sin 2\theta\dot{\theta}^2 + 0.2 \cos 2\theta\ddot{\theta}$

$\dot{\theta} = 6$ rad/s

$\ddot{\theta} = 0$

$\ddot{z} = -14.4 \sin 2\theta$

$\sum F_z = ma_z;\quad F_A - 12(z + 0.1) = m\ddot{z}$

$$F_A - 12(0.1 \sin 2\theta + 0.1) = \frac{0.75}{32.2}(-14.4 \sin 2\theta)$$

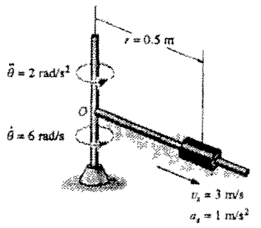
$F_A = 1.2 - 0.8646 \sin 2\theta$

$(F_A)_{max} = 2.06$ lb **Ans**

$(F_A)_{min} = 0.335$ lb **Ans**

43

13-87. The 4-kg spool slides along the rotating rod. At the instant shown, the angular rate of rotation of the rod is $\dot{\theta} = 6$ rad/s, which is increasing at $\ddot{\theta} = 2$ rad/s^2. At this same instant, the spool is moving outward along the rod at 3 m/s, which is increasing at 1 m/s^2 when $r = 0.5$ m. Determine the radial frictional force and the normal force of the rod on the spool at this instant.



$r = 0.5 \quad \dot{r} = 3 \quad \dot{r} = 1$

$\dot{\theta} = 6 \quad \dot{\theta} = 2$

$a_r = \ddot{r} - r\dot{\theta}^2 = 1 - 0.5(6)^2 = -17$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(2) + 2(3)(6) = 37$

Hence,

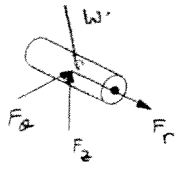
$\Sigma F_r = ma_r;\quad F_r = 4(-17) = -68$ N

$\Sigma F_\theta = ma_\theta;\quad F_\theta = 4(37) = 148$ N

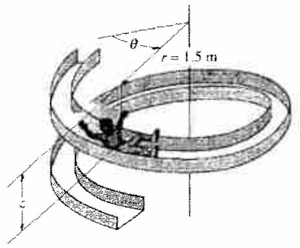
$\Sigma F_z = ma_z;\quad F_z - 4(9.81) = 0 \quad F_z = 39.24$ N

$F_{frict} = 68$ N **Ans**

Normal force is $N_C = \sqrt{(148)^2 + (39.24)^2} = 153$ N **Ans**



13-88. The boy of mass 40 kg is sliding down the spiral slide at a constant speed such that his position, measured from the top of the chute, has components $r = 1.5$ m, $\theta = (0.7t)$ rad, and $z = (-0.5t)$ m, where t is in seconds. Determine the components of force $\mathbf{F}_r$, $\mathbf{F}_\theta$, and $\mathbf{F}_z$ which the slide exerts on him at the instant $t = 2$ s. Neglect the size of the boy.



$r = 1.5 \qquad \theta = 0.7t \qquad z = -0.5t$

$\dot{r} = \ddot{r} = 0 \qquad \dot\theta = 0.7 \qquad \dot z = -0.5$

$\ddot\theta = 0 \qquad \ddot z = 0$

$a_r = \ddot r - r(\dot\theta)^2 = 0 - 1.5(0.7)^2 = -0.735$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0$

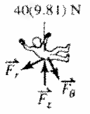
$a_z = \ddot z = 0$

$\sum F_r = ma_r; \quad F_r = 40(-0.735) = -29.4$ N **Ans**

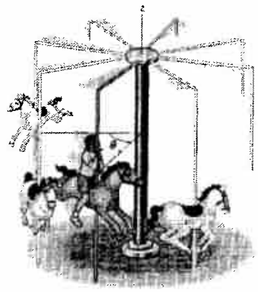
$\sum F_\theta = ma_\theta; \quad F_\theta = 0$ **Ans**

$\sum F_z = ma_z; \quad F_z - 40(9.81) = 0$

$F_z = 392$ N **Ans**



13-89. The girl has a mass of 50 kg. She is seated on the horse of the merry-go-round which undergoes constant rotational motion $\dot\theta = 1.5$ rad/s. If the path of the horse is defined by $r = 4$ m, $z = (0.5 \sin \theta)$ m, determine the maximum and minimum force F_z the horse exerts on her during the motion.



$\theta = 1.5 \qquad \theta = 0$

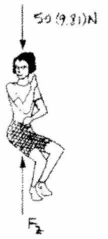
$z = 0.5\sin\theta \qquad \dot z = 0.5\cos\theta\theta \qquad \ddot z = -0.5\sin\theta\theta^2 + 0.5\cos\theta\theta$

$+\uparrow \Sigma F_z = ma_z; \quad F_z - 50(9.81) = 50[-0.5\sin\theta(1.5)^2 + 0]$

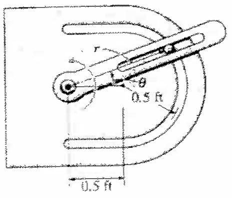
$F_z = 490.5 - 56.25\sin\theta$

Max. when $\sin\theta = -1$, $\quad (F_z)_{\max} = 547$ N **Ans**

Min. when $\sin\theta = 1$, $\quad (F_z)_{\min} = 434$ N **Ans**



13-90. The 0.5-lb particle is guided along the circular path using the slotted arm guide. If the arm has an angular velocity $\dot\theta = 4$ rad/s and an angular acceleration $\ddot\theta = 8$ rad/s^2 at the instant $\theta = 30°$, determine the force of the guide on the particle. Motion occurs in the *horizontal plane*.



$r = 2(0.5\cos\theta) = 1\cos\theta$

$\dot r = -\sin\theta\dot\theta$

$\ddot r = -\cos\theta\dot\theta^2 - \sin\theta\ddot\theta$

At $\theta = 30°$, $\dot\theta = 4$ rad/s and $\ddot\theta = 8$ rad/s^2

$r = 1\cos 30° = 0.8660$ ft

$\dot r = -\sin 30°(4) = -2$ ft/s

$\ddot r = -\cos 30°(4)^2 - \sin 30°(8) = -17.856$ ft/s^2

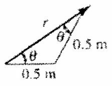
$a_r = \ddot r - r\dot\theta^2 = -17.856 - 0.8660(4)^2 = -31.713$ ft/s^2

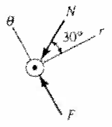
$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0.8660(8) + 2(-2)(4) = -9.072$ ft/s^2

$\nearrow + \sum F_r = ma_r; \quad -N\cos 30° = \frac{0.5}{32.2}(-31.713) \quad N = 0.5686$ lb

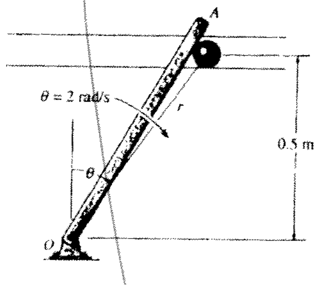
$+\nwarrow \sum F_\theta = ma_\theta; \quad F - 0.5686\sin 30° = \frac{0.5}{32.2}(-9.072)$

$F = 0.143$ lb **Ans**

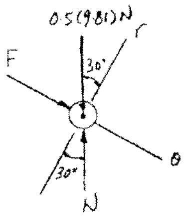




13-91. The particle has a mass of 0.5 kg and is confined to move along the smooth horizontal slot due to the rotation of the arm *OA*. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot{\theta} = 2$ rad/s. Assume the particle contacts only one side of the slot at any instant.



$$r = \frac{0.5}{\cos\theta} = 0.5\sec\theta$$



$$\dot{r} = 0.5\sec\theta\tan\theta\dot{\theta}$$

$$\ddot{r} = 0.5\left\{\left[\left(\sec\theta\tan\theta\dot{\theta}\right)\tan\theta + \sec\theta\left(\sec^2\theta\dot{\theta}\right)\right]\dot{\theta} + \sec\theta\tan\theta\ddot{\theta}\right\}$$

$$= 0.5\left[\sec\theta\tan^2\theta\dot{\theta}^2 + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan\theta\ddot{\theta}\right]$$

When $\theta = 30°$, $\dot{\theta} = 2\text{rad/s}$ and $\ddot{\theta} = 0$

$$r = 0.5\sec 30° = 0.5774 \text{ m}$$

$$\dot{r} = 0.5\sec 30°\tan 30°(2) = 0.6667 \text{ m/s}$$

$$\ddot{r} = 0.5\left[\sec 30°\tan^2 30°(2)^2 + \sec^3 30°(2)^2 + \sec 30°\tan 30°(0)\right]$$

$$= 3.849 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 3.849 - 0.5774(2)^2 = 1.540 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(0) + 2(0.6667)(2) = 2.667 \text{ m/s}^2$$

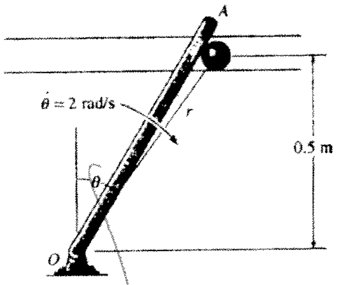
$\nearrow{+}\Sigma F_r = ma_r;$ $\quad N\cos 30° - 0.5(9.81)\cos 30° = 0.5(1.540)$

$N = 5.79$ N **Ans**

$+\searrow\Sigma F_\theta = ma_\theta;$ $\quad F + 0.5(9.81)\sin 30° - 5.79\sin 30° = 0.5(2.667)$

$F = 1.78$ N **Ans**

***13-92.** Solve Problem 13–91 if the arm has an angular acceleration of $\ddot{\theta} = 3\ \text{rad/s}^2$ and $\dot{\theta} = 2\ \text{rad/s}$ at this instant. Assume the particle contacts only one side of the slot at any instant.

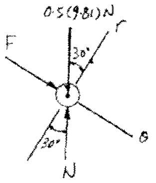


$$r = \frac{0.5}{\cos\theta} = 0.5\sec\theta$$

$$\dot{r} = 0.5\sec\theta\tan\theta\dot{\theta}$$

$$\ddot{r} = 0.5\left\{\left[\left(\sec\theta\tan\theta\dot{\theta}\right)\tan\theta + \sec\theta\left(\sec^2\theta\dot{\theta}\right)\right]\dot{\theta} + \sec\theta\tan\theta\ddot{\theta}\right\}$$

$$= 0.5\left[\sec\theta\tan^2\theta\dot{\theta}^2 + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan\theta\ddot{\theta}\right]$$



When $\theta = 30°$, $\dot{\theta} = 2\text{rad/s}$ and $\ddot{\theta} = 3\ \text{rad/s}^2$

$$r = 0.5\sec 30° = 0.5774\ \text{m}$$

$$\dot{r} = 0.5\sec 30°\tan 30°(2) = 0.6667\ \text{m/s}$$

$$\ddot{r} = 0.5\left[\sec 30°\tan^2 30°(2)^2 + \sec^3 30°(2)^2 + \sec 30°\tan 30°(3)\right]$$

$$= 4.849\ \text{m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4.849 - 0.5774(2)^2 = 2.5396\ \text{m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(3) + 2(0.6667)(2) = 4.3987\ \text{m/s}^2$$

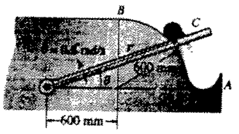
$\nearrow + \Sigma F_r = m a_r;$ $\quad N\cos 30° - 0.5(9.81)\cos 30° = 0.5(2.5396)$

$N = 6.3712 = 6.37\ \text{N}$ **Ans**

$+\searrow \Sigma F_\theta = m a_\theta;$ $\quad F + 0.5(9.81)\sin 30° - 6.3712\sin 30° = 0.5(4.3987)$

$F = 2.93\ \text{N}$ **Ans**

13-93. A smooth can C, having a mass of 3 kg, is lifted from a feed at A to a ramp at B by a rotating rod. If the rod maintains a constant angular velocity of $\dot{\theta} = 0.5$ rad/s, determine the force which the rod exerts on the can at the instant $\theta = 30°$. Neglect the effects of friction in the calculation and the size of the can so that $r = (1.2 \cos \theta)$ m. The ramp from A to B is circular, having a radius of 600 mm.



$r = 2(0.6\cos\theta) = 1.2\cos\theta$

$\dot{r} = -1.2\sin\theta\dot{\theta}$

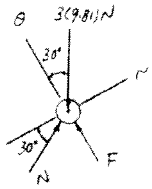
$\ddot{r} = -1.2\cos\theta\dot{\theta}^2 - 1.2\sin\theta\ddot{\theta}$

At $\theta = 30°$, $\dot{\theta} = 0.5$ rad/s and $\ddot{\theta} = 0$

$r = 1.2\cos 30° = 1.0392$ m

$\dot{r} = -1.2\sin 30°(0.5) = -0.3$ m/s

$\ddot{r} = -1.2\cos 30°(0.5)^2 - 1.2\sin 30°(0) = -0.2598$ m/s^2



$a_r = \ddot{r} - r\dot{\theta}^2 = -0.2598 - 1.0392(0.5)^2 = -0.5196$ m/s^2

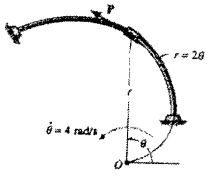
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.0392(0) + 2(-0.3)(0.5) = -0.3$ m/s^2

$+\nearrow \Sigma F_r = ma_r$; $N\cos 30° - 3(9.81)\sin 30° = 3(-0.5196)$ $N = 15.19$ N

$+\nwarrow \Sigma F_\theta = ma_\theta$; $F + 15.19\sin 30° - 3(9.81)\cos 30° = 3(-0.3)$

$F = 17.0$ N **Ans**

13-94. The 2-lb collar slides along the smooth *horizontal* spiral rod, $r = (2\theta)$ ft, where θ is in radians. If its angular rate of rotation is constant and equals $\dot{\theta} = 4$ rad/s, determine the tangential force P needed to cause the motion and the normal force that the spool exerts on the rod at the instant $\theta = 90°$.



$r = 2\theta$

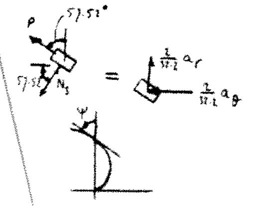
$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{2\theta}{2} = \dfrac{\pi}{2}$ $\psi = 57.52°$

$\dot{\theta} = 4$ $\ddot{\theta} = 0$

$r = 2\theta = 2\left(\dfrac{\pi}{2}\right) = \pi$

$\dot{r} = 2\dot{\theta} = 2(4) = 8$

$\ddot{r} = 2\ddot{\theta} = 0$



$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - \pi(4)^2 = -50.27$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(8)(4) = 64$

$+\uparrow \Sigma F_r = ma_r$; $P\cos 57.52° - N_s\sin 57.52° = \left(\dfrac{2}{32.2}\right)(-50.27)$

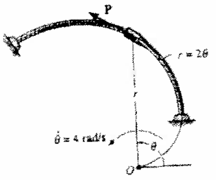
$\overset{+}{\leftarrow} \Sigma F_\theta = ma_\theta$; $P\sin 57.52° + N_s\cos 57.52° = \left(\dfrac{2}{32.2}\right)(64)$

Solving :

$P = 1.68$ lb **Ans**

$N_s = 4.77$ lb **Ans**

13–95. Solve Prob. 13–94 if the spiral rod is *vertical*.



$r = 2\theta$

$\tan\psi = \frac{r}{dr/d\theta} = \frac{2\theta}{2} = \frac{\pi}{2} \qquad \psi = 57.52^\circ$

$\dot{\theta} = 4 \qquad \ddot{\theta} = 0$

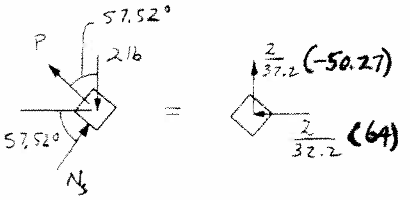
$r = 2\theta = 2\left(\frac{\pi}{2}\right) = \pi$

$r = 2\dot{\theta} = 2(4) = 8$

$\dot{r} = 2\dot{\theta} = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - \pi(4)^2 = -50.27$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(8)(4) = 64$



$+\uparrow \Sigma F_r = ma_r; \qquad P\cos 57.52^\circ - N_r \sin 57.52^\circ - 2 = \left(\frac{2}{32.2}\right)(-50.27)$

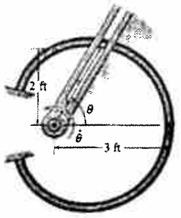
$\overset{+}{\leftarrow} \Sigma F_\theta = ma_\theta; \qquad P\sin 57.52^\circ + N_r\cos 57.52^\circ = \left(\frac{2}{32.2}\right)(64)$

Solving :

$P = 2.75$ lb **Ans**

$N_r = 3.08$ lb **Ans**

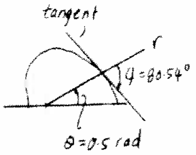
***13-96.** The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If $\theta = (0.5t^2)$ rad, where t is in seconds, determine the force which the rod exerts on the particle at the instant $t = 1$ s. The fork and path contact the particle on only one side.



$r = 2 + \cos\theta \qquad \theta = 0.5t^2$

$\dot{r} = -\sin\theta\dot{\theta} \qquad \dot{\theta} = t$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta} \qquad \ddot{\theta} = 1 \text{ rad/s}^2$

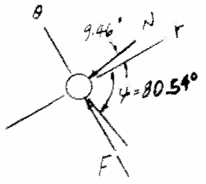


At $t = 1$ s, $\theta = 0.5$ rad, $\dot{\theta} = 1$ rad/s and $\ddot{\theta} = 1$ rad/s^2

$r = 2 + \cos 0.5 = 2.8776$ ft

$\dot{r} = -\sin 0.5(1) = -0.4794$ ft/s

$\ddot{r} = -\cos 0.5(1)^2 - \sin 0.5(1) = -1.357$ ft/s^2



$a_r = \ddot{r} - r\dot{\theta}^2 = -1.357 - 2.8776(1)^2 = -4.2346$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.8776(1) + 2(-0.4794)(1) = 1.9187$ ft/s^2

$\tan\psi = \frac{r}{dr/d\theta} = \left.\frac{2 + \cos\theta}{-\sin\theta}\right|_{\theta = 0.5 \text{ rad}} = -6.002 \qquad \psi = -80.54^\circ$

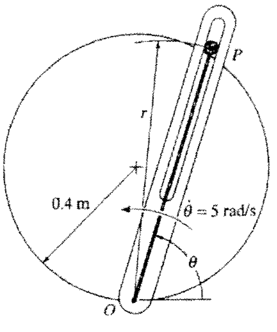
$+\swarrow \Sigma F_r = ma_r; \qquad -N\cos 9.46^\circ = \frac{2}{32.2}(-4.2346) \qquad N = 0.2666$ lb

$+\nwarrow \Sigma F_\theta = ma_\theta; \qquad F - 0.2666\sin 9.46^\circ = \frac{2}{32.2}(1.9187)$

$F = 0.163$ lb **Ans**

46

13-97. The smooth particle has a mass of 80 g. It is attached to an elastic cord extending from O to P and due to the slotted arm guide moves along the *horizontal* circular path $r = (0.8 \sin\theta)$ m. If the cord has a stiffness $k = 30$ N/m and an unstretched length of 0.25 m, determine the force of the guide on the particle when $\theta = 60°$ The guide has a constant angular velocity $\dot{\theta} = 5$ rad/s.



$r = 0.8 \sin\theta$

$r = 0.8 \cos\theta\ \theta$

$r = -0.8 \sin\theta\ (\theta)^2 + 0.8 \cos\theta\ \theta$

$\theta = 5, \quad \theta = 0$

At $\theta = 60°$, $r = 0.6928$

$r = 2$

$\dot{r} = -17.321$

$a_r = r - r(\theta)^2 = -17.321 - 0.6928(5)^2 = -34.641$

$a_\theta = r\theta + 2\,r\theta = 0 + 2(2)(5) = 20$

$F_s = ks; \qquad F_s = 30(0.6928 - 0.25) = 13.284$ N

$\nearrow + \Sigma F_r = m\,a_r; \qquad -13.284 + N_P \cos 30° = 0.08(-34.641)$

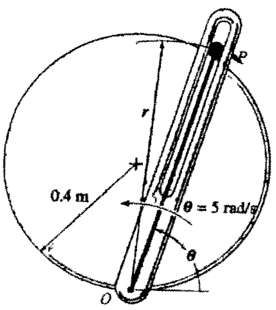
$\nwarrow + \Sigma F_\theta = ma_\theta; \qquad F - N_P \sin 30° = 0.08(20)$

$F = 7.67$ N **Ans**

$N_P = 12.1$ N

47

13-98. Solve Prob. 13-97 if $\theta = 2$ rad/s^2 when $\theta = 5$ rad/s and $\theta = 60°$.



$r = 0.8 \sin\theta$

$r = 0.8 \cos\theta\ \theta$

$r = -0.8 \sin\theta\ (\theta)^2 + 0.8 \cos\theta\ \dot{\theta}$

$\dot{\theta} = 5, \quad \theta = 2$

At $\theta = 60°$, $r = 0.6928$

$r = 2$

$\ddot{r} = -16.521$

$a_r = r - r(\theta)^2 = -16.521 - 0.6928(5)^2 = -33.841$

$a_\theta = r\dot{\theta} + 2\,r\theta = 0.6928(2) + 2(2)(5) = 21.386$

$F_s = ks; \qquad F_s = 30(0.6928 - 0.25) = 13.284$ N

$\nearrow + \Sigma F_r = m\,a_r, \qquad -13.284 + N_P \cos 30° = 0.08(-33.841)$

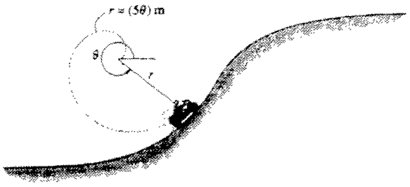
$\nwarrow\ \Sigma F_\theta = ma_\theta; \qquad F - N_P \sin 30° = 0.08(21.386)$

$F = 7.82$ N **Ans**

$N_P = 12.2$ N

48

13-~~99~~. Determine the normal and frictional driving forces that the partial spiral track exerts on the 200-kg motorcycle at the instant $\theta = \frac{5}{3}\pi$ rad, $\dot{\theta} = 0.4$ rad/s, and $\ddot{\theta} = 0.8$ rad/s². Neglect the size of the motorcycle.



$\theta = \left(\frac{5}{3}\pi\right) = 300° \qquad \dot{\theta} = 0.4 \qquad \theta = 0.8$

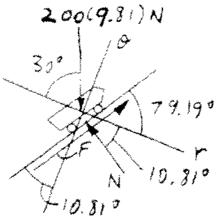
$r = 5\theta = 5\left(\frac{5}{3}\pi\right) = 26.18$

$r = 5\theta = 5(0.4) = 2$

$\dot{r} = 5\dot{\theta} = 5(0.8) = 4$

$a_r = \dot{r} - r\dot{\theta}^2 = 4 - 26.18(0.4)^2 = -0.1888$

$a_\theta = r\theta + 2\dot{r}\dot{\theta} = 26.18(0.8) + 2(2)(0.4) = 22.54$



$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{5\left(\frac{5}{3}\pi\right)}{5} = 5.236 \qquad \psi = 79.19°$

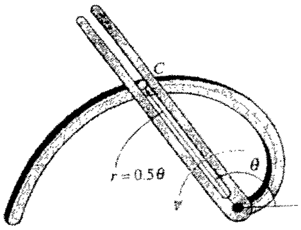
$+\searrow\Sigma F_r = ma_r;\qquad F\sin 10.81° - N\cos 10.81° + 200(9.81)\cos 30° = 200(-0.1888)$

$+\nearrow\Sigma F_\theta = ma_\theta;\qquad F\cos 10.81° - 200(9.81)\sin 30° + N\sin 10.81° = 200(22.54)$

$F = 5.07$ kN **Ans**

$N = 2.74$ kN **Ans**

***13-100.** Using a forked rod, a smooth cylinder C having a mass of 0.5 kg is forced to move along the *vertical slotted* path $r = (0.5\theta)$ m, where θ is in radians. If the angular position of the arm is $\theta = (0.5t^2)$ rad, where t is in seconds, determine the force of the rod on the cylinder and the normal force of the slot on the cylinder at the instant $t = 2$ s. The cylinder is in contact with only *one edge* of the rod and slot at any instant.



$r = 0.5\theta \qquad r = 0.5\dot{\theta} \qquad \dot{r} = 0.5\theta$

$\theta = 0.5t^2 \qquad \theta = t \qquad \dot{\theta} = 1$

At $t = 2$ s,

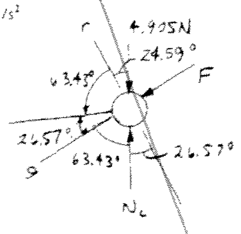
$\theta = 2 \text{ rad} = 114.59° \qquad \theta = 2 \text{ rad/s} \qquad \theta = 1 \text{ rad/s}^2$

$r = 1 \text{ m} \qquad r = 1 \text{ m/s} \qquad \dot{r} = 0.5 \text{ m/s}^2$

$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{0.5(2)}{0.5} \qquad \psi = 63.43°$

$a_r = r - r\dot{\theta}^2 = 0.5 - 1(2)^2 = -3.5$

$a_\theta = r\dot{\theta} + 2r\theta = 1(1) + 2(1)(2) = 5$



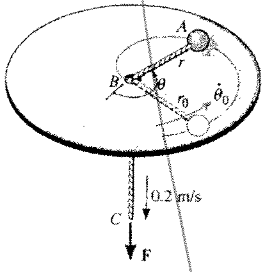
$+\nwarrow\Sigma F_r = ma_r;\qquad N_C\cos 26.57° - 4.905\cos 24.59° = 0.5(-3.5)$

$N_C = 3.030 = 3.03$ N **Ans**

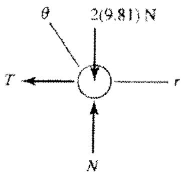
$+\nearrow\Sigma F_\theta = ma_\theta;\qquad F - 3.030\sin 26.57° + 4.905\sin 24.59° = 0.5(5)$

$F = 1.81$ N **Ans**

13-101. The ball has a mass of 2 kg and a negligible size. It is originally traveling around the horizontal circular path of radius $r_0 = 0.5$ m such that the angular rate of rotation is $\dot{\theta}_0 = 1$ rad/s. If the attached cord ABC is drawn down through the hole at a constant speed of 0.2 m/s, determine the tension the cord exerts on the ball at the instant $r = 0.25$ m. Also, compute the angular velocity of the ball at this instant. Neglect the effects of friction between the ball and horizontal plane. *Hint:* First show that the equation of motion in the θ direction yields $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1/r)(d(r^2\dot{\theta})/dt) = 0$. When integrated, $r^2\dot{\theta} = c$, where the constant c is determined from the problem data.



$\sum F_\theta = ma_\theta; \quad 0 = m[r\ddot{\theta} + 2\dot{r}\dot{\theta}] = m\left[\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\right] = 0$



Thus,

$d(r^2\dot{\theta}) = 0$

$r^2\dot{\theta} = c$

$(0.5)^2(1) = c = (0.25)^2\dot{\theta}$

$\theta = 4.00$ rad/s **Ans**

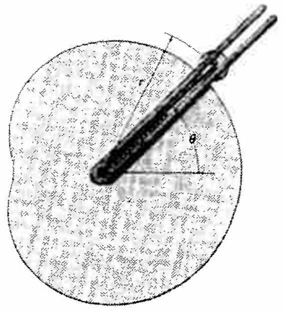
Since $\dot{r} = -0.2$ m/s, $\ddot{r} = 0$

$a_r = \ddot{r} - \dot{r}(\theta)^2 = 0 - 0.25(4.00)^2 = -4$ m/s^2

$\sum F_r = ma_r; \quad -T = 2(-4)$

$T = 8$ N **Ans**

13-102. The smooth surface of the vertical cam is defined in part by the curve $r = (0.2\cos\theta + 0.3)$ m. If the forked rod is rotating with a constant angular velocity of $\dot{\theta} = 4$ rad/s, determine the force the cam and the rod exert on the 2-kg roller when $\theta = 30°$. The attached spring has a stiffness $k = 30$ N/m and an unstretched length of 0.1 m.

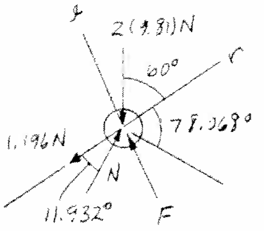


$r = 0.2\cos\theta + 0.3$

$\dot{r} = -0.2\sin\theta\dot{\theta}$

$\ddot{r} = -0.2\cos\theta\dot{\theta}^2 - 0.2\sin\theta\ddot{\theta}$

$\theta = 30°\qquad \dot{\theta} = 4\qquad \ddot{\theta} = 0$



Thus,

$r = 0.47321\qquad \dot{r} = -0.4\qquad \ddot{r} = -2.77128$

$a_r = \ddot{r} - r\dot{\theta}^2 = -2.77128 - 0.47321(4)^2 = -10.3426$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-0.4)(4) = -3.20$

$F_s = kx = 30(0.47321 - 0.1) = 11.196\text{ N}$

$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{0.47321}{-0.2\sin 30°} = -4.7321\qquad \psi = -78.068°$

$\nearrow + \Sigma F_r = ma_r;\qquad N\cos 11.932° - 11.196 - 2(9.81)\cos 60° = 2(-10.3426)$

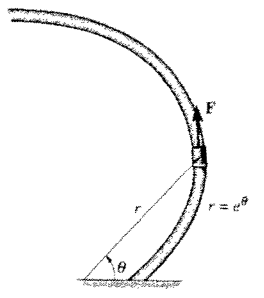
$N = 0.3281\text{ N} = 0.328\text{ N}$ **Ans**

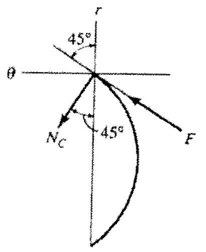
$\nwarrow + \Sigma F_\theta = ma_\theta;\qquad F + 0.3281\sin 11.932° - 2(9.81)\sin 60° = 2(-3.20)$

$F = 10.5\text{ N}$ **Ans**

49

13-103. The collar has a mass of 2 kg and travels along the smooth horizontal rod defined by the equiangular spiral $r = (e^{\theta})$ m, where θ is in radians. Determine the tangential force F and the normal force N acting on the collar when $\theta = 90°$, if the force F maintains a constant angular motion $\dot{\theta} = 2$ rad/s.





$r = e^{\theta}$

$\dot{r} = e^{\theta}\dot{\theta}$

$\ddot{r} = e^{\theta}(\dot{\theta})^2 + e^{\theta}\ddot{\theta}$

At $\theta = 90°$

$\dot{\theta} = 2$ rad/s

$\ddot{\theta} = 0$

$r = 4.8105$

$\dot{r} = 9.6210$

$\ddot{r} = 19.242$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 19.242 - 4.8105(2)^2 = 0$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(9.6210)(2) = 38.4838$ m/s²

$\tan\psi = \dfrac{r}{\left(\frac{dr}{d\theta}\right)} = e^{\theta}/e^{\theta} = 1$

$\psi = 45°$

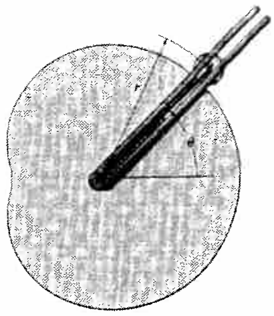
$+\uparrow \sum F_r = ma_r; \quad -N_C\cos 45° + F\cos 45° = 2(0)$

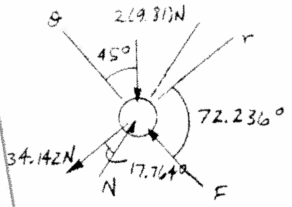
$\overset{+}{\leftarrow} \sum F_\theta = ma_\theta; \quad F\sin 45° + N_C\sin 45° = 2(38.4838)$

$N_C = 54.4$ N **Ans**

$F = 54.4$ N **Ans**

•13-104. The smooth surface of the vertical cam is defined in part by the curve $r = (0.2\cos\theta + 0.3)$ m. The forked rod is rotating with an angular acceleration of $\ddot{\theta} = 2\ \text{rad/s}^2$, and when $\theta = 45°$ the angular velocity is $\dot{\theta} = 6$ rad/s. Determine the force the cam and the rod exert on the 2-kg roller at this instant. The attached spring has a stiffness $k = 100$ N/m and an unstretched length of 0.1 m.





$r = 0.2\cos\theta + 0.3$

$\dot{r} = -0.2\sin\theta\dot{\theta}$

$\ddot{r} = -0.2\cos\theta\dot{\theta}^2 - 0.2\sin\theta\ddot{\theta}$

$\theta = 45°\qquad \dot{\theta} = 6\qquad \ddot{\theta} = 2$

Thus,

$r = 0.44142\qquad \dot{r} = -0.84853\qquad \ddot{r} = -5.37401$

$a_r = \ddot{r} - r\dot{\theta}^2 = -5.37401 - 0.44142(6)^2 = -21.265$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.44142(2) + 2(-0.84853)(6) = -9.2995$

$F_s = kx = 100(0.44142 - 0.1) = 34.142\ \text{N}$

$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{0.44142}{-0.2\sin 45°} = -3.1213\qquad \psi = -72.236°$

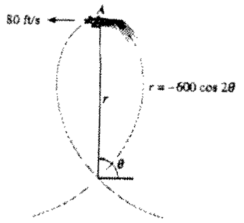
$+\nearrow \Sigma F_r = ma_r;\qquad N\cos 17.764° - 34.142 - 2(9.81)\sin 45° = 2(-21.265)$

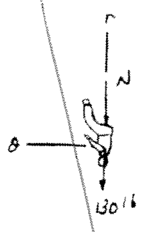
$N = 5.7598\ \text{N} = 5.76\ \text{N}$ **Ans**

$+\nwarrow \Sigma F_\theta = ma_\theta;\qquad 5.7598\sin 17.764° - 2(9.81)\cos 45° + F = 2(-9.2995)$

$F = -6.48\ \text{N} = 6.48\ \text{N}$ **Ans**

13–105. The pilot of an airplane executes a vertical loop which in part follows the path of a "four-leaved rose," $r = (-600\cos 2\theta)$ ft, where θ is in radians. If his speed at A is a constant $v_P = 80$ ft/s, determine the vertical reaction the seat of the plane exerts on the pilot when the plane is at A. He weighs 130 lb. *Hint:* To determine the time derivatives necessary to compute the acceleration components a_r and a_θ, take the first and second time derivatives of $r = 400(1 + \cos\theta)$. Then, for further information, use Eq. 12–26 to determine $\dot{\theta}$. Also, take the time derivative of Eq. 12–26, noting that $\dot{v}_C = 0$, to determine $\ddot{\theta}$.





$r = -600\cos 2\theta \qquad \dot{r} = 1200\sin 2\theta\dot{\theta} \qquad \ddot{r} = 1200\left(2\cos 2\theta\dot{\theta}^2 + \sin 2\theta\ddot{\theta}\right)$

At $\theta = 90°$

$r = -600\cos 180° = 600$ ft $\qquad \dot{r} = 1200\sin 180°\dot{\theta} = 0$

$\ddot{r} = 1200\left(2\cos 180°\dot{\theta}^2 + \sin 180°\ddot{\theta}\right) = -2400\dot{\theta}^2$

$\upsilon_r = \dot{r} = 0 \qquad \upsilon_\theta = r\dot{\theta} = 600\dot{\theta}$

$\upsilon_p^2 = \upsilon_r^2 + \upsilon_\theta^2$

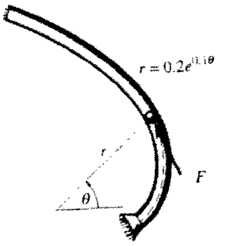
$80^2 = 0^2 + \left(600\dot{\theta}\right)^2 \qquad \dot{\theta} = 0.1333$ rad/s

$\ddot{r} = -2400(0.1333)^2 = -42.67$ ft/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = -42.67 - 600(0.1333)^2 = -53.33$ ft/s^2

$+\uparrow \Sigma F_r = m a_r; \qquad -N - 130 = \frac{130}{32.2}(-53.33) \qquad N = 85.3$ lb **Ans**

13-106. Using air pressure, the 0.5-kg ball is forced to move through the tube lying in the *horizontal plane* and having the shape of a logarithmic spiral. If the tangential force exerted on the ball due to the air is 6 N, determine the rate of increase in the ball's speed at the instant $\theta = \pi/2$. What direction does it act

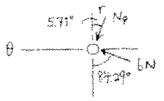


$r = 0.2\, e^{0.1\theta}$

$\dfrac{dr}{d\theta} = 0.02\, e^{0.1\theta}$

$\tan\psi = \dfrac{r}{\frac{dr}{d\theta}} = \dfrac{0.2\, e^{0.1\theta}}{0.02\, e^{0.1\theta}} = 10$

$\psi = 84.29^\circ$

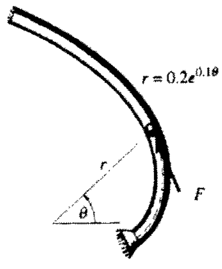


Rate of increase in speed is equivalent to the tangential component of acceleration.

$\Sigma F_t = m\, a_t;$ $6 = 0.5\, a_t$

$a_t = 12 \text{ m/s}^2 \;\underset{5.71^\circ}{\theta\measuredangle}$ **Ans**

13-107. Solve Prob. 13–106 if the tube lies in a *vertical plane.*



$r = 0.2\, e^{0.1\theta}$

$\dfrac{dr}{d\theta} = 0.02\, e^{0.1\theta}$

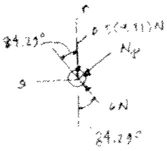
$\tan\psi = \dfrac{r}{\frac{dr}{d\theta}} = \dfrac{0.2\, e^{0.1\theta}}{0.02\, e^{0.1\theta}} = 10$

$\psi = 84.29^\circ$

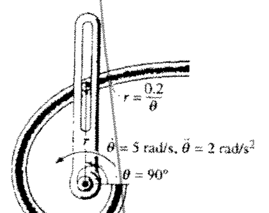
Rate of increase in speed is equivalent to the tangential component of acceleration.

$\Sigma F_t = 0.5\, a_t;$ $6 - 0.5(9.81)\cos 84.29^\circ = 0.5\, a_t$

$a_t = 11.0 \text{ m/s}^2 \;\underset{5.71^\circ}{\theta\measuredangle}$ **Ans**



***13-108.** The arm is rotating at a rate of $\dot{\theta} = 5$ rad/s when $\ddot{\theta} = 2$ rad/s^2 and $\theta = 90°$. Determine the normal force it must exert on the 0.5-kg particle if the particle is confined to move along the slotted path defined by the *horizontal* hyperbolic spiral $r\theta = 0.2$ m.



$$\theta = \frac{\pi}{2} = 90°$$

$$\dot{\theta} = 5 \text{ rad/s}$$

$$\ddot{\theta} = 2 \text{ rad/s}^2$$

$$r = 0.2/\theta = 0.12732 \text{ m}$$

$$\dot{r} = -0.2\,\theta^{-2}\dot{\theta} = -0.40528 \text{ m/s}$$

$$\ddot{r} = -0.2[-2\theta^{-3}(\dot{\theta})^2 + \theta^{-2}\ddot{\theta}] = 2.41801$$

$$a_r = \ddot{r} - r(\dot{\theta})^2 = 2.41801 - 0.12732(5)^2 = -0.7651 \text{ m/s}^2$$

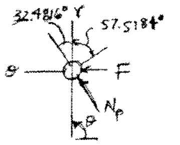
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.12732(2) + 2(-0.40528)(5) = -3.7982 \text{ m/s}^2$$

$$\tan\psi = \frac{r}{\left(\frac{dr}{d\theta}\right)} = \frac{0.2/\theta}{-0.2\theta^{-2}}$$

$$\psi = \tan^{-1}(-\frac{\pi}{2}) = -57.5184°$$

$+\uparrow \Sigma F_r = m\,a_r;\quad N_P\cos 32.4816° = 0.5(-0.7651)$

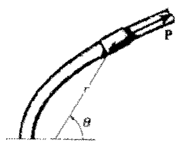
$\overset{+}{\leftarrow} \Sigma F_\theta = m\,a_\theta;\quad F + N_P \sin 32.4816° = 0.5(-3.7982)$



$N_P = -0.453$ N

$F = -1.66$ N **Ans**

13-109. The collar, which has a weight of 3 lb, slides along the smooth rod lying in the *horizontal plane* and having the shape of a parabola $r = 4/(1 - \cos\theta)$, where θ is in radians and r is in feet. If the collar's angular rate is constant and equals $\dot{\theta} = 4$ rad/s, determine the tangential retarding force P needed to cause the motion and the normal force that the collar exerts on the rod at the instant $\theta = 90°$.



$$r = \frac{4}{1-\cos\theta}$$

$$\dot{r} = \frac{-4\sin\theta\ \dot{\theta}}{(1-\cos\theta)^2}$$

$$\ddot{r} = \frac{-4\sin\theta\ \ddot{\theta}}{(1-\cos\theta)^2} + \frac{-4\cos\theta\ (\dot{\theta})^2}{(1-\cos\theta)^2} + \frac{8\sin^2\theta\ \dot{\theta}^2}{(1-\cos\theta)^3}$$

At $\theta = 90°$, $\quad\dot{\theta} = 4, \quad \ddot{\theta} = 0$

$r = 4$
$\dot{r} = -16$
$\ddot{r} = 128$

$$a_r = \ddot{r} - r(\dot{\theta})^2 = 128 - 4(4)^2 = 64$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-16)(4) = -128$$

$$r = \frac{4}{1-\cos\theta}$$

$$\frac{dr}{d\theta} = \frac{-4\sin\theta}{(1-\cos\theta)^2}$$

$$\tan\psi = \frac{r}{\left(\frac{dr}{d\theta}\right)} = \left.\frac{\frac{4}{1-\cos\theta}}{\frac{-4\sin\theta}{(1-\cos\theta)^2}}\right|_{\theta=90°} = \frac{4}{-4} = -1$$

$$\psi = -45° = 135°$$

$+\uparrow \Sigma F_r = m\,a_r,\quad P\sin 45° - N\cos 45° = \frac{3}{32.2}(64)$

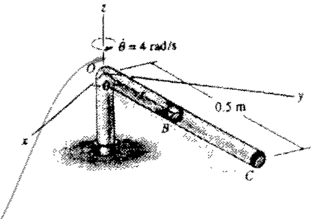
$\overset{+}{\leftarrow} \Sigma F_\theta = m a_\theta,\quad -P\cos 45° - N\sin 45° = \frac{3}{32.2}(-128)$

Solving,

$P = 12.6$ lb **Ans**

$N = 4.22$ lb **Ans**

13-110. The tube rotates in the horizontal plane at a constant rate of $\dot{\theta} = 4$ rad/s. If a 0.2-kg ball B starts at the origin O with an initial radial velocity of $\dot{r} = 1.5$ m/s and moves outward through the tube, determine the radial and transverse components of the ball's velocity at the instant it leaves the outer end at C, $r = 0.5$ m. *Hint:* Show that the equation of motion in the r direction is $\ddot{r} - 16r = 0$. The solution is of the form $r = Ae^{-4t} + Be^{4t}$. Evaluate the integration constants A and B, and determine the time t when $r = 0.5$ m. Proceed to obtain v_r and v_θ.

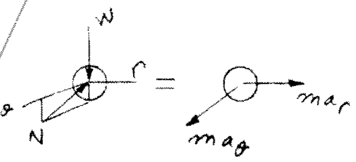


$\dot{\theta} = 4 \quad \ddot{\theta} = 0$

$\Sigma F_r = ma_r; \quad 0 = 0.2\left[\ddot{r} - r(4)^2\right]$

$\ddot{r} - 16r = 0$

Solving this second - order differential equation,



$r = Ae^{-4t} + Be^{4t} \qquad (1)$

$\dot{r} = -4Ae^{-4t} + 4Be^{4t} \qquad (2)$

At $t = 0, \; r = 0, \; \dot{r} = 1.5$

$0 = A + B \qquad \dfrac{1.5}{4} = -A + B$

$A = -0.1875 \qquad B = 0.1875$

From Eq. (1) at $r = 0.5$ m,

$0.5 = 0.1875\left(-e^{-4t} + e^{4t}\right)$

$\dfrac{2.667}{2} = \dfrac{(-e^{-4t} + e^{4t})}{2}$

$1.333 = \sinh(4t)$

$t = \dfrac{1}{4}\sinh^{-1}(1.333) \qquad t = 0.275$ s

Using Eq. (2),

$\dot{r} = 4(0.1875)\left(e^{-4t} + e^{4t}\right)$

$\dot{r} = 8(0.1875)\left(\dfrac{e^{-4t} + e^{4t}}{2}\right) = 8(0.1875)(\cosh(4t))$

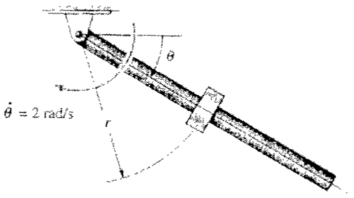
At $t = 0.275$ s

$\dot{r} = 1.5\cosh[4(0.275)]$

$v_r = \dot{r} = 2.50$ m/s **Ans**

$v_\theta = r\dot{\theta} = 0.5(4) = 2$ m/s **Ans**

13-111. A 0.2-kg spool slides down along a smooth rod. If the rod has a constant angular rate of rotation $\dot{\theta} = 2$ rad/s in the vertical plane, show that the equations of motion for the spool are $\ddot{r} - 4r - 9.81 \sin\theta = 0$ and $0.8\dot{r} + N_s - 1.962\cos\theta = 0$, where N_s is the magnitude of the normal force of the rod on the spool. Using the methods of differential equations, it can be shown that the solution of the first of these equations is $r = C_1 e^{-2t} + C_2 e^{2t} - (9.81/8)\sin 2t$. If r, $\dot{r}$, and θ are zero when $t = 0$, evaluate the constants C_1 and C_2 determine r at the instant $\theta = \pi/4$ rad.



Kinematic : Here, $\dot{\theta} = 2$ rad/s and $\ddot{\theta} = 0$. Applying Eqs. 12 – 29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = \ddot{r} - r\left(2^2\right) = \ddot{r} - 4r$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = r(0) + 2\dot{r}(2) = 4\dot{r}$$

Equation of Motion : Applying Eq. 13 – 9, we have

$$\Sigma F_r = ma_r; \qquad 1.962\sin\theta = 0.2(\ddot{r} - 4r)$$

$$\ddot{r} - 4r - 9.81\sin\theta = 0 \qquad (\boldsymbol{Q.E.D.}) \qquad [1]$$

$$\Sigma F_\theta = ma_\theta, \qquad 1.962\cos\theta - N_s = 0.2(4\dot{r})$$

$$0.8\dot{r} + N_s - 1.962\cos\theta = 0 \qquad (\boldsymbol{Q.E.D.}) \qquad [2]$$

Since $\dot{\theta} = 2$ rad/s, then, $\int_0^\theta \dot{\theta} = \int_0^t 2dt$, $\theta = 2t$. The solution of the differential equation (Eq.[1]) is given by

$$r = C_1 e^{-2t} + C_2 e^{2t} - \frac{9.81}{8}\sin 2t \qquad [3]$$

Thus,

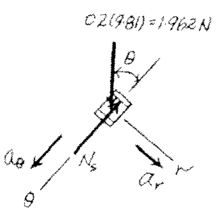
$$\dot{r} = -2C_1 e^{-2t} + 2C_2 e^{2t} - \frac{9.81}{4}\cos 2t \qquad [4]$$

At $t = 0$, $r = 0$. From Eq.[3] $\qquad 0 = C_1(1) + C_2(1) - 0 \qquad [5]$

At $t = 0$, $r = 0$. From Eq.[4] $\qquad 0 = -2C_1(1) + 2C_2(1) - \frac{9.81}{4} \qquad [6]$

Solving Eqs.[5] and [6] yields

$$C_1 = -\frac{9.81}{16} \qquad C_2 = \frac{9.81}{16}$$



Thus,

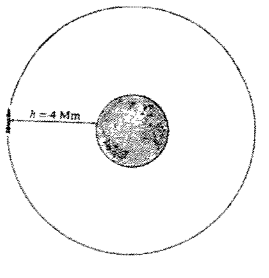
$$r = -\frac{9.81}{16}e^{-2t} + \frac{9.81}{16}e^{2t} - \frac{9.81}{8}\sin 2t$$

$$= \frac{9.81}{8}\left(\frac{-e^{-2t} + e^{2t}}{2} - \sin 2t\right)$$

$$= \frac{9.81}{8}(\sinh 2t - \sin 2t)$$

At $\theta = 2t = \frac{\pi}{4}$, $\qquad r = \frac{9.81}{8}\left(\sinh\frac{\pi}{4} - \sin\frac{\pi}{4}\right) = 0.198$ m $\qquad$ **Ans**

***13-112.** The rocket is in circular orbit about the earth at an altitude of $h = 4$ Mm. Determine the minimum increment in speed it must have in order to escape the earth's gravitational field.



Circular orbit :

$$v_C = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})5.976(10^{24})}{4000(10^3) + 6378(10^3)}} = 6198.8 \text{ m/s}$$

Parabolic orbit :

$$v_e = \sqrt{\frac{2GM_e}{r_0}} = \sqrt{\frac{2(66.73)(10^{-12})5.976(10^{24})}{4000(10^3) + 6378(10^3)}} = 8766.4 \text{ m/s}$$

$\Delta v = v_e - v_C = 8766.4 - 6198.8 = 2567.6$ m/s

$\Delta v = 2.57$ km/s $\qquad$ **Ans**

13-113. Prove Kepler's third law of motion. *Hint:* Use Eqs. 13–19, 13–28, 13–29, and 13–31.

From Eq. 13 – 19,

$$\frac{1}{r} = C\cos\theta + \frac{GM_s}{h^2}$$

For $\theta = 0°$ and $\theta = 180°$

$$\frac{1}{r_p} = C + \frac{GM_s}{h^2}$$

$$\frac{1}{r_a} = -C + \frac{GM_s}{h^2}$$

Eliminating C,

From Eqs. 13 – 28 and 13 – 29,

$$\frac{2a}{b^2} = \frac{2GM_s}{h^2}$$

From Eq. 13 – 31,

$$T = \frac{\pi}{h}(2a)(b)$$

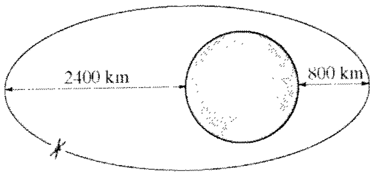
Thus,

$$b^2 = \frac{T^2h^2}{4\pi^2a^2}$$

$$\frac{4\pi^2a^3}{T^2h^2} = \frac{GM_s}{h^2}$$

$$T^2 = (\frac{4\pi^2}{GM_s})a^3 \qquad \textbf{Q.E.D.}$$

13-114. A satellite is to be placed into an elliptical orbit about the earth such that at the perigee of its orbit it has an *altitude* of 800 km, and at apogee its *altitude* is 2400 km. Determine its required launch velocity tangent to the earth's surface at perigee and the period of its orbit.



$r_p = 800 + 6378 = 7178$ km $\qquad r_a = 2400 + 6378 = 8778$ km

$$r_a = \frac{r_p}{\frac{2GM_e}{r_p v_0^2} - 1}$$

$$v_0 = \sqrt{\frac{2GM_e r_a}{r_p(r_a + r_p)}}$$

$$= \sqrt{\frac{2(66.73)(10^{-12})(5.976)(10^{24})(8778)(10^3)}{7178(10^3)\left[8778(10^3) + 7178(10^3)\right]}}$$

$= 7818$ m/s $= 7.82$ km/s **Ans**

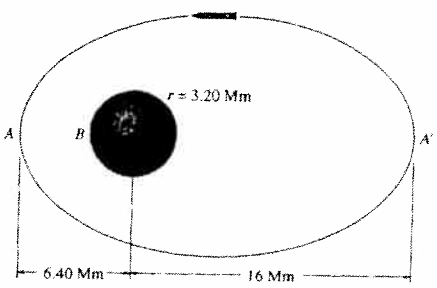
$h = r_p v_0 = 7178(10^3)(7818) = 56.12(10^9)$ m^2/s

$$T = \frac{\pi}{h}(r_p + r_a)\sqrt{r_p r_a}$$

$$= \frac{\pi}{56.12(10^9)}\left[7178(10^3) + 8778(10^3)\right]\sqrt{\left[7178(10^3)\right]\left[8778(10^3)\right]}$$

$= 7090$ s $= 1.97$ h **Ans**

13-115. The rocket is traveling in free flight along an elliptical trajectory $A'A$. The planet has a mass 0.60 times that of the earth's. If the rocket has an apoapsis and periapsis as shown in the figure, determine the speed of the rocket when it is at point A.

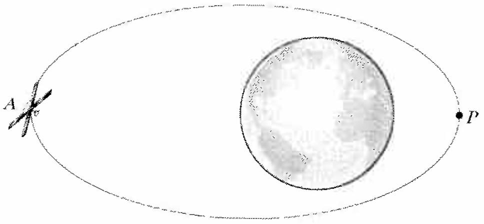


Central - Force Motion : Substitute Eq. 13 − 27, $r_a = \dfrac{r_0}{\left(2GM/r_0 v_0^2\right) - 1}$, with $r_a = 16\left(10^6\right)$ m , $r_0 = r_p = 6.40\left(10^6\right)$ m and $M = 0.60M_e$, we have

$$16\left(10^6\right) = \frac{6.40(10^6)}{\left(\dfrac{2(66.73)(10^{-12})(0.6)[5.976(10^{24})]}{6.40(10^6)\,v_p^2}\right) - 1}$$

$v_p = 7308.07\text{m/s} = 7.31$ km/s **Ans**

***13–116.** An elliptical path of a satellite has an eccentricity $e = 0.130$. If it has a speed of 15 Mm/h when it is at perigee, P, determine its speed when it arrives at apogee, A. Also, how far is it from the earth's surface when it is at A?



$e = 0.130$

$v_p = v_0 = 15$ Mm/h $= 4.167$ km/s

$$e = \frac{Ch^2}{GM_e} = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\left(\frac{r_0^2 v_0^2}{GM_e}\right)$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1$$

$$r_0 = \frac{(e+1)GM_e}{v_0^2}$$

$$= \frac{1.130(66.73)(10^{-12})(5.976)(10^{24})}{\left[4.167(10^3)\right]^2}$$

$= 25.96$ Mm

$$\frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1}$$

$$r_A = \frac{r_0}{\frac{2GM_e}{r_0 v_0^2} - 1} = \frac{r_0}{\left(\frac{2}{e+1}\right) - 1}$$

$$r_A = \frac{r_0\,(e+1)}{1-e}$$

$$= \frac{25.96(10^6)(1.130)}{0.870}$$

$= 33.71\left(10^6\right)\text{m} = 33.7$ Mm

$$v_A = \frac{v_0 r_0}{r_A}$$

$$= \frac{15(25.96)(10^6)}{33.71(10^6)}$$

$= 11.5$ Mm/h **Ans**

$d = 33.71\left(10^6\right) - 6.378\left(10^6\right)$

$= 27.3$ Mm **Ans**

13-117. A communications satellite is to be placed into an equatorial circular orbit around the earth so that it always remains directly over a point on the earth's surface. If this requires the period to be 24 hours (approximately), determine the radius of the orbit and the satellite's velocity.

$$\frac{GM_sM_e}{r^2} = \frac{M_sv^2}{r}$$

$$\frac{GM_e}{r} = v^2$$

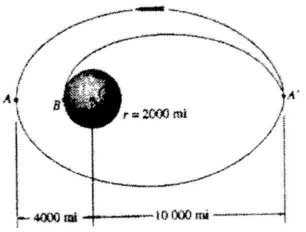
$$\frac{GM_e}{r} = \left[\frac{2\pi r}{24(3600)}\right]^2$$

$$\frac{66.73(10^{-12})(5.976)(10^{24})}{\left[\frac{2\pi}{24(3600)}\right]^2} = r^3$$

$$r = 42.25(10^6)\text{ m} = 42.2\text{ Mm} \qquad \textbf{Ans}$$

$$v = \frac{2\pi(42.25)(10^6)}{24(3600)} = 3.07\text{ km/s} \qquad \textbf{Ans}$$

13-118. The rocket is traveling in free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.60 times that of the earth's. If the rocket has the apogee and perigee shown, determine the rocket's velocity when it is at point A. Take $G = 34.4(10^{-9})$ (lb·ft²)/slug², $M_e = 409(10^{21})$ slug, 1 mi = 5280 ft.



$$r_0 = OA = (4000)(5280) = 21.12\left(10^6\right)\text{ ft} \qquad OA' = (10\,000)(5280) = 52.80\left(10^6\right)\text{ ft}$$

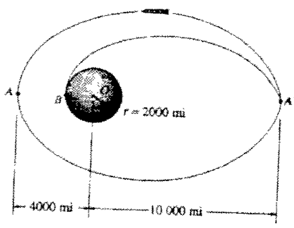
$$M_P = \left(409\left(10^{21}\right)\right)(0.6) = 245.4\left(10^{21}\right)\text{ slug}$$

$$OA' = \frac{OA}{\left(\frac{2GM_P}{OAv_0^2} - 1\right)}$$

$$v_0 = \sqrt{\frac{2GM_P}{OA\left(\frac{OA}{OA'} + 1\right)}} = \sqrt{\frac{2(34.4)(10^{-9})(245.4)(10^{21})}{21.12(10^6)\left(\frac{21.12}{52.80} + 1\right)}}$$

$$v_0 = 23.9\left(10^3\right)\text{ ft/s} \qquad \textbf{Ans}$$

13–119. If the rocket is to land on the surface of the planet, determine the required free-flight speed it must have at A' so that the landing occurs at B. How long does it take for the rocket to land, in going from A' to B? The planet has no atmosphere, and its mass is 0.6 times that of the earth's. Take $G = 34.4(10^{-9})(\text{lb}\cdot\text{ft}^2)/\text{slug}^2$, $M_e = 409(10^{21})$ slug, 1 mi = 5280 ft.



$$M_P = 409(10^{21})(0.6) = 245.4(10^{21})\text{ slug}$$

$$OA' = (10\,000)(5280) = 52.80(10^6)\text{ ft} \qquad OB = (2000)(5280) = 10.56(10^6)\text{ ft}$$

$$OA' = \frac{OB}{\left(\frac{2GM_P}{OBv_0^2} - 1\right)}$$

$$v_0 = \sqrt{\frac{2GM_P}{OB\left(\frac{OB}{OA'}+1\right)}} = \sqrt{\frac{2(34.4(10^{-9}))245.4(10^{21})}{10.56(10^6)\left(\frac{10.56}{52.80}+1\right)}}$$

$$v_0 = 36.50(10^3)\text{ ft/s} \qquad \text{(speed at } B\text{)}$$

$$v_{A'} = \frac{OBv_0}{OA'}$$

$$v_{A'} = \frac{10.56(10^6)36.50(10^3)}{52.80(10)^6}$$

$$v_{A'} = 7.30(10^3)\text{ ft/s} \qquad \textbf{Ans}$$

$$T = \frac{\pi}{h}(OB + OA')\sqrt{(OB)(OA')}$$

$$h = (OB)(v_0) = 10.56(10^6)36.50(10^3) = 385.5(10^9)$$

Thus,

$$T = \frac{\pi(10.56+52.80)(10^6)}{385.5(10^9)}\left(\sqrt{(10.56)(52.80)}\right)(10^6)$$

$$T = 12.20(10^3)\text{ s}$$

$$t = \frac{T}{2} = 6.10(10^3)\text{ s} = 1.69\text{ h} \qquad \textbf{Ans}$$

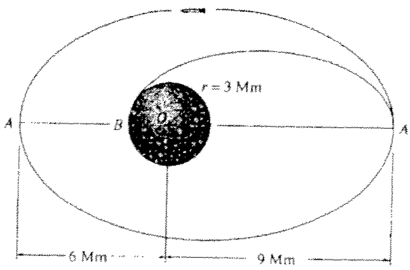
***13–120.** The speed of a satellite launched into a circular orbit about the earth is given by Eq. 13–25. Determine the speed of a satellite launched parallel to the surface of the earth so that it travels in a circular orbit 800 km from the earth's surface.

For a 800 - km orbit

$$v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{(800+6378)(10^3)}}$$

$$= 7453.6\text{ m/s} = 7.45\text{ km/s} \qquad \textbf{Ans}$$

13-121. The rocket is traveling in free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.70 times that of the earth's. If the rocket has an apogee and perigee as shown in the figure, determine the speed of the rocket when it is at point A.



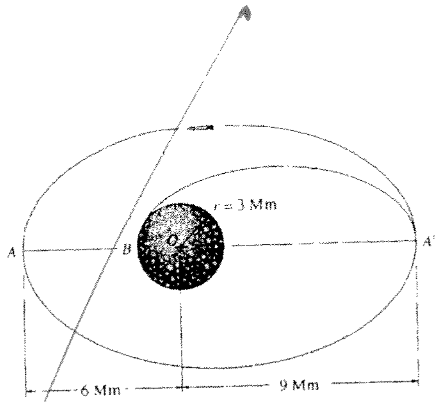
Central - Force Motion : Use $r_a = \dfrac{r_0}{(2GM/r_0 v_0^2) - 1}$,

with $r_a = 9(10^6)$ m, $r_0 = r_p = 6(10^6)$ m and $M = 0.70M_e$, we have

$$9(10^6) = \frac{6(10^6)}{\left(\frac{2(66.73)(10^{-12})(0.7)[5.976(10^{24})]}{6(10^6)\,v_p^2}\right) - 1}$$

$$v_p = 7471\ 89\text{ m/s} = 7.47\text{ km/s} \qquad \textbf{Ans}$$

13-122. If the rocket in Prob. 13–121 is to land on the surface of the planet, determine the required free-flight speed it must have at A' so that it strikes the planet at B. How long does it take for the rocket to land, going from A' to B along an elliptical path?



Central - Force Motion : Use $r_a = \dfrac{r_0}{\left(2GM/r_0 v_0^2\right) - 1}$,

with $r_a = 9\left(10^6\right)$ m , $r_0 = r_p = 3\left(10^6\right)$ m and $M = 0.70M_e$, we have

$$9\left(10^6\right) = \frac{3(10^6)}{\left(\dfrac{2(66.73)(10^{-12})(0.7)[5.976(10^{24})]}{3(10^6)\,v_p^2}\right) - 1}$$

$$v_p = 11814.08 \text{ m/s}$$

Applying Eq. 13 − 20, we have

$$v_a = \left(\frac{r_p}{r_a}\right)v_p = \left[\frac{3(10^6)}{9(10^6)}\right](11814.08) = 3938.03 \text{ m/s} = 3.94 \text{ km/s} \qquad \textbf{Ans}$$

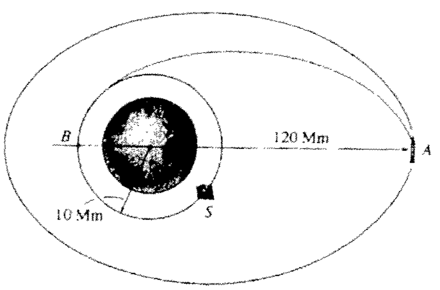
Eq. 13 − 20 gives $h = r_p v_p = 3\left(10^6\right)(11814.08) = 35.442\left(10^9\right)$ m²/s. Thus, applying Eq. 13 − 31, we have

$$\begin{aligned} T &= \frac{\pi}{h}\left(r_p + r_a\right)\sqrt{r_p r_a} \\ &= \frac{\pi}{35.442(10^9)}\left[(9+3)\left(10^6\right)\right]\sqrt{3(10^6)9(10^6)} \\ &= 5527.03 \text{ s} \end{aligned}$$

The time required for the rocket to go from A' to B (half the orbit) is given by

$$t = \frac{T}{2} = 2763.51 \text{ s} = 46.1 \text{ min} \qquad \textbf{Ans}$$

13-123. A satellite S travels in a circular orbit around the earth. A rocket is located at the apogee of its elliptical orbit for which $e = 0.58$. Determine the sudden change in speed that must occur at A so that the rocket can enter the satellite's orbit while in free flight along the dashed elliptical trajectory. When it arrives at B, determine the sudden adjustment in speed that must be given to the rocket in order to maintain the circular orbit.



Central - Force Motion : Here, $C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)$ [Eq. 13 − 21] and $h = r_0 v_0$ [Eq. 13 − 20]. Substitute these values into Eq. 13 − 17 gives

$$e = \frac{ch^2}{GM_e} = \frac{\frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\left(r_0^2 v_0^2\right)}{GM_e} = \frac{r_0 v_0^2}{GM_e} - 1 \qquad [1]$$

Rearrange Eq. [1] gives

$$\frac{1}{1+e} = \frac{GM_e}{r_0 v_0^2} \qquad [2]$$

Rearrange Eq. [2] , we have

$$v_0 = \sqrt{\frac{(1+e)\,GM_e}{r_0}} \qquad [3]$$

Substitute Eq. [2] into Eq. 13 − 27, $r_a = \frac{r_0}{\left(2GM_e / r_0 v_0^2\right) - 1}$, we have

$$r_a = \frac{r_0}{2\left(\frac{1}{1+e}\right) - 1} \quad \text{or} \quad r_0 = \left(\frac{1-e}{1+e}\right) r_a \qquad [4]$$

For the first elliptical orbit $e = 0.58$, from Eq. [4]

$$\left(r_p\right)_1 = r_0 = \left(\frac{1-0.58}{1+0.58}\right)\left[120\left(10^6\right)\right] = 31.899\left(10^6\right)\ \text{m}$$

Substitute $r_0 = \left(r_p\right)_1 = 31.899\left(10^6\right)$ m into Eq. [3] yields

$$\left(v_p\right)_1 = \sqrt{\frac{(1+0.58)(66.73)(10^{-12})(5.976)(10^{24})}{31.899(10^6)}} = 4444.34\ \text{m/s}$$

Applying Eq. 13 − 20, we have

$$(v_a)_1 = \left(\frac{r_p}{r_a}\right)\left(v_p\right)_1 = \left[\frac{31.899(10^6)}{120(10^6)}\right](4444.34) = 1181.41\ \text{m/s}$$

When the rocket travels along the second elliptical orbit, from Eq. [4], we have

$$10\left(10^6\right) = \left(\frac{1-e}{1+e}\right)\left[120\left(10^6\right)\right] \qquad e = 0.8462$$

Substitute $r_0 = \left(r_p\right)_2 = 10\left(10^6\right)$ m into Eq. [3] yields

$$\left(v_p\right)_2 = \sqrt{\frac{(1+0.8462)(66.73)(10^{-12})(5.976)(10^{24})}{10(10^6)}} = 8580.25\ \text{m/s}$$

Applying Eq. 13 − 20, we have

$$(v_a)_2 = \left[\frac{\left(r_p\right)_2}{(r_a)_2}\right]\left(v_p\right)_2 = \left[\frac{10(10^6)}{120(10^6)}\right](8580.25) = 715.02\ \text{m/s}$$

For the rocket to enter into orbit two from orbit one at A, its speed must be decreased by

$$\Delta v = (v_e)_1 - (v_a)_2 = 1184.41 - 715.02 = 466\ \text{m/s} \qquad \textbf{Ans}$$

If the rocket travels in a cicular free - flight trajectory, its speed is given by Eq. 13 − 25.

$$v_c = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{10(10^6)}} = 6314.89\ \text{m/s}$$

The speed for which the rocket must be decreased in order to have a circular orbit is

$$\Delta v = \left(v_p\right)_2 - v_c = 8580.25 - 6314.89 = 2265.36\ \text{m/s} = 2.27\ \text{km/s} \qquad \textbf{Ans}$$

***13-124.** An asteroid is in an elliptical orbit about the sun such that its periapsis is $9.30(10^9)$ km. If the eccentricity of the orbit is $e = 0.073$, determine the apoapsis of the orbit.

$$r_p = r_0 = 9.30(10^9)\ \text{km}$$

$$e = \frac{ch^2}{GM_s} = \frac{1}{r_0}(1 - \frac{GM_s}{r_0 v_0^2})(\frac{r_0 v_0^2}{G M_s})$$

$$e = (\frac{r_0 v_0^2}{G M_s} - 1)$$

$$\frac{r_0 v_0^2}{G M_s} = e + 1 \qquad (1)$$

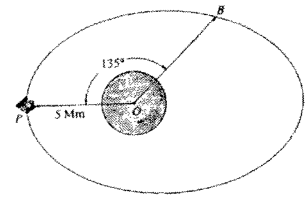
$$\frac{GM_s}{r_0 v_0^2} = (\frac{1}{e+1})$$

$$r_a = \frac{r_0}{\frac{2GM_s}{r_0 v_0^2} - 1} = \frac{r_0}{(\frac{2}{e+1}) - 1} \qquad (2)$$

$$r_a = \frac{r_0(e+1)}{(1-e)} = \frac{9.30(10^9)(1.073)}{0.927}$$

$r_a = 10.8(10^9)$ km **Ans**

13-125. A satellite is in an elliptical orbit around the earth such that $e = 0.156$. If its perigee is 5 Mm, determine its velocity at this point and also the distance OB when it is at point B, located 135° away as shown.



$$e = \frac{Ch^2}{GM_e} = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\left(\frac{r_0^2 v_0^2}{GM_e}\right)$$

$$= \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1$$

$$\frac{5(10^6) v_0^2}{66.73(10^{-12})(5.976)(10^{24})} = 1.156$$

$v_0 = 9602$ m/s $= 9.60$ km/s **Ans**

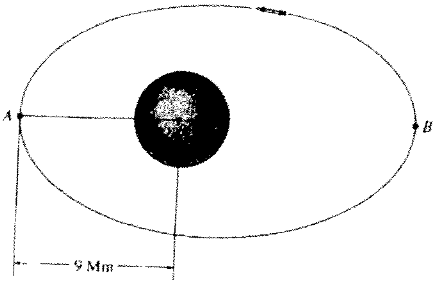
$$\frac{1}{r} = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\cos\theta + \frac{GM_e}{r_0^2 v_0^2}$$

$$= \frac{1}{r_0}\left(1 - \frac{1}{e+1}\right)\cos\theta + \frac{1}{r_0}\left(\frac{1}{e+1}\right)$$

$$= \frac{1}{5(10^6)}\left(1 - \frac{1}{1.156}\right)\cos 135^\circ + \frac{1}{5(10^6)}\left(\frac{1}{1.156}\right)$$

$r = 6.50$ Mm **Ans**

13-126. The rocket is traveling in a free-flight elliptical orbit about the earth such that $e = 0.76$ and its perigee is 9 Mm as shown. Determine its speed when it is at point B. Also determine the sudden decrease in speed the rocket must experience at A in order to travel in a circular orbit about the earth.



Central - Force Motion : Here, $C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)$ [Eq. 13 − 21] and $h = r_0 v_0$ [Eq. 13 − 20] Substitute these values into Eq. 13 − 17 gives

$$e = \frac{ch^2}{GM_e} = \frac{\frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\left(r_0^2 v_0^2\right)}{GM_e} = \frac{r_0 v_0^2}{GM_e} - 1 \qquad [1]$$

Rearrange Eq. [1] gives

$$\frac{1}{1+e} = \frac{GM_e}{r_0 v_0^2} \qquad [2]$$

Rearrange Eq. [2] , we have

$$v_0 = \sqrt{\frac{(1+e)GM_e}{r_0}} \qquad [3]$$

Substitute Eq. [2] into Eq. 13 − 27, $r_a = \frac{r_0}{\left(2GM_e/r_0 v_0^2\right) - 1}$, we have

$$r_a = \frac{r_0}{2\left(\frac{1}{1+e}\right) - 1} \qquad [4]$$

Rearrange Eq. [4] , we have

$$r_a = \left(\frac{1+e}{1-e}\right) r_0 = \left(\frac{1+0.76}{1-0.76}\right)\left[9\left(10^6\right)\right] = 66.0\left(10^6\right) \text{ m}$$

Substitute $r_0 = r_p = 9\left(10^6\right)$ m into Eq. [3] yields

$$v_p = \sqrt{\frac{(1+0.76)(66.73)(10^{-12})(5.976)(10^{24})}{9(10^6)}} = 8830.82 \text{ m/s}$$

Applying Eq. 13 − 20, we have

$$v_a = \left(\frac{r_p}{r_a}\right) v_p = \left[\frac{9(10^6)}{66.0(10^6)}\right](8830.82) = 1204.2 \text{ m/s} = 1.20 \text{ km/s} \qquad \textbf{Ans}$$

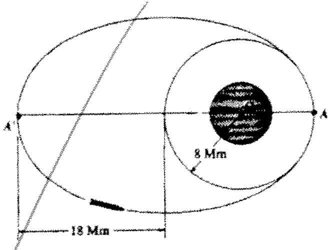
If the rocket travels in a cicular free - flight trajectory, its speed is given by Eq. 13 − 25.

$$v_c = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{9(10^6)}} = 6656.48 \text{ m/s}$$

The speed for which the rocket must be decreased in order to have a circular orbit is

$$\Delta v = v_p - v_c = 8830.82 - 6656\,48 = 2174.34 \text{ m/s} = 2.17 \text{ km/s} \qquad \textbf{Ans}$$

13-127. A rocket is in free-flight elliptical orbit around the planet Venus. Knowing that the periapsis and apoapsis of the orbit are 8 Mm and 26 Mm, respectively, determine (a) the speed of the rocket at point A', (b) the required speed it must attain at A just after braking so that it undergoes an 8-Mm free-flight circular orbit around Venus, and (c) the periods of both the circular and elliptical orbits. The mass of Venus is 0.816 times the mass of the earth.



a)

$$M_v = 0.816\left(5.976\left(10^{24}\right)\right) = 4.876\left(10^{24}\right)$$

$$OA' = \frac{OA}{\left(\frac{2GM_v}{OA\,v_A^2} - 1\right)}$$

$$26(10)^6 = \frac{8(10^6)}{\left(\frac{2(66.73)(10^{-12})4.876(10^{24})}{8(10^6)v_A^2} - 1\right)}$$

$$\frac{81.35(10^6)}{v_A^2} = 1.307$$

$v_A = 7887.3\text{ m/s} = 7.89\text{ km/s}$

$v_A' = \frac{OA\,v_A}{OA'} = \frac{8(10^6)(7887.3)}{26(10^6)} = 2426.9\text{ m/s} = 2.43\text{ km/s}$ **Ans**

b)

$$v_A'' = \sqrt{\frac{GM_v}{OA}} = \sqrt{\frac{66.73(10^{-12})4.876(10^{24})}{8(10^6)}}$$

$v_A'' = 6377.7\text{ m/s} = 6.38\text{ km/s}$ **Ans**

c)

Circular orbit :

$T_c = \frac{2\pi\,OA}{v_A''} = \frac{2\pi\,8(10^6)}{6377.7} = 7881.41\text{ s} = 2.19\text{ h}$ **Ans**

Elliptic orbit :

$$T_e = \frac{\pi}{OA\,v_A}(OA + OA')\sqrt{(OA)(OA')} = \frac{\pi}{8(10^6)(7886.8)}(8+26)\left(10^6\right)\left(\sqrt{(8)(26)}\right)\left(10^6\right)$$

$T_e = 24\,414.2\text{ s} = 6.78\text{ h}$ **Ans**