12–1. A truck, traveling along a straight road at 20 km/h, increases its speed to 120 km/h in 15 s. If its acceleration is constant, determine the distance traveled.

$$v^2 = v_0^2 + 2a(s - s_0)$$

$$\left[\frac{(120\times10^3)}{3600}\right]^2 = \left(\frac{(20\times10^3)}{3600}\right)^2 + 2\left(\frac{(100\times10^3)}{3600\cdot 15}\right)s$$

$s = 292$ m **Ans**

12-2. A car starts from rest and reaches a speed of 80 ft/s after traveling 500 ft along a straight road. Determine its constant acceleration and the time of travel.

$$v_2^2 = v_1^2 + 2\,a_c(s_2 - s_1)$$

$$(80)^2 = 0 + 2\,a_c(500 - 0)$$

$a_c = 6.40\ \text{ft/s}^2$ **Ans**

$$v_2 = v_1 + a_c\,t$$

$$80 = 0 + 6.4(t)$$

$t = 12.5$ s **Ans**

12-3. A baseball is thrown downward from a 50-ft tower with an initial speed of 18 ft/s. Determine the speed at which it hits the ground and the time of travel.

$$v_2^2 = v_1^2 + 2\,a_c(s_2 - s_1)$$

$$v_2^2 = (18)^2 + 2\,(32.2)(50 - 0)$$

$v_2 = 59.532 = 59.5$ ft/s **Ans**

$$v_2 = v_1 + a_c\,t$$

$$59.532 = 18 + 32.2\,(t)$$

$t = 1.29$ s **Ans**

***12-4.** Starting from rest, a particle moving in a straight line has an acceleration of $a = (2t - 6)$ m/s², where t is in seconds. What is the particle's velocity when $t = 6$ s, and what is its position when $t = 11$ s?

$a = 2t - 6$

$dv = a\,dt$

$\int_0^v dv = \int_0^t (2t - 6)\,dt$

$v = t^2 - 6t$

$ds = v\,dt$

$\int_0^s ds = \int_0^t (t^2 - 6t)\,dt$

$s = \dfrac{t^3}{3} - 3t^2$

When $t = 6$ s,

$v = 0$ **Ans**

When $t = 11$ s,

$s = 80.7$ m **Ans**

12-5. Traveling with an initial speed of 70 km/h, a car accelerates at 6000 km/h^2 along a straight road. How long will it take to reach a speed of 120 km/h? Also, through what distance does the car travel during this time?

$$v = v_1 + a_c t$$

$$120 = 70 + 6000(t)$$

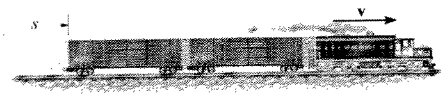
$$t = 8.33(10^{-3}) \text{ hr} = 30 \text{ s} \qquad \textbf{Ans}$$

$$v^2 = v_1^2 + 2a_c(s - s_1)$$

$$(120)^2 = 70^2 + 2(6000)(s - 0)$$

$$s = 0.792 \text{ km} = 792 \text{ m} \qquad \textbf{Ans}$$

12–6. A freight train travels at $v = \sout{60}\,20(1 - e^{-t})$ ~~ft/s~~ m/s, where t is the elapsed time in seconds. Determine the distance traveled in three seconds, and the acceleration at this time.



$$v = \sout{60}\,20(1 - e^{-t})$$

$$\int_0^s ds = \int v\,dt = \int_0^3 \sout{60}\,20(1 - e^{-t})dt$$

$$s = 60(t + e^{-t})|_0^3$$

s = ~~123 ft~~ 41 m **Ans**

$$a = \frac{dv}{dt} = \sout{60}\,20(e^{-t})$$

At $t = 3$ s

$a = \sout{60}\,20\, e^{-3}$ = ~~2.99 ft/s^2~~ 1.00 m/s^2 **Ans**

12-7. The position of a particle along a straight line is given by $s = (t^3 - 9t^2 + 15t)$ ft, where t is in seconds. Determine its maximum acceleration and maximum velocity during the time interval $0 \leq t \leq 10$ s.

$$s = t^3 - 9t^2 + 15t$$

$$v = \frac{ds}{dt} = 3t^2 - 18t + 15$$

$$a = \frac{dv}{dt} = 6t - 18$$

a_{max} occurs at $t = 10$ s,

$$a_{max} = 6(10) - 18 = 42 \text{ ft/s}^2 \qquad \textbf{Ans}$$

v_{max} occurs when $t = 10$ s

$$v_{max} = 3(10)^2 - 18(10) + 15 = 135 \text{ ft/s} \qquad \textbf{Ans}$$

***12-8.** From approximately what floor of a building must a car be dropped from an at-rest position so that it reaches a speed of 80.7 ft/s (55 mi/h) when it hits the ground? Each floor is 12 ft higher than the one below it. (*Note:* You may want to remember this when traveling 55 mi/h.)

$(+\downarrow) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$

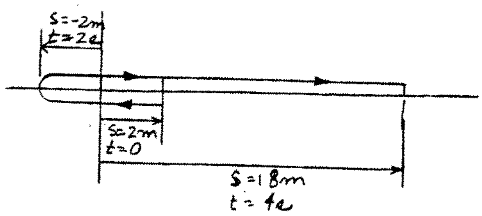
$80.7^2 = 0 + 2(32.2)(s - 0)$

$s = 101.13 \text{ ft}$

$\# \text{ of floors} = \frac{101.13}{12} = 8.43$

The car must be dropped from the 9th floor. **Ans**

12-9. A particle moves along a straight line such that its position is defined by $s = (t^3 - 3t^2 + 2)$ m. Determine the average velocity, the average speed, and the acceleration of the particle when $t = 4$ s.



$s = t^3 - 3t^2 + 2$

$v = \frac{ds}{dt} = 3t^2 - 6t$

$v = 0 \text{ at } t = 0, \; t = 2$

$a = \frac{dv}{dt} = 6t - 6$

$s|_{t=0} = 2$

$s|_{t=2} = -2$

$s|_{t=4} = 18$

$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{18 - 2}{4 - 0} = 4 \text{ m/s}$ **Ans**

$(v_{sp})_{avg} = \frac{s_T}{\Delta t} = \frac{4 + 20}{4 - 0} = 6 \text{ m/s}$ **Ans**

$a|_{t=4} = 6(4) - 6 = 18 \text{ m/s}^2$ **Ans**

12-10. A particle is moving along a straight line such that its acceleration is defined as $a = (-2v)\ \text{m/s}^2$, where v is in meters per second. If $v = 20$ m/s when $s = 0$ and $t = 0$, determine the particle's velocity as a function of position and the distance the particle moves before it stops.

$a = -2v$

$a\,ds = v\,dv$

$-2v\,ds = v\,dv$

$-2\int_0^s ds = \int_{20}^{v} dv$

$-2s = v - 20$

$v = (20 - 2s)$ m/s **Ans**

When $v = 0$,

$s = 10$ m **Ans**

Note : $t \to \infty$ for this to occur.

12-11. The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)\ \text{m/s}^2$, where t is in seconds. If $s = 1$ m and $v = 2$ m/s when $t = 0$, determine the particle's velocity and position when $t = 6$ s. Also, determine the total distance the particle travels during this time period.

$\int_2^v dv = \int_0^t (2t - 1)\,dt$

$v = t^2 - t + 2$

$\int_1^s ds = \int_0^t (t^2 - t + 2)\,dt$

$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2t + 1$

When $t = 6$ s ,

$v = 32$ m/s **Ans**

$s = 67$ m **Ans**

Since $v \neq 0$ then

$d = 67 - 1 = 66$ m **Ans**

12-12. A particle, initially at the origin, moves along a straight line through a fluid medium such that its velocity is defined as $v = 1.8(1 - e^{-0.3t})$ m/s, where t is in seconds. Determine the displacement of the particle during the first 3 s.

$$v = 1.8\left(1 - e^{-0.3t}\right)$$

$$ds = v\,dt$$

$$\Delta s = \int_0^3 1.8\left(1 - e^{-0.3t}\right)dt = 1.8\left(t + \frac{1}{0.3}e^{-0.3t}\right)\Big|_0^3$$

$$\Delta s = 1.8\left(3 + \frac{1}{0.3}e^{-0.3(3)}\right) - 1.8\left(0 + \frac{1}{0.3}\right)$$

$\Delta s = 1.84$ m **Ans**

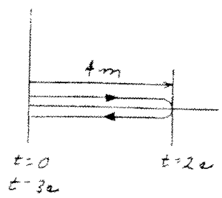
12-13. The velocity of a particle traveling in a straight line is given by $v = (6t - 3t^2)$ m/s, where t is in seconds. If $s = 0$ when $t = 0$, determine the particle's deceleration and position when $t = 3$ s. How far has the particle traveled during the 3-s time interval, and what is its average speed?

$$v = 6t - 3t^2$$

$$a = \frac{dv}{dt} = 6 - 6t$$

At $t = 3$ s

$a = -12$ m/s^2 **Ans**



$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \left(6t - 3t^2\right)dt$$

$$s = 3t^2 - t^3$$

At $t = 3$ s

$s = 0$ **Ans**

Since $v = 0 = 6t - 3t^2$, when $t = 0$ and $t = 2$ s,

when $t = 2$ s, $s = 3(2)^2 - (2)^3 = 4$ m

$s_T = 4 + 4 = 8$ m **Ans**

$(v_{sp})_{avg} = \dfrac{s_t}{t} = \dfrac{8}{3} = 2.67$ m/s **Ans**

12-14. A particle moves along a straight line such that its position is defined by $s = (t^2 - 6t + 5)$ m. Determine the average velocity, the average speed, and the acceleration of the particle when $t = 6$ s.

$s = t^2 - 6t + 5$

$v = \dfrac{ds}{dt} = 2t - 6$

$a = \dfrac{dv}{dt} = 2$

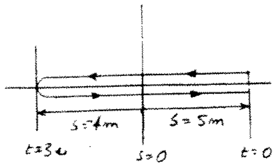
$v = 0$ when $t = 3$

$s|_{t=0} = 5$

$s|_{t=3} = -4$

$s|_{t=6} = 5$

$v_{avg} = \dfrac{\Delta s}{\Delta t} = \dfrac{0}{6} = 0$ **Ans**



$(v_{sp})_{avg} = \dfrac{s_T}{\Delta t} = \dfrac{9+9}{6} = 3$ m/s **Ans**

$a|_{t=6} = 2$ m/s² **Ans**

12-15. A particle is moving along a straight line such that when it is at the origin it has a velocity of 4 m/s. If it begins to decelerate at the rate of $a = (-1.5v^{1/2})$ m/s², where v is in m/s, determine the particle's position and velocity when $t = 2$ s.

$a = \dfrac{dv}{dt} = -1.5v^{\frac{1}{2}}$

$\int_4^v v^{-\frac{1}{2}}\,dv = \int_0^t -1.5\,dt$

$2v^{\frac{1}{2}}\Big|_4^v = -1.5t\Big|_0^t$

$2\left(v^{\frac{1}{2}} - 2\right) = -1.5t$

$v = (2 - 0.75t)^2$ m/s

$v|_{t=2} = (2 - 0.75(2))^2 = 0.25$ m/s **Ans**

$\int_0^s ds = \int_0^t (2 - 0.75t)^2\,dt = \int_0^t \left(4 - 3t + 0.5625t^2\right) dt$

$s = 4t - 1.5t^2 + 0.1875t^3$

$s|_{t=2} = 4(2) - 1.5(2)^2 + 0.1875(2)^3 = 3.5$ m **Ans**

*12-16. A particle travels to the right along a straight line with a velocity $v = [5/(4 + s)]$ m/s, where s is in meters. Determine its deceleration when $s = 2$ m.

$v = \dfrac{5}{4+s}$

$v\,dv = a\,ds$

$dv = \dfrac{-5\,ds}{(4+s)^2}$

$\dfrac{5}{(4+s)}\left(\dfrac{-5\,ds}{(4+s)^2}\right) = a\,ds$

$a = \dfrac{-25}{(4+s)^3}$

When $s = 2$ m,

$a = -0.116$ m/s² **Ans**

12-17. Two particles A and B start from rest at the origin $s = 0$ and move along a straight line such that $a_A = (6t - 3)\ \text{ft/s}^2$ and $a_B = (12t^2 - 8)\ \text{ft/s}^2$, where t is in seconds. Determine the distance between them when $t = 4$ s and the total distance each has traveled in $t = 4$ s.

Velocity: The velocity of particles A and B can be determine using Eq. 12-2.

$$dv_A = a_A dt$$

$$\int_0^{v_A} dv_A = \int_0^t (6t - 3)dt$$

$$v_A = 3t^2 - 3t$$

$$dv_B = a_B dt$$

$$\int_0^{v_B} dv_B = \int_0^t (12t^2 - 8)dt$$

$$v_B = 4t^3 - 8t$$

The times when particle A stops are

$$3t^2 - 3t = 0 \qquad t = 0 \text{ s and } = 1 \text{ s}$$

The times when particle B stops are

$$4t^3 - 8t = 0 \qquad t = 0 \text{ s and } t = \sqrt{2} \text{ s}$$

Position: The position of particles A and B can be determine using Eq. 12-1.

$$ds_A = v_A dt$$

$$\int_0^{s_A} ds_A = \int_0^t (3t^2 - 3t)dt$$

$$s_A = t^3 - \frac{3}{2}t^2$$

$$ds_B = v_B dt$$

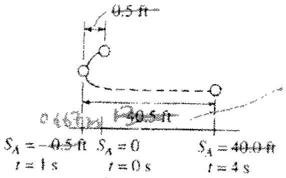
$$\int_0^{s_B} ds_B = \int_0^t (4t^3 - 8t)dt$$

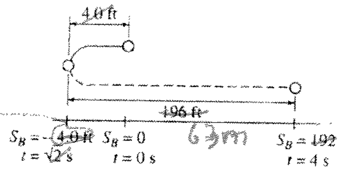
$$s_B = t^4 - 4t^2$$

The positions of particle A at $t = 1$ s and 4 s are

$$s_A|_{t=1\,s} = 1^3 - \frac{3}{2}(1^2) = -0.500 \text{ ft}$$

$$s_A|_{t=4\,s} = 4^3 - \frac{3}{2}(4^2) = 40.0 \text{ ft}$$





Particle A has traveled

$$d_A = 2(0.5) + 40.0 = 41.0 \text{ ft} \quad \textbf{Ans}$$

The positions of particle B at $t = \sqrt{2}$ s and 4 s are

$$s_B|_{t=\sqrt{2}} = (\sqrt{2})^4 - 4(\sqrt{2})^2 = -4 \text{ ft}$$

$$s_B|_{t=4} = (4)^4 - 4(4)^2 = 192 \text{ ft}$$

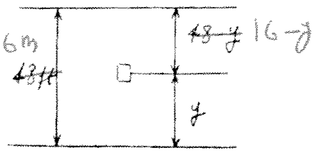
Particle B has traveled

$$d_B = 2(4) + 192 = 200 \text{ ft} \quad \textbf{Ans}$$

At $t = 4$ s, the distance between A and B is

$$\Delta s_{AB} = 192 - 40 = 152 \text{ ft} \quad \textbf{Ans}$$

12-18. A car is to be hoisted by elevator to the fourth floor of a parking garage, which is 48 ft above the ground. If the elevator can accelerate at 0.6 ft/s², decelerate at 0.3 ft/s², and reach a maximum speed of 8 ft/s, determine the shortest time to make the lift, starting from rest and ending at rest.



$+\uparrow \; v^2 = v_0^2 + 2a_c(s - s_0)$

$v_{max}^2 = 0 + 2(0.6)(y - 0)$

$0 = v_{max}^2 + 2(-0.3)(48 - y)$

$0 = 1.2y - 0.6(48 - y)$

$y = 16.0$ ft, $v_{max} = 4.382$ ft/s < 8 ft/s

$+\uparrow \; v = v_0 + a_c t$

$4.382 = 0 + 0.6t_1$

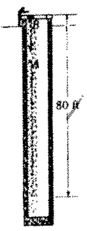
$t_1 = 7.303$ s

$0 = 4.382 - 0.3t_2$

$t_2 = 14.61$ s

$t = t_1 + t_2 = 21.9$ s **Ans**

12-19. A stone A is dropped from rest down a well, and in 1 s another stone B is dropped from rest. Determine the distance between the stones another second later.



$+\downarrow \; s = s_1 + v_1 t + \frac{1}{2}a_c t^2$

$s_A = 0 + 0 + \frac{1}{2}(32.2)(2)^2$

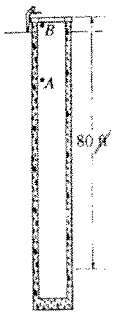
$s_A = 64.4$ ft

$s_B = 0 + 0 + \frac{1}{2}(32.2)(1)^2$

$s_B = 16.1$ ft

$\Delta s = 64.4 - 16.1 = 48.3$ ft **Ans**

***12-20.** A stone A is dropped from rest down a well, and in 1 s another stone B is dropped from rest. Determine the time interval between the instant A strikes the water and the instant B strikes the water. Also, at what speed do they strike the water?



B is dropped one second after A, so that

$\Delta t = 1$ s **Ans**

$+\downarrow \; s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$80 = 0 + 0 + \frac{1}{2}(32.2)(t^2)$

$t = 2.2291$ s

$+\downarrow \; v = v_0 + a_c t$

$v = 0 + 32.2(2.2291)$

$v = 71.8$ ft/s **Ans**

Also,

$v^2 = v_0^2 + 2a_c s$

$v^2 = 0^2 + 2(32.2)(80)$

$v = 71.8$ ft/s **Ans**

12-21. A particle has an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t)$ m/s^2, where t is in seconds, determine the distance traveled before it stops.

$a = -6t$

$dv = a\,dt$

$\int_{27}^{v} dv = -\int_0^t 6t\,dt$

$v - 27 = -3t^2$

$v = 27 - 3t^2$

$v = 0$ at $t = 3$

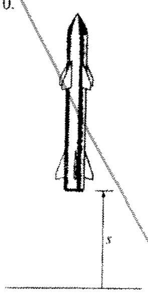
$ds = v\,dt$

$\int_0^s ds = \int_0^t (27 - 3t^2)\,dt$

$s = 27t - t^3$

$s|_{t=3} = 27(3) - (3)^3 = 54$ m **Ans**

12-22. The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)$ m/s^2, where s is in meters. Determine the rocket's velocity when $s = 2$ km and the time needed to reach this altitude. Initially, $v = 0$ and $s = 0$ when $t = 0$.



$a\,ds = v\,dv$

$\int_0^s (6 + 0.02\,s)\,ds = \int_0^v v\,dv$

$6\,s + 0.01\,s^2 = \frac{1}{2}v^2$

$v = \sqrt{12\,s + 0.02\,s^2}$

When $s = 2000$ m,

$v = 322$ m/s **Ans**

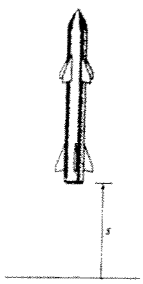
$\int_0^s \frac{ds}{\sqrt{12\,s + 0.02\,s^2}} = \int_0^t dt$

$\frac{1}{\sqrt{0.02}} \ln\left(\sqrt{12\,s + 0.02\,s^2} + s\sqrt{0.02} + \frac{12}{2\sqrt{0.02}}\right|_0^s = t$

Set $s = 2000$ m

$t = 19.3$ s **Ans**

12-23. The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)$ m/s^2, where s is in meters. Determine the time needed for the rocket to reach an altitude of $s = 100$ m. Initially, $v = 0$ and $s = 0$ when $t = 0$.



$a\,ds = v\,dv$

$\int_0^s (6 + 0.02\,s)\,ds = \int_0^v v\,dv$

$6\,s + 0.01\,s^2 = \frac{1}{2}v^2$

$v = \sqrt{12\,s + 0.02\,s^2}$

$ds = v\,dt$

$\int_0^{100} \frac{ds}{\sqrt{12\,s + 0.02\,s^2}} = \int_0^t dt$

$\frac{1}{\sqrt{0.02}} \ln\left[\sqrt{12\,s + 0.02\,s^2} + s\sqrt{0.02} + \frac{12}{2\sqrt{0.02}}\right]_0^{100} = t$

$t = 5.62$ s **Ans**

***12-24.** A particle is moving with a velocity of v_0 when $s = 0$ and $t = 0$. If it is subjected to a deceleration of $a = -kv^3$, where k is a constant, determine its velocity and position as functions of time.

$$a = \frac{dv}{dt} = -kv^3$$

$$\int_{v_0}^{v} v^{-3}\, dv = \int_0^t -k\, dt$$

$$-\frac{1}{2}\left(v^{-2} - v_0^{-2}\right) = -kt$$

$$v = \left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{-\frac{1}{2}} \quad \textbf{Ans}$$

$$ds = v\, dt$$

$$\int_0^s ds = \int_0^t \frac{dt}{\left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{\frac{1}{2}}}$$

$$s = \left.\frac{2\left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{\frac{1}{2}}}{2k}\right|_0^t$$

$$s = \frac{1}{k}\left[\left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{\frac{1}{2}} - \frac{1}{v_0}\right] \quad \textbf{Ans}$$

***12-25.** A particle has an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t)\ \text{m/s}^2$, where t is in seconds, determine its velocity when it travels 10 m. How much time does this take?

$a = -6t$

$dv = a\, dt$

$\int_{27}^{v} dv = -\int_0^t 6t\, dt$

$v - 27 = -3t^2$

$v = 27 - 3t^2$

$v = 0$ when $t = 3$

$ds = v\, dt$

$\int_0^s ds = \int_0^t (27 - 3t^2)\, dt$

$s = 27t - t^3$

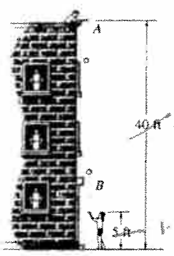
At $s = 10$ m $\qquad 10 = 27t - t^3 \qquad t^3 - 27t + 10 = 0$

Solving for the smallest positive root $t < 3$ s

$t = 0.3723 = 0.372$ s **Ans**

$v|_{t=0.3723} = 27 - 3(0.3723)^2 = 26.6$ m/s **Ans**

12-26. Ball A is released from rest at a height of 40 ft at the same time that a second ball B is thrown upward 5 ft from the ground. If the balls pass one another at a height of 20 ft, determine the speed at which ball B was thrown upward.



For ball # 1 :

$(+\downarrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$20 = 0 + 0 + \frac{1}{2}(32.2)t^2$

$t = 1.1146$ s

For ball # 2 :

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$15 = 0 + v_B(1.1146) + \frac{1}{2}(-32.2)(1.1146)^2$

$v_B = 31.4$ ft/s **Ans**

12-27. A car starts from rest and moves along a straight line with an acceleration of $a = (3s^{-1/3})$ m/s², where s is in meters. Determine the car's velocity and position when $t = 6$ s.

$$a = 3s^{-\frac{1}{3}}$$

$$a\,ds = v\,dv$$

$$\int_0^s 3s^{-\frac{1}{3}}\,ds = \int_0^v v\,dv$$

$$\frac{3}{2}(3)s^{\frac{2}{3}} = \frac{1}{2}v^2$$

$$v = 3s^{\frac{1}{3}}$$

$$\frac{ds}{dt} = 3s^{\frac{1}{3}}$$

$$\int_0^s s^{-\frac{1}{3}}\,ds = \int_0^t 3\,dt$$

$$\frac{3}{2}s^{\frac{2}{3}} = 3t$$

$$s = (2t)^{\frac{3}{2}}$$

$$s|_{t=6} = (2(6))^{\frac{3}{2}} = 41.57 = 41.6 \text{ m} \qquad \textbf{Ans}$$

$$v|_{t=6} = 3(41.57)^{\frac{1}{3}} = 10.4 \text{ m/s} \qquad \textbf{Ans}$$

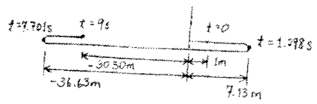
***12-28.** The acceleration of a particle along a straight line is defined by $a = (2t - 9)$ m/s², where t is in seconds. At $t = 0$, $s = 1$ m and $v = 10$ m/s. When $t = 9$ s, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

$$a = 2t - 9$$

$$\int_{10}^{v} dv = \int_0^t (2t - 9)\,dt$$

$$v - 10 = t^2 - 9t$$

$$v = t^2 - 9t + 10$$



$$\int_1^s ds = \int_0^t (t^2 - 9t + 10)\,dt$$

$$s - 1 = \frac{1}{3}t^3 - 4.5t^2 + 10t$$

$$s = \frac{1}{3}t^3 - 4.5t^2 + 10t + 1$$

Note when $v = 0$ at $t^2 - 9t + 10 = 0$

$$t = 1.298 \text{ s and } t = 7.701 \text{ s}$$

When $t = 1.298$ s, $\quad s = 7.13$ m

When $t = 7.701$ s, $\quad s = -36.63$ m

When $t = 9$ s, $\quad s = -30.50$ m

(a) $s = -30.5$ m **Ans**

(b) $s_{Tot} = (7.13 - 1) + 7.13 + 36.63 + (36.63 - 30.50)$

$s_{Tot} = 56.0$ m **Ans**

(c) $v = 10$ m/s **Ans**

12-29. A particle is moving along a straight line such that its acceleration is defined as $a = (4s^2)$ m/s^2, where s is in meters. If $v = -100$ m/s when $s = 10$ m and $t = 0$, determine the particle's velocity as a function of position.

$a = 4s^2$

$v\,dv = a\,ds$

$$\int_{-100}^{v} v\,dv = \int_{10}^{s} 4s^2\,ds$$

$$\frac{1}{2}v^2\Big|_{-100}^{v} = \frac{4}{3}s^3\Big|_{10}^{s}$$

$$\frac{1}{2}\left(v^2-(-100)^2\right) = \frac{4}{3}\left(s^3-(10)^3\right)$$

$v = -\left(2.667s^3 + 7333.3\right)^{\frac{1}{2}}$ m/s **Ans**

12-30. A car can have an acceleration and a deceleration of 5 m/s^2. If it starts from rest, and can have a maximum speed of 60 m/s, determine the shortest time it can travel a distance of 1200 m when it stops.

Time and distance to reach 60 m/s :

$a = 5$ m/s^2

$v = v_0 + a_c t$

$60 = 0 + 5t$

$t = 12$ s

$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$s_1 = 0 + 0 + \frac{1}{2}(5)(12)^2 = 360$ m

This is the same time and distance to stop.

Time and distance to travel at 60 m/s :

$s_2 = 1200 - 2(360) = 480$ m

$s = vt$

$480 = 60t$

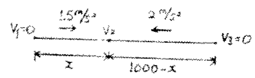
$t = 8$ s

Total time :

$t = 12 + 8 + 12 = 32$ s **Ans**

12-31. Determine the time required for a car to travel 1 km along a road if the car starts from rest, reaches a maximum speed at some intermediate point, and then stops at the end of the road. The car can accelerate at $1.5\ \text{m/s}^2$ and decelerate at $2\ \text{m/s}^2$.

Using formulas of constant acceleration:



$$v_2 = 1.5\,t_1$$

$$x = \frac{1}{2}(1.5)(t_1^2)$$

$$0 = v_2 - 2\,t_2$$

$$1000 - x = v_2 t_2 - \frac{1}{2}(2)(t_2^2)$$

Combining equations:

$$t_1 = 1.33\,t_2; \qquad v_2 = 2\,t_2$$

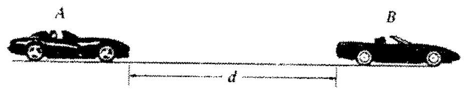
$$x = 1.33\,t_2^2$$

$$1000 - 1.33\,t_2^2 = 2\,t_2^2 - t_2^2$$

$$t_2 = 20.702\ \text{s} \quad ; t_1 = 27.603\ \text{s}$$

$$t = t_1 + t_2 = 48.3\ \text{s}$$ **Ans**

***12-32.** When two cars A and B are next to one another, they are traveling in the same direction with speeds v_A and v_B, respectively. If B maintains its constant speed, while A begins to decelerate at a_A, determine the distance d between the cars at the instant A stops.



Motion of car A :

$$v = v_0 + a_c t$$

$$0 = v_A - a_A t \qquad t = \frac{v_A}{a_A}$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = v_A^2 + 2(-a_A)(s_A - 0)$$

$$s_A = \frac{v_A^2}{2a_A}$$

Motion of car B :

$$s_B = v_B t = v_B\left(\frac{v_A}{a_A}\right) = \frac{v_A v_B}{a_A}$$

The distance between cars A and B is

$$s_{BA} = |s_B - s_A| = \left|\frac{v_A v_B}{a_A} - \frac{v_A^2}{2a_A}\right| = \left|\frac{2v_A v_B - v_A^2}{2a_A}\right|$$ **Ans**

12-33. If the effects of atmospheric resistance are accounted for, a freely falling body has an acceleration defined by the equation $a = 9.81[1 - v^2(10^{-4})]$ m/s², where v is in m/s and the positive direction is downward. If the body is released from rest at a *very high altitude*, determine (a) the velocity when $t = 5$ s, and (b) the body's terminal or maximum attainable velocity (as $t \rightarrow \infty$).

(a) $a = \dfrac{dv}{dt} = 9.81\left[1 - v^2\left(10^{-4}\right)\right]$

$$\int_0^v \frac{dv}{\left[10^4 - v^2\right]} = \int_0^t 9.81\left(10^{-4}\right)dt \qquad (1)$$

$$\frac{1}{100}\tanh^{-1}\left(\frac{v}{100}\right)\Big|_0^v = 9.81\left(10^{-4}\right)t$$

$$\tanh^{-1}\left(\frac{v}{100}\right) = \left(9.81\left(10^{-2}\right)t\right) \qquad (2)$$

$$v = 100\tanh\left(9.81\left(10^{-2}\right)(5)\right)$$

$$= 100\tanh(0.4905) = 45.5 \text{ m/s} \qquad \textbf{Ans}$$

(b) From Eq. (2), with $t \rightarrow \infty$,

$$v = 100\tanh\infty = 100 \text{ m/s} \qquad \textbf{Ans}$$

Also note that Eq. (1) can be written

$$10^4\int_0^v \frac{dv}{\left[10^4 - v^2\right]} = 9.81t$$

$$10^4\left[\left(\frac{1}{2(10^2)}\right)\ln\left(\frac{10^2 + v}{10^2 - v}\right)\right]_0^v = 9.81t$$

$$50\left[\ln\left(\frac{100 + v}{100 - v}\right) - \ln 1\right] = 9.81t$$

When $t = 5$ s,

$$\frac{100 + v}{100 - v} = e^{0.981} = 2.667$$

$$100 + v = 266.7 - 2.667v$$

$$v = \frac{166.7}{3.667} = 45.5 \text{ m/s} \qquad \textbf{Ans}$$

12-34. As a body is projected to a high altitude above the earth's *surface*, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0[R^2/(R + y)^2]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g_0 = 9.81$ m/s² and $R = 6356$ km, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \rightarrow \infty$.

$$v\,dv = a\,dy$$

$$\int_v^0 v\,dv = -g_0R^2\int_0^\infty \frac{dy}{(R + y)^2}$$

$$\frac{v^2}{2}\Big|_v^0 = \frac{g_0R^2}{R + y}\Big|_0^\infty$$

$$v = \sqrt{2g_0R}$$

$$= \sqrt{2(9.81)(6356)(10)^3}$$

$$= 11167 \text{ m/s} = 11.2 \text{ km/s} \qquad \textbf{Ans}$$

12-35. Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12-34), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude $y_0 = 500$ km? Use the numerical data in Prob. 12-34.

From Prob. 12-34,

$$(+\uparrow) \qquad g = -g_0 \frac{R^2}{(R+y)^2}$$

Since $g\,dy = v\,dv$

then

$$-g_0 R^2 \int_{y_0}^{y} \frac{dy}{(R+y)^2} = \int_0^v v\,dv$$

$$g_0 R^2 \left[\frac{1}{R+y}\right]_{y_0}^{y} = \frac{v^2}{2}$$

$$g_0 R^2 \left[\frac{1}{R+y} - \frac{1}{R+y_0}\right] = \frac{v^2}{2}$$

Thus

$$v = R\sqrt{\frac{2g_0(y_0-y)}{(R+y)(R+y_0)}} \qquad \textbf{Ans}$$

When $y_0 = 500$ km, $\quad y = 0$,

$$v = 6356(10^3)\sqrt{\frac{2(9.81)(500)(10^3)}{6356(6356+500)(10^6)}}$$

$$v = 3016 \text{ m/s} = 3.02 \text{ km/s} \qquad \textbf{Ans}$$

***12-36.** When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f. If this variation of the acceleration can be expressed as $a = (g/v_f^2)(v_f^2 - v^2)$, determine the time needed for the velocity to become $v < v_f$. Initially the particle falls from rest.

$$\frac{dv}{dt} = a = \left(\frac{g}{v_f^2}\right)\left(v_f^2 - v^2\right)$$

$$\int_0^v \frac{dv}{v_f^2 - v^2} = \frac{g}{v_f^2}\int_0^t dt$$

$$\frac{1}{2v_f}\ln\left(\frac{v_f+v}{v_f-v}\right)\Bigg|_0^v = \frac{g}{v_f^2}t$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f+v}{v_f-v}\right) \qquad \textbf{Ans}$$

12-37. An airplane starts from rest, travels 5000 ft down a runway, and after uniform acceleration, takes off with a speed of 162 mi/h. It then climbs in a straight line with a uniform acceleration of 3 ft/s² until it reaches a constant speed of 220 mi/h. Draw the *s–t*, *v–t*, and *a–t* graphs that describe the motion.

$v_1 = 0$

$v_2 = 162 \frac{\text{mi}}{\text{h}} \frac{(1\ \text{h})\,5280\ \text{ft}}{(3600\ \text{s})(1\ \text{mi})} = 237.6\ \text{ft/s}$

$v_2^2 = v_1^2 + 2\,a_c(s_2 - s_1)$

$(237.6)^2 = 0^2 + 2(a_c)(5000 - 0)$

$a_c = 5.64538\ \text{ft/s}^2$

$v_2 = v_1 + a_c t$

$237.6 = 0 + 5.64538\,t$

$t = 42.09 = 42.1\ \text{s}$

$v_3 = 220 \frac{\text{mi}}{\text{h}} \frac{(1\ \text{h})\,5280\ \text{ft}}{(3600\ \text{s})(1\ \text{mi})} = 322.67\ \text{ft/s}$

$v_3^2 = v_2^2 + 2\,a_c(s_3 - s_2)$

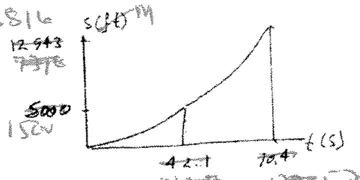
$(322.67)^2 = (237.6)^2 + 2(3)(s - 5000)$

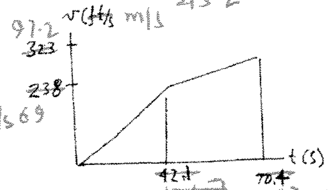
$s = 12\,943.34\ \text{ft}$

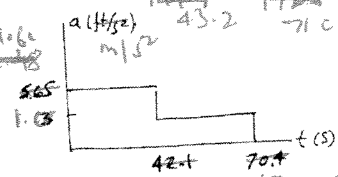
$v_3 = v_2 + a_c t$

$322.67 = 237.6 + 3t$

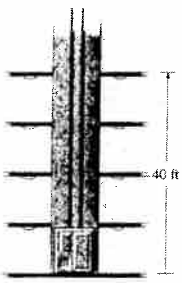
$t = 28.4\ \text{s}$







12-38. The elevator starts from rest at the first floor of the building. It can accelerate at 5 ft/s^2 and then decelerate at 2 ft/s^2. Determine the shortest time it takes to reach a floor 40 ft above the ground. The elevator starts from rest and then stops. Draw the a–t, v–t, and s–t graphs for the motion.



$$+\uparrow \quad v_2 = v_1 + a_c t_1$$

$$v_{max} = 0 + 5t_1$$

$$+\uparrow \quad v_3 = v_1 + a_c t$$

$$0 = v_{max} - 2t_2$$

Thus

$$t_1 = 0.4\,t_2$$

$$+\uparrow \quad s_2 = s_1 + v_1 t + \frac{1}{2}a_c t_1^2$$

$$h = 0 + 0 + \frac{1}{2}(5)(t_1^2) = 2.5\,t_1^2$$

$$+\uparrow \quad 40 - h = 0 + v_{max} t_2 - \frac{1}{2}(2)\,t_2^2$$

$$+\uparrow \quad v^2 = v_1^2 + 2\,a_c(s - s_1)$$

$$v_{max}^2 = 0 + 2(5)(h - 0)$$

$$v_{max}^2 = 10\,h$$

$$0 = v_{max}^2 + 2(-2)(40 - h)$$

$$v_{max}^2 = 160 - 4\,h$$

Thus,

$$10\,h = 160 - 4h$$

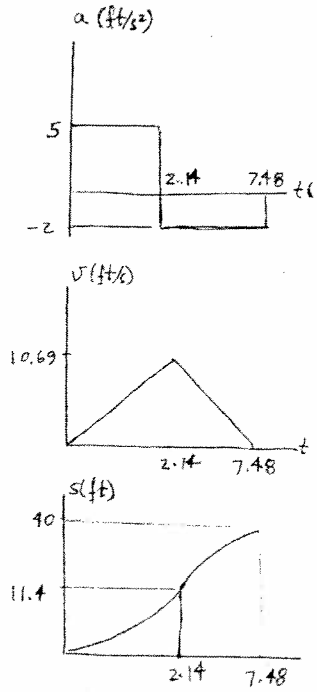
$$h = 11.429 \text{ ft}$$

$$v_{max} = 10.69 \text{ ft/s}$$

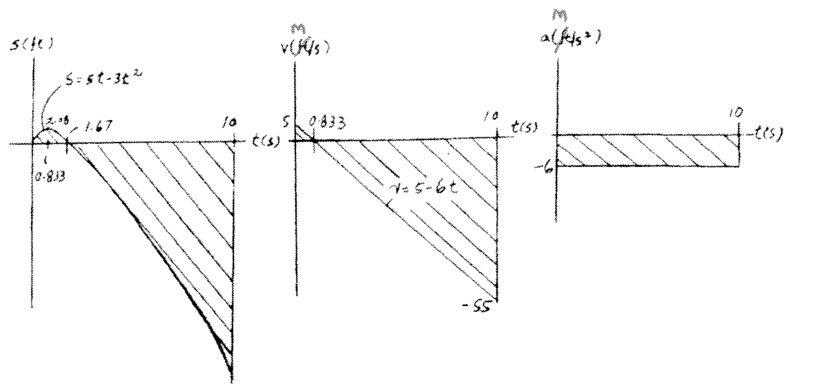
$$t_1 = 2.138 \text{ s}$$

$$t_2 = 5.345 \text{ s}$$

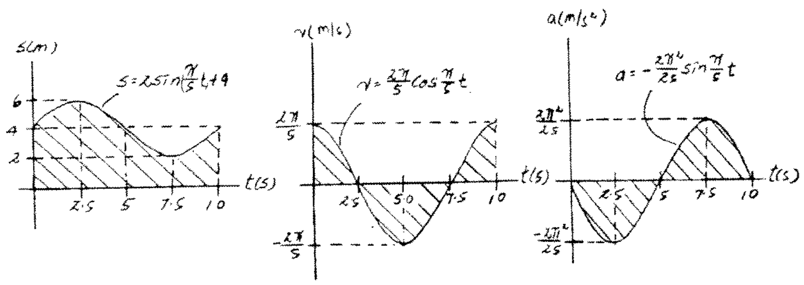
$$t = t_1 + t_2 = 7.48 \text{ s} \qquad \textbf{Ans}$$



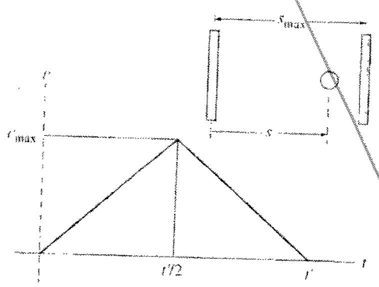
12-39. If the position of a particle is defined as $s = (5t - 3t^2)$ ft, where t is in seconds, construct the s–t, v–t, and a–t graphs for $0 \le t \le 10$ s.



***12-40.** If the position of a particle is defined by $s = [2 \sin (\pi/5)t + 4]$ m, where t is in seconds, construct the s–t, v–t, and a–t graphs for $0 \le t \le 10$ s.



12-41. The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure. The acceleration and deceleration that occur are constant and both have a magnitude of 4 m/s^2. If the plates are spaced 200 mm apart, determine the maximum velocity v_{max} and the time t' for the particle to travel from one plate to the other. Also draw the s–t graph. When $t = t'/2$ the particle is at s = 100 mm.



$a_c = 4\text{ m/s}^2$

$\frac{s}{2} = 100\text{ mm} = 0.1\text{ m}$

$v^2 = v_0^2 + 2\,a_c(s - s_0)$

$v_{max}^2 = 0 + 2(4)(0.1 - 0)$

$v_{max} = 0.89442\text{ m/s} \qquad = 0.894\text{ m/s}$ **Ans**

$v = v_0 + a_c\, t'$

$0.89442 = 0 + 4(\frac{t'}{2})$

$t' = 0.44721\text{ s} \qquad = 0.447\text{ s}$ **Ans**

$s = s_0 + v_0\, t + \frac{1}{2}\, a_c\, t^2$

$s = 0 + 0 + \frac{1}{2}(4)(t)^2$

$s = 2\,t^2$

When $t = \frac{0.44721}{2} = 0.2236 = 0.224\text{ s}$,

$s = 0.1\text{ m}$

$\int_{0.894}^{v} dv = -\int_{0.2235}^{t} 4\,dt$

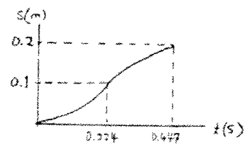
$v = -4\,t + 1.788$

$\int_{0.1}^{s} ds = \int_{0.2235}^{t} (-4t + 1.788)\,dt$

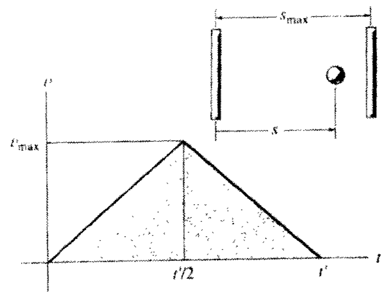
$s = -2\,t^2 + 1.788\,t - 0.2$

When $t = 0.447$ s,

$s = 0.2$ m



12-42. The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure, where $t' = 0.2$ s and $v_{max} = 10$ m/s. Draw the s–t and a–t graphs for the particle. When $t = t'/2$ the particle is at $s = 0.5$ m.



For $0 < t < 0.1$ s,

$$v = 100\,t$$

$$a = \frac{dv}{dt} = 100$$

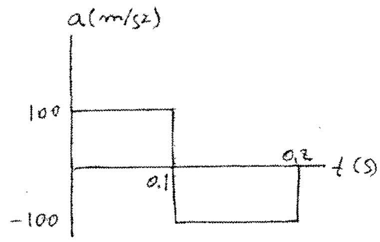
$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 100\,t\,dt$$

$$s = 50\,t^2$$

When $t = 0.1$ s,

$$s = 0.5 \text{ m}$$

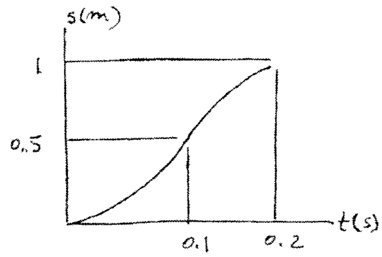


For $0.1 \text{ s} < t < 0.2$ s,

$$v = -100\,t + 20$$

$$a = \frac{dv}{dt} = -100$$

$$ds = v\,dt$$



$$\int_{0.5}^s ds = \int_{0.1}^t (-100\,t + 20)\,dt$$

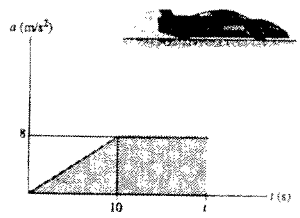
$$s - 0.5 = (-50\,t^2 + 20\,t - 1.5)$$

$$s = -50\,t^2 + 20\,t - 1$$

When $t = 0.2$ s,

$s = 1$ m **Ans**

12-43. A car starting from rest moves along a straight track with an acceleration as shown. Determine the time t for the car to reach a speed of 50 m/s and construct the v–t graph that describes the motion until the time t.



For $0 \le t \le 10$ s,

$$a = \frac{8}{10}t$$

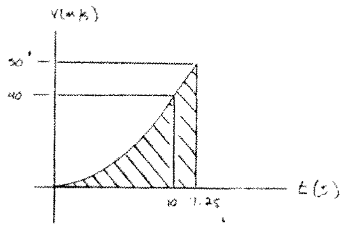
$$dv = a\,dt$$

$$\int_0^v dv = \int_0^t \frac{8}{10}t\,dt$$

$$v = \frac{8}{20}t^2$$

At $t = 10$ s,

$$v = \frac{8}{20}(10)^2 = 40 \text{ m/s}$$



For $t > 10$ s,

$$a = 8$$

$$dv = a\,dt$$

$$\int_{40}^v dv = \int_{10}^t 8\,dt$$

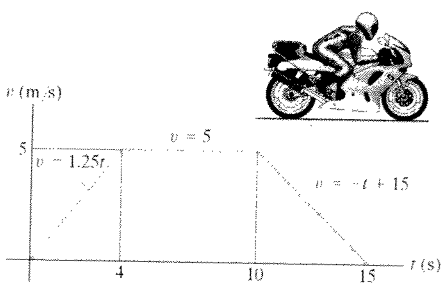
$$v - 40 = 8t - 80$$

$$v = 8t - 40$$

When $v = 50$ m/s

$$t = \frac{50+40}{8} = 11.25 \text{ s} \qquad \textbf{Ans}$$

***12-44.** A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v–t graph. Determine the motorcycle's acceleration and position when $t = 8$ s and $t = 12$ s.



At $t = 8$ s

$$a = \frac{dv}{dt} = 0 \qquad \textbf{Ans}$$

$$\Delta s = \int v\,dt$$

$$s - 0 = \frac{1}{2}(4)(5) + (8-4)(5) = 30$$

$$s = 30 \text{ m} \qquad \textbf{Ans}$$

At $t = 12$ s

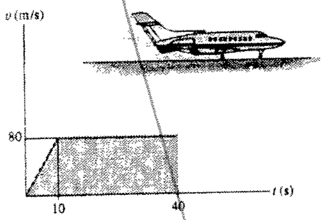
$$a = \frac{dv}{dt} = \frac{-5}{5} = -1 \text{ m/s}^2 \qquad \textbf{Ans}$$

$$\Delta s = \int v\,dt$$

$$s - 0 = \frac{1}{2}(4)(5) + (10-4)(5) + \frac{1}{2}(15-10)(5) - \frac{1}{2}(\frac{3}{5})(5)(\frac{3}{5})(5)$$

$$s = 48 \text{ m} \qquad \textbf{Ans}$$

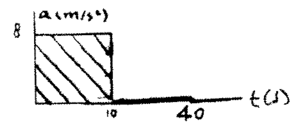
12-45. From experimental data, the motion of a jet plane while traveling along a runway is defined by the v–t graph shown. Construct the s–t and a–t graphs for the motion.



From the $v-t$ graph :

$0 \leq t \leq 10$ s $\quad a = \frac{\Delta v}{\Delta t} = \frac{80}{10} = 8 \text{ m/s}^2$

$10 \leq t \leq 40$ s $\quad a = \frac{\Delta v}{\Delta t} = \frac{(80-80)}{(40-10)} = 0$



From the $v-t$ graph at $t_1 = 10$ s and $t_2 = 40$ s :

$s_1 = A_1 = \frac{1}{2}(10)(80) = 400$ m $\qquad s_2 = A_1 + A_2 = 400 + 80(40-10) = 2800$ m

The equations defining the $s-t$ graph are :

$0 \leq t \leq 10$:

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 8t\,dt$$

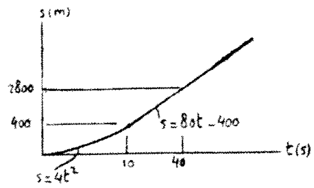
$$s = 4t^2$$

$10 \leq t \leq 40$:

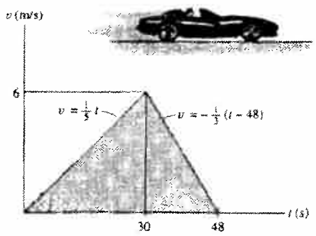
$$ds = v\,dt$$

$$\int_{400}^s ds = \int_{10}^t 80\,dt$$

$$s = 80t - 400$$



12-46. A car travels along a straight road with the speed shown by the v–t graph. Determine the total distance the car travels until it stops when $t = 48$ s. Also plot the s–t and a–t graphs.



For $0 \le t \le 30$ s,

$$v = \frac{1}{5}t$$

$$a = \frac{dv}{dt} = \frac{1}{5}$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \frac{1}{5}t\,dt$$

$$s = \frac{1}{10}t^2$$

When $t = 30$ s, $s = 90$ m,

$$v = -\frac{1}{3}(t-48)$$

$$a = \frac{dv}{dt} = -\frac{1}{3}$$

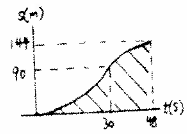
$$ds = v\,dt$$

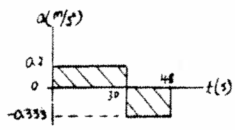
$$\int_{90}^s ds = \int_{30}^t -\frac{1}{3}(t-48)\,dt$$

$$s = -\frac{1}{6}t^2 + 16t - 240$$

When $t = 48$ s,

$s = 144$ m **Ans**

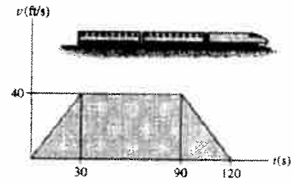




Also, from the $v-t$ graph

$\Delta s = \int v\,dt \qquad s - 0 = \frac{1}{2}(6)(48) = 144$ m **Ans**

12-47. The v–t graph for the motion of a train as it moves from station A to station B is shown. Draw the a–t graph and determine the average speed and the distance between the stations.



For $0 \le t < 30$ s

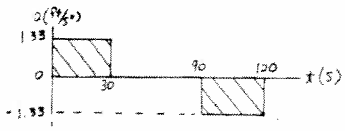
$$a = \frac{\Delta v}{\Delta t} = \frac{40}{30} = 1.33 \text{ ft/s}^2$$

For $30 \text{ s} < t < 90$ s

$$a = \frac{\Delta v}{\Delta t} = 0$$

For $90 \text{ s} < t < 120$ s

$$a = \frac{\Delta v}{\Delta t} = \frac{0-40}{120-90} = -1.33 \text{ ft/s}^2$$



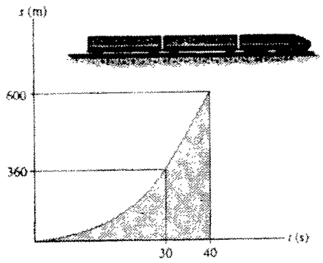
$$\Delta s = \int v\,dt$$

$$s - 0 = \frac{1}{2}(30)(40) + 40(90-30) + \frac{1}{2}(40)(120-90)$$

$s = 3600$ ft **Ans**

$(v_{sp})\quad v_{avg} = \frac{\Delta s}{\Delta t} = \frac{3600}{120} = 30$ ft/s **Ans**

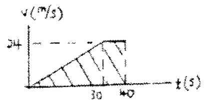
•12-48. The s–t graph for a train has been experimentally determined. From the data, construct the v–t and a–t graphs for the motion; $0 \le t \le 40$ s. For $0 \le t \le 30$ s, the curve is $s = (0.4t^2)$ m, and then it becomes straight for $t \ge 30$ s.



$0 \le t \le 30$:

$$s = 0.4t^2$$

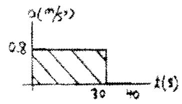
$$v = \frac{ds}{dt} = 0.8t$$



$$a = \frac{dv}{dt} = 0.8$$

$30 \le t \le 40$:

$$s - 360 = \left(\frac{600-360}{40-30}\right)(t-30)$$

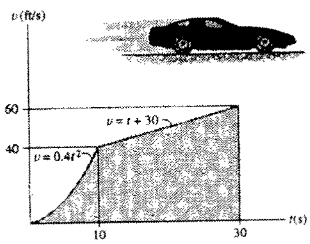


$$s = 24(t-30) + 360$$

$$v = \frac{ds}{dt} = 24$$

$$a = \frac{dv}{dt} = 0$$

12-49. The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the a–t graph and determine the maximum acceleration during the 30-s time interval. The car starts from rest at $s = 0$.



For $t < 10$ s :

$$v = 0.4t^2$$

$$a = \frac{dv}{dt} = 0.8t$$

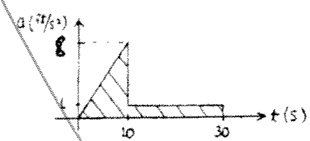
At $t = 10$ s :

$$a = 8 \text{ ft/s}^2$$

For $10 < t \le 30$ s :

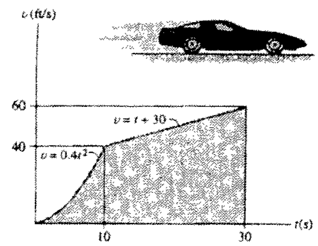
$$v = t + 30$$

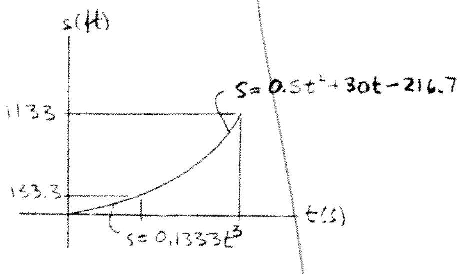
$$a = \frac{dv}{dt} = 1$$



$a_{max} = 8 \text{ ft/s}^2$ **Ans**

12-50. The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the s–t graph and determine the average speed and the distance traveled for the 30-s time interval. The car starts from rest at $s = 0$.





For $t < 10$ s,

$$v = 0.4t^2$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 0.4t^2\,dt$$

$$s = 0.1333t^3$$

At $t = 10$ s,

$$s = 133.3 \text{ ft}$$

For $10 < t < 30$ s,

$$v = t + 30$$

$$ds = v\,dt$$

$$\int_{133.3}^{s} ds = \int_{10}^{t} (t + 30)\,dt$$

$$s = 0.5t^2 + 30t - 216.7$$

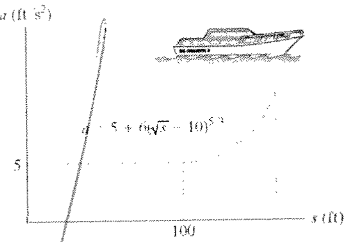
At $t = 30$ s,

$$s = 1133 \text{ ft}$$

$$(v_{sp})_{Avg} = \frac{\Delta s}{\Delta t} = \frac{1133}{30} = 37.8 \text{ ft/s} \quad \textbf{Ans}$$

$$s_T = 1133 \text{ ft} = 1.13(10^3) \text{ ft} \quad \textbf{Ans}$$

***12–51.** The a–s graph for a boat moving along a straight path is given. If the boat starts at $s = 0$ when $v = 0$, determine its speed when it is at $s = 75$ ft, and 125 ft, respectively. Use Simpson's rule with $n = 100$ to evaluate v at $s = 125$ ft.



Velocity: The velocity v in terms of s can be obtained by applying $v\,dv = a\,ds$. For the interval **0 ft ≤ s < 100 ft,**

$$v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_0^s 5\,ds$$

$$v = (\sqrt{10s})\ \text{ft/s}$$

At $s = 75$ ft, $\quad v = \sqrt{10(75)} = 27.4$ ft/s **Ans**

At $s = 100$ ft, $\quad v = \sqrt{10(100)} = 31.62$ ft/s **Ans**

For the interval **100 ft < s ≤ 125 ft,**

$$v\,dv = a\,ds$$

$$\int_{31.62\ \text{ft/s}}^{v} v\,dv = \int_{100\ \text{ft}}^{125\ \text{ft}} \left[5 + 6(\sqrt{s} - 10)^{5/3}\right] ds$$

Evaluating the integral on the right using Simpson's rule, we have

$$\left.\frac{v^2}{2}\right|_{31.62\ \text{ft/s}}^{v} = 201.032$$

$$v = 37.4\ \text{ft/s} \quad \textbf{Ans}$$

***12-52.** A man riding upward in a freight elevator accidentally drops a package off the elevator when it is 100 ft from the ground. If the elevator maintains a constant upward speed of 4 ft/s, determine how high the elevator is from the ground the instant the package hits the ground. Draw the v–t curve for the package during the time it is in motion. Assume that the package was released with the same upward speed as the elevator.

For package :

$$(+\uparrow) \quad v^2 = v_0^2 + 2a_c(s_2 - s_0)$$

$$v^2 = (4)^2 + 2(-32.2)(-100 - 0)$$

$$v = 80.35\ \text{ft/s} \downarrow$$

$$(+\uparrow) \quad v = v_0 + a_c t$$

$$-80.35 = 4 + (-32.2)t$$

$$t = 2.620\ \text{s}$$

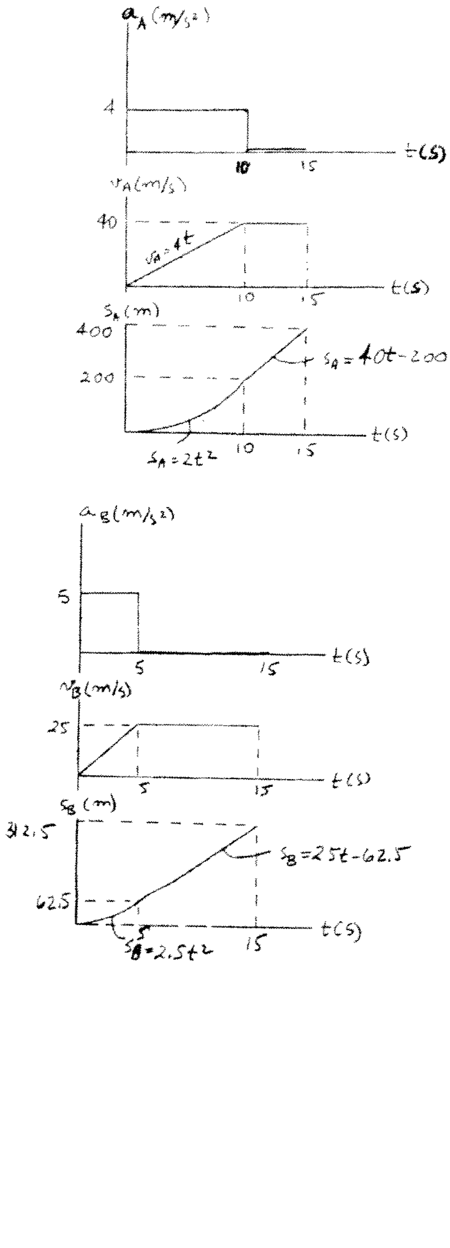
For elevator :

$$(+\uparrow) \quad s_2 = s_0 + vt$$

$$s = 100 + 4(2.620)$$

$$s = 110\ \text{ft} \quad \textbf{Ans}$$

12-53. Two cars start from rest side by side and travel along a straight road. Car *A* accelerates at 4 m/s² for 10 s and then maintains a constant speed. Car *B* accelerates at 5 m/s² until reaching a constant speed of 25 m/s and then maintains this speed. Construct the a–t, v–t, and s–t graphs for each car until $t = 15$ s. What is the distance between the two cars when $t = 15$ s?



Car *A*:

$$v = v_0 + a_c t$$

$$v_A = 0 + 4t$$

At $t = 10$ s, $v_A = 40$ m/s

$$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(4)t^2 = 2t^2$$

At $t = 10$ s, $s_A = 200$ m

$t > 10$ s, $ds = v\,dt$

$$\int_{200}^{s_A} ds = \int_{10}^{t} 40\,dt$$

$$s_A = 40t - 200$$

At $t = 15$ s, $s_A = 400$ m

Car *B*:

$$v = v_0 + a_c t$$

$$v_B = 0 + 5t$$

When $v_B = 25$ m/s, $t = \frac{25}{5} = 5$ s

$$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$s_B = 0 + 0 + \frac{1}{2}(5)t^2 = 2.5t^2$$

At $t = 5$ s, $s_B = 62.5$ m

$t > 5$ s, $ds = v\,dt$

$$\int_{62.5}^{s_B} ds = \int_{5}^{t} 25\,dt$$

$$s_B - 62.5 = 25t - 125$$

$$s_B = 25t - 62.5$$

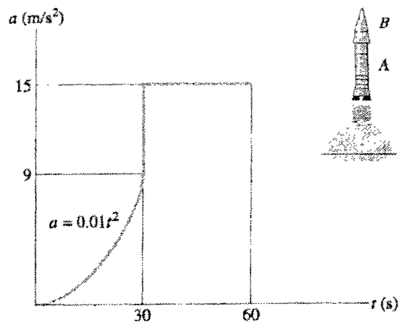
When $t = 15$ s, $s_B = 312.5$

Distance between the cars is

$$\Delta s = s_A - s_B = 400 - 312.5 = 87.5 \text{ m} \quad \textbf{Ans}$$

Car *A* is ahead of car *B*.

12-34. A two-stage rocket is fired vertically from rest at $s = 0$ with an acceleration as shown. After 30 s the first stage A burns out and the second stage B ignites. Plot the v–t and s–t graphs which describe the motion of the second stage for $0 \leq t \leq 60$ s.



For $0 \leq t < 30$ s

$$\int_0^v dv = \int_0^t 0.01t^2 dt$$

$$v = 0.00333t^3$$

When $t = 30$ s, $v = 90$ m/s

$$\int_0^s ds = \int_0^t 0.00333t^3 dt$$

$$s = 0.000833t^4$$

When $t = 30$ s, $s = 675$ m

For $30 \text{ s} \leq t < 60$ s

$$\int_{90}^v dv = \int_{30}^t 15 dt$$

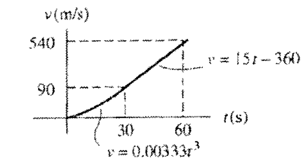
$$v = 15t - 360$$

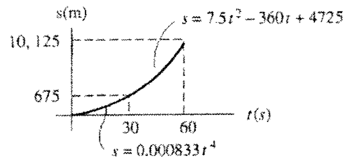
When $t = 60$ s, $v = 540$ m/s

$$\int_{675}^s ds = \int_{30}^t (15t - 360) dt$$

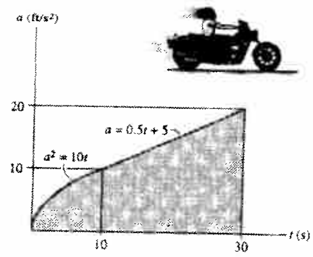
$$s = 7.5t^2 - 360t + 4725$$

When $t = 60$ s, $s = 10\,125$ m





12-55. The a–t graph for a motorcycle traveling along a straight road has been estimated as shown. Determine the time needed for the motorcycle to reach a maximum speed of 100 ft/s and the distance traveled in this time. Draw the v–t and s–t graphs. The motorcycle starts from rest at $s = 0$.



$0 \le t < 10$ s :

$$a = \sqrt{10}\, t^{\frac{1}{2}}$$

$$dv = a\, dt$$

$$\int_0^v dv = \int_0^t \sqrt{10}\, t^{\frac{1}{2}}\, dt$$

$$v = \frac{2}{3}\sqrt{10}\, t^{\frac{3}{2}}$$

When $t = 10$ s, $\quad v = \frac{2}{3}\sqrt{10}\,(10)^{\frac{3}{2}} = 66.67 \text{ ft/s} < 100 \text{ ft/s}$

$$ds = v\, dt$$

$$\int_0^s ds = \int_0^t \frac{2}{3}\sqrt{10}\, t^{\frac{3}{2}}\, dt$$

$$s = \frac{4}{15}\sqrt{10}\, t^{\frac{5}{2}}$$

When $t = 10$ s, $\quad s = \frac{4}{15}\sqrt{10}\,(10)^{\frac{5}{2}} = 266.67 \text{ ft}$

$t > 10$ s :

$$a = 0.5t + 5$$

$$dv = a\, dt$$

$$\int_{66.67}^{v} dv = \int_{10}^{t} (0.5t + 5)\, dt$$

$$v - 66.67 = 0.25t^2 + 5t - 75$$

$$v = 0.25t^2 + 5t - 8.333$$

When $v = 100$ ft/s,

$0.25t^2 + 5t - 108.333 = 0$

Solving for the positive root : $\quad t = 13.09 \text{ s} = 13.1 \text{ s}$ **Ans**

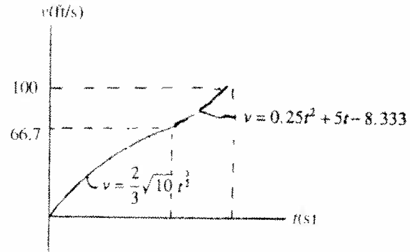
$$ds = v\, dt$$

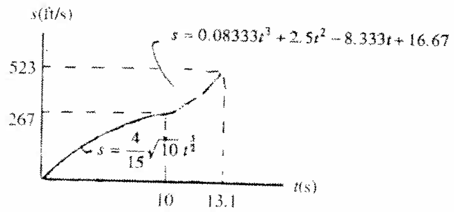
$$\int_{266.67}^{s} ds = \int_{10}^{t} v\, dt = \int_{10}^{t} \left(0.25t^2 + 5t - 8.333\right) dt$$

$$s - 266.67 = 0.08333t^3 + 2.5t^2 - 8.333t - 250$$

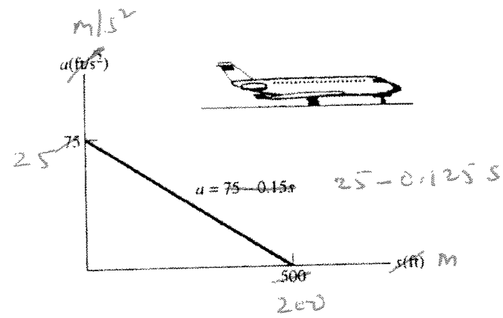
$$s = 0.08333t^3 + 2.5t^2 - 8.333t + 16.67$$

$s|_{t=13.09} = 0.08333(13.09)^3 + 2.5(13.09)^2 - 8.333(13.09) + 16.67 = 523 \text{ ft}$ **Ans**





***12-56.** The jet plane starts from rest at $s = 0$ and is subjected to the acceleration shown. Determine the speed of the plane when it has traveled 200 ft. Also, how much time is required for it to travel 200 ft?



$a = 75 - 0.15s$

$\int_0^v v\,dv = \int_0^s (75 - 0.15s)\,ds$

$v = \sqrt{150\,s - 0.15\,s^2}$

At $s = 200$ ft

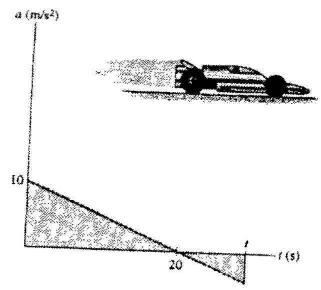
$v = \sqrt{150(200) - 0.15(200)^2} = 155$ ft/s **Ans**

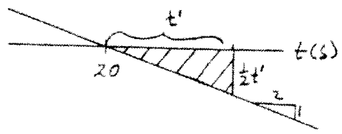
$v = \frac{ds}{dt}$

$\int_0^t dt = \int_0^{200} \frac{ds}{\sqrt{150\,s - 0.15\,s^2}}$

$t = 2.582 \sin^{-1}\left(\frac{0.3\,s - 150}{150}\right)\Big|_0^{200} = 2.39$ s **Ans**

12-57. The jet car is originally traveling at a speed of 20 m/s when it is subjected to the acceleration shown in the graph. Determine the car's maximum speed and the time t when it stops.





$a = -0.5t + 10$

$dv = a\,dt$

$\int_{20}^{v} dv = \int_{0}^{t} (-0.5t + 10)\,dt$

$v = -0.25t^2 + 10t + 20$

Maximum speed :

$a = \frac{dv}{dt} = -0.5t + 10 = 0$

$t = 20$ s

$v_{max} = -0.25(20)^2 + 10(20) + 20 = 120$ m/s **Ans**

Also, find the area under the $a - t$ graph $0 \le t \le 20$ s,

$\Delta v = v_{max} - 20 = \frac{1}{2}(10)(20)$ $v_{max} = 120$ m/s **Ans**

When the car stops, $v = 0$,

$0 = -0.25t^2 + 10t + 20$

Solve for the positive root,

$t = 41.9$ s **Ans**

Also, use $a - t$ graph, require negative area to equal 120.

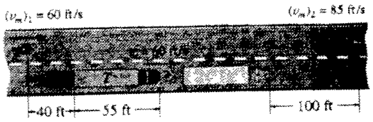
$120 = \frac{1}{2}(t')\left[\frac{1}{2}(t')\right]$

$t' = 21.9$ s

Thus,

$t' + 20 = 41.9$ s **Ans**

12-58. A motorcyclist at A is traveling at 60 ft/s when he wishes to pass the truck T which is traveling at a constant speed of 60 ft/s. To do so the motorcyclist accelerates at 6 ft/s² until reaching a maximum speed of 85 ft/s. If he then maintains this speed, determine the time needed for him to reach a point located 100 ft in front of the truck. Draw the v–t and s–t graphs for the motorcycle during this time.



Motorcycle :

Time to reach 85 ft/s,

$$v = v_0 + a_c t$$

$$85 = 60 + 6t$$

$$t = 4.167 \text{ s}$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

Distance traveled,

$$(85)^2 = (60)^2 + 2(6)(s_m - 0)$$

$$s_m = 302.08 \text{ ft}$$

In $t = 4.167$ s, truck travels

$$s_t = 60(4.167) = 250 \text{ ft}$$

Further distance for motorcycle to travel : $40 + 55 + 250 + 100 - 302.08 = 142.92$ ft

Motorcycle :

$$s = s_0 + v_0 t$$

$$(s + 142.92) = 0 + 85t'$$

Truck :

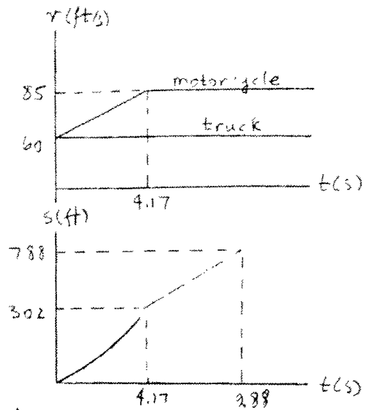
$$s = 0 + 60t'$$

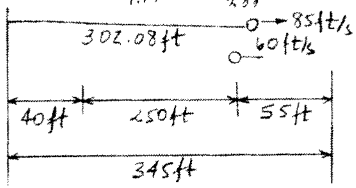
Thus $t' = 5.717$ s

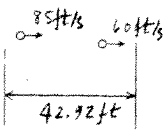
$t = 4.167 + 5.717 = 9.88$ s **Ans**

Total distance motorcycle travels

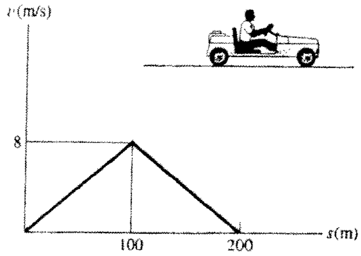
$$s_T = 302.08 + 85(5.717) = 788 \text{ ft}$$







12-59. The v–s graph for a go-cart traveling on a straight road is shown. Determine the acceleration of the go-cart at $s = 50$ m and $s = 150$ m. Draw the a–s graph.



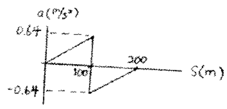
For $0 \le s < 100$

$v = 0.08\,s, \qquad dv = 0.08\,ds$

$a\,ds = (0.08\,s)(0.08\,ds)$

$a = 6.4(10^{-3})\,s$

At $s = 50$ m, $\quad a = 0.32\ \text{m/s}^2$ **Ans**



For $100 < s < 200$

$v = -0.08\,s + 16,$

$dv = -0.08\,ds$

$a\,ds = (-0.08\,s + 16)(-0.08\,ds)$

$a = 0.08(0.08\,s - 16)$

At $s = 150$ m, $\quad a = -0.32\ \text{m/s}^2$ **Ans**

Also,

$v\,dv = a\,ds$

$a = v(\frac{dv}{ds})$

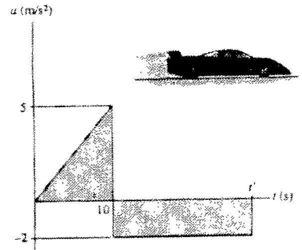
At $s = 50$ m,

$a = 4(\frac{8}{100}) = 0.32\ \text{m/s}^2$ **Ans**

At $s = 150$ m,

$a = 4(\frac{-8}{100}) = -0.32\ \text{m/s}^2$ **Ans**

•12-60. The a–t graph for a car is shown. Construct the v–t and s–t graphs if the car starts from rest at $t = 0$. At what time t' does the car stop?



Velocity :

For $0 \le t \le 10$ s $\quad \dfrac{dv}{dt} = 0.5t \quad \displaystyle\int_0^v dv = \int_0^t 0.5t\,dt \quad v = 0.25t^2$

At $t = 10$ s $\quad v = 0.25(10)^2 = 25$ m/s

For $10\text{ s} \le t \le t'$ $\quad \dfrac{dv}{dt} = -2 \quad \displaystyle\int_{25}^v dv = \int_{10}^t -2dt \quad v = -2t + 45$

At $t = t'$, $v = 0$ $\quad v = -2t' + 45 = 0 \quad t' = 22.5$ s **Ans**

Position :

For $0 \le t \le 10$ s $\quad \dfrac{ds}{dt} = 0.25t^2 \quad \displaystyle\int_0^s ds = \int_0^t 0.25t^2 dt$

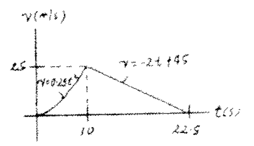
$s = 0.0833t^3$

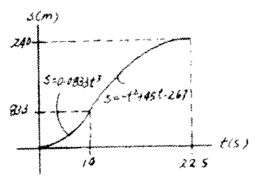
At $t = 10$ s $\quad s = 0.0833(10)^3 = 83.3$ m

For $10\text{ s} \le t \le t' = 22.5$ s $\quad \dfrac{ds}{dt} = -2t + 45 \quad \displaystyle\int_{83.3}^s ds = \int_{10}^t (-2t + 45)dt$

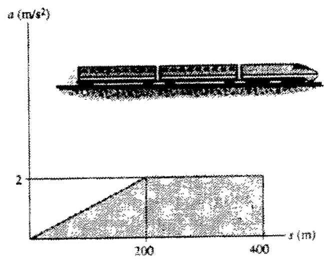
$s = -t^2 + 45t - 267$

At $t = 22.5$ s $\quad s = -22.5^2 + 45(22.5) - 267 = 240$ m





12-61. The a–s graph for a train traveling along a straight track is given for the first 400 m of its motion. Plot the v–s graph. $v = 0$ at $s = 0$.



$0 \le s \le 200: \quad a = \dfrac{1}{100}s$

$a\,ds = v\,dv$

$$\int_0^s \frac{1}{100}s\,ds = \int_0^v v\,dv$$

$$\frac{1}{200}s^2 = \frac{1}{2}v^2$$

$v = 0.1s$

At $s = 200$, $\quad v = 20$ m/s

$200 \le s \le 400: \quad a = 2$

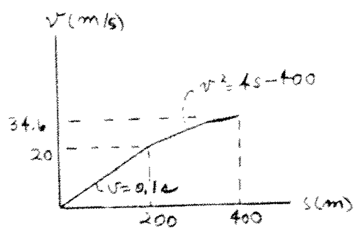
$a\,ds = v\,dv$

$$\int_{200}^s 2\,ds = \int_{20}^v v\,dv$$

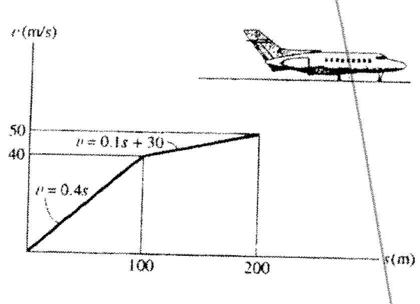
$$2(s - 200) = \frac{1}{2}\left(v^2 - 400\right)$$

$v^2 = 4s - 400$

At $s = 400$ m, $\quad v = \sqrt{4(400) - 400} = 34.6$ m/s



12-62. The v–s graph for an airplane traveling on a straight runway is shown. Determine the acceleration of the plane at $s = 100$ m and $s = 150$ m. Draw the a–s graph.



For $0 \le s < 100$ m

$a\,ds = v\,dv$

$a\,ds = 0.4\,s\,(0.4\,ds)$

$a = 0.16\,s$

At $s = 100$ m, $a = 16\ \text{m/s}^2$ **Ans**

For $100\text{ m} < s \le 200$ m

$a\,ds = v\,dv$

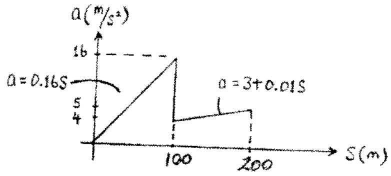
$a\,ds = (30 + 0.1\,s)(0.1\,ds)$

$a = 3 + 0.01\,s$

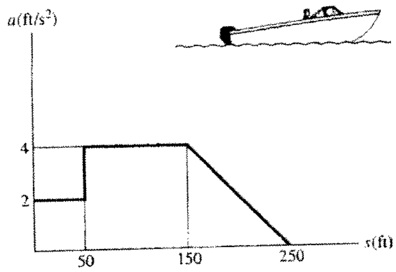
At $s = 150$ m, $a = 4.5\ \text{m/s}^2$ **Ans**

At $s = 100$ m, $a = 4\ \text{m/s}^2$

At $s = 200$ m, $a = 5\ \text{m/s}^2$



12-63. Starting from rest at $s = 0$, a boat travels in a straight line with an acceleration as shown by the a–s graph. Determine the boat's speed when $s = 40$, 90, and 200 ft.



Since $\int a\,ds = \int_0^v v\,dv$

$\int a\,ds = \frac{1}{2}v^2$

$v = \sqrt{2\int a\,ds}$

$\int a\,ds$ = area under $a-s$ graph.

For $s = 40$ ft

$v = \sqrt{2(2)(40)} = 12.7$ ft/s **Ans**

For $s = 90$ ft

$v = \sqrt{2[2(50) + 4(40)]} = 22.8$ ft/s **Ans**

For $s = 200$ ft

$v = \sqrt{2[2(50) + 4(100) + \frac{1}{2}(50)(4+2)]}$

$v = 36.1$ ft/s **Ans**

Also,

For $0 \le s < 50$ ft

$a = 2, \quad v\,dv = a\,ds$

$\int_0^s 2\,ds = \int_0^v v\,dv$

$v = \sqrt{4s}$

When $s = 40$ ft, $v = \sqrt{4(40)} = 12.7$ ft/s **Ans**

When $s = 50$ ft, $v = \sqrt{4(50)} = 14.14$ ft/s

For 50 ft $< s < 150$ ft

$a = 4,$

$\int_{50}^s 4\,ds = \int_{14.14}^v v\,dv$

$v = \sqrt{8s - 200}$

When $s = 90$ ft, $v = \sqrt{8(90) - 200} = 22.8$ ft/s **Ans**

When $s = 150$ ft, $v = \sqrt{8(90) - 200} = 31.62$ ft/s

For 150 ft $< s < 250$ ft

$a = -\frac{4}{100}s + 10$

$\int_{150}^s (-\frac{1}{25}s + 10)\,ds = \int_{31.62}^v v\,dv$

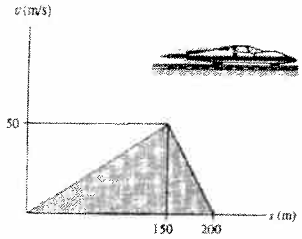
$v = (-\frac{1}{25}s^2 + 20s - 1100)^{1/2}$

When $s = 200$ ft

$v = [-\frac{1}{25}(200)^2 + 20(200) - 1100]^{1/2}$

$v = 36.1$ ft/s **Ans**

***12-64.** The v–s graph for a test vehicle is shown. Determine its acceleration when $s = 100$ m and when $s = 175$ m.



$0 \le s \le 150$ m : $\quad v = \frac{1}{3}s,$

$$dv = \frac{1}{3}ds$$

$$v\,dv = a\,ds$$

$$\frac{1}{3}s\left(\frac{1}{3}ds\right) = a\,ds$$

$$a = \frac{1}{9}s$$

At $s = 100$ m, $\quad a = \frac{1}{9}(100) = 11.1 \text{ m/s}^2$ **Ans**

$150 \le s \le 200$ m : $\quad v = 200 - s,$

$$dv = -ds$$

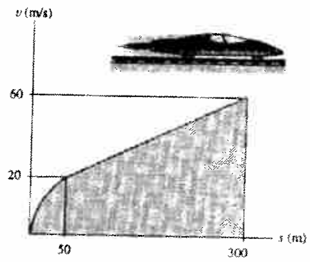
$$v\,dv = a\,ds$$

$$(200 - s)(-ds) = a\,ds$$

$$a = s - 200$$

At $s = 175$ m, $\quad a = 175 - 200 = -25 \text{ m/s}^2$ **Ans**

12-65. The v–s graph was determined experimentally to describe the straight-line motion of a rocket sled. Determine the acceleration of the sled when $s = 100$ m, and when $s = 200$ m.



$$a = v\frac{dv}{ds}$$

The two points on the $v - s$ graph are

(50, 20) and (300, 60). The slope of the line is

$$\frac{dv}{ds} = \frac{60 - 20}{300 - 50} = 0.160$$

From the line segment on the graph at $s = 100$ m, $v = 28$ m/s,

$a = 28(0.160) = 4.48 \text{ m/s}^2$ **Ans**

In a similar manner, $s = 200$ m, $v = 44.0$ m/s, so that

$a = 44.0(0.160) = 7.04 \text{ m/s}^2$ **Ans**

12-66. A particle, originally at rest and located at point (3 ft, 2 ft, 5 ft), is subjected to an acceleration of $\mathbf{a} = \{6t\mathbf{i} + 12t^2\mathbf{k}\}$ ft/s². Determine the particle's position (x, y, z) at $t = 1$ s.

Velocity : The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12 – 9.

$$d\mathbf{v} = \mathbf{a}dt$$

$$\int_0^{\mathbf{v}} d\mathbf{v} = \int_0^t \left(6t\mathbf{i} + 12t^2\mathbf{k}\right) dt$$

$$\mathbf{v} = \{3t^2\mathbf{i} + 4t^3\mathbf{k}\} \text{ ft/s}$$

Position : The position expressed in Cartesian vector form can be obtained by applying Eq. 12 – 7.

$$d\mathbf{r} = \mathbf{v}dt$$

$$\int_{\mathbf{r}_1}^{\mathbf{r}} d\mathbf{r} = \int_0^t \left(3t^2\mathbf{i} + 4t^3\mathbf{k}\right) dt$$

$$\mathbf{r} - (3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) = t^3\mathbf{i} + t^4\mathbf{k}$$

$$\mathbf{r} = \left\{\left(t^3 + 3\right)\mathbf{i} + 2\mathbf{j} + \left(t^4 + 5\right)\mathbf{k}\right\} \text{ ft}$$

When $t = 1$ s, $\mathbf{r} = \left(1^3 + 3\right)\mathbf{i} + 2\mathbf{j} + \left(1^4 + 5\right)\mathbf{k} = \{4\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}$ ft.

The coordinates of the particle are

(4, 2, 6) ft **Ans**

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12-67. The velocity of a particle is given by $\mathbf{v} = \{16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\}$ m/s, where t is in seconds. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = 2$ s. Also, what is the x, y, z coordinate position of the particle at this instant?

Acceleration : The acceleration expressed in Cartesian vector form can be obtained by applying Eq. 12 – 9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{32t\mathbf{i} + 12t^2\mathbf{j} + 5\mathbf{k}\} \text{ m/s}^2$$

When $t = 2$ s, $\mathbf{a} = 32(2)\mathbf{i} + 12\left(2^2\right)\mathbf{j} + 5\mathbf{k} = \{64\mathbf{i} + 48\mathbf{j} + 5\mathbf{k}\}$ m/s². The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{64^2 + 48^2 + 5^2} = 80.2 \text{ m/s}^2 \quad \textbf{Ans}$$

Position : The position expressed in Cartesian vector form can be obtained by applying Eq. 12 – 7.

$$d\mathbf{r} = \mathbf{v}\,dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left(16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\right) dt$$

$$\mathbf{r} = \left[\frac{16}{3}t^3\mathbf{i} + t^4\mathbf{j} + \left(\frac{5}{2}t^2 + 2t\right)\mathbf{k}\right] \text{ m}$$

When $t = 2$ s,

$$\mathbf{r} = \frac{16}{3}\left(2^3\right)\mathbf{i} + \left(2^4\right)\mathbf{j} + \left[\frac{5}{2}\left(2^2\right) + 2(2)\right]\mathbf{k} = \{42.7\mathbf{i} + 16.0\mathbf{j} + 14.0\mathbf{k}\} \text{ m}.$$

Thus, the coordinate of the particle is

(42.7, 16.0, 14.0) m **Ans**

***12-68.** A particle is traveling with a velocity of $\mathbf{v} = \{3\sqrt{t}e^{-0.2t}\mathbf{i} + 4e^{-0.8t^2}\mathbf{j}\}$ m/s, where t is in seconds. Determine the magnitude of the particle's displacement from $t = 0$ to $t = 3$ s. Use Simpson's rule with $n = 100$ to evaluate the integrals. What is the magnitude of the particle's acceleration when $t = 2$ s?

$ds = v\,dt$

$\Delta s_x = \int_0^3 3\sqrt{t}\,e^{-0.2t}\,dt = 7.341$

$\Delta s_y = \int_0^3 4\,e^{-0.8t^2}\,dt = 3.963$

Thus,

$\Delta s = \sqrt{(7.341)^2 + (3.963)^2} = 8.34$ m **Ans**

$a_x = \dot{v}_x = 3\left(\frac{1}{2}\right)t^{-\frac{1}{2}}e^{-0.2t} + 3\sqrt{t}\,e^{-0.2t}(-0.2)\Big|_{t=2} = 0.1422$

$a_y = \dot{v}_y = 4\,e^{-0.8t^2}(-0.8)(2t)\Big|_{t=2} = -0.5218$

$a = \sqrt{(0.1422)^2 + (-0.5218)^2} = 0.541$ m/s^2 **Ans**

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12-69. The position of a particle is defined by $r = \{5(\cos 2t)\,\mathbf{i} + 4(\sin 2t)\mathbf{j}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. Determine the magnitudes of the velocity and acceleration of the particle when $t = 1$ s. Also, prove that the path of the particle is elliptical.

Velocity : The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12 – 7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{-10\sin 2t\mathbf{i} + 8\cos 2t\mathbf{j}\}\ \text{m/s}$$

When $t = 1$ s, $\mathbf{v} = -10\sin 2(1)\,\mathbf{i} + 8\cos 2(1)\,\mathbf{j} = \{-9.093\mathbf{i} - 3.329\mathbf{j}\}$ m/s. Thus, the magnitude of the velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-9.093)^2 + (-3.329)^2} = 9.68\ \text{m/s} \qquad \textbf{Ans}$$

Acceleration : The acceleration express in Cartesian vector form can be obtained by applying Eq. 12 – 9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{-20\cos 2t\mathbf{i} - 16\sin 2t\mathbf{j}\}\ \text{m/s}^2$$

When $t = 1$ s, $\mathbf{a} = -20\cos 2(1)\,\mathbf{i} - 16\sin 2(1)\,\mathbf{j} = \{8.323\mathbf{i} - 14.549\mathbf{j}\}$ m/s^2. Thus, the magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8.323^2 + (-14.549)^2} = 16.8\ \text{m/s}^2 \qquad \textbf{Ans}$$

Travelling Path : Here, $x = 5\cos 2t$ and $y = 4\sin 2t$. Then,

$$\frac{x^2}{25} = \cos^2 2t \qquad [1]$$

$$\frac{y^2}{16} = \sin^2 2t \qquad [2]$$

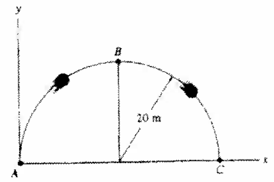
Adding Eqs[1] and [2] yields

$$\frac{x^2}{25} + \frac{y^2}{16} = \cos^2 2t + \sin^2 2t$$

However, $\cos^2 2t + \sin^2 2t = 1$. Thus,

$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \ \textbf{(Equation of an Ellipse)}\ (Q.E.D.)$$

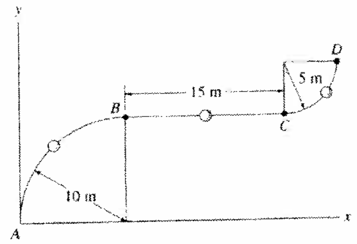
12-70. A particle travels along the curve from A to B in 1 s. If it takes 3 s for it to go from A to C, determine its *average velocity* when it goes from B to C.



Time from B to C is $3 - 1 = 2$ s

$$\mathbf{v}_{avg} = \frac{\Delta \mathbf{r}}{\Delta t} = \frac{(\mathbf{r}_{AC} - \mathbf{r}_{AB})}{\Delta t} = \frac{40\mathbf{i} - (20\mathbf{i} + 20\mathbf{j})}{2} = \{10\mathbf{i} - 10\mathbf{j}\}\ \text{m/s} \qquad \textbf{Ans}$$

12-71. A particle travels along the curve from A to B in 2 s. It takes 4 s for it to go from B to C and then 3 s to go from C to D. Determine its average speed when it goes from A to D.



$$s_T = \frac{1}{4}(2\pi)(10)) + 15 + \frac{1}{4}(2\pi(5)) = 38.56$$

$$v_{sp} = \frac{s_T}{t_t} = \frac{38.56}{2+4+3} = 4.28\ \text{m/s} \qquad \textbf{Ans}$$

***12-72.** A car travels east 2 km for 5 minutes, then north 3 km for 8 minutes, and then west 4 km for 10 minutes. Determine the total distance traveled and the magnitude of displacement of the car. Also, what is the magnitude of the average velocity and the average speed?

Total Distance Traveled and Displacement : The total distance traveled is

$$s = 2 + 3 + 4 = 9\ \text{km} \qquad \textbf{Ans}$$

and the magnitude of the displacement is

$$\Delta r = \sqrt{2^2 + 3^2} = 3.606\ \text{km} = 3.61\ \text{km} \qquad \textbf{Ans}$$

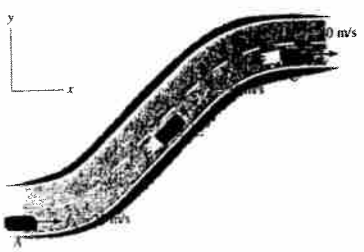
Average Velocity and Speed : The total time is $\Delta t = 5 + 8 + 10 = 23$ min $= 1380$ s. The magnitude of average velocity is

$$v_{avg} = \frac{\Delta r}{\Delta t} = \frac{3.606(10^3)}{1380} = 2.61\ \text{m/s} \qquad \textbf{Ans}$$

and the average speed is

$$(v_{sp})_{avg} = \frac{s}{\Delta t} = \frac{9(10^3)}{1380} = 6.52\ \text{m/s} \qquad \textbf{Ans}$$

12-73. A car traveling along the straight portions of the road has the velocities indicated in the figure when it arrives at points A, B, and C. If it takes 3 s to go from A to B, and then 5 s to go from B to C, determine the average acceleration between points A and B and between points A and C.



$$\mathbf{v}_A = 20\,\mathbf{i}$$

$$\mathbf{v}_B = 21.21\,\mathbf{i} + 21.21\,\mathbf{j}$$

$$\mathbf{v}_C = 40\,\mathbf{i}$$

$$\mathbf{a}_{AB} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{21.21\,\mathbf{i} + 21.21\mathbf{j} - 20\,\mathbf{i}}{3}$$

$$\mathbf{a}_{AB} = \{0.404\,\mathbf{i} + 7.07\,\mathbf{j}\}\ \text{m/s}^2 \qquad \text{Ans}$$

$$\mathbf{a}_{AC} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{40\,\mathbf{i} - 20\,\mathbf{i}}{8}$$

$$\mathbf{a}_{AC} = \{2.50\,\mathbf{i}\,\}\ \text{m/s}^2 \qquad \text{Ans}$$

12-74. A particle moves along the curve $y = e^{2x}$ such that its velocity has a constant magnitude of $v = 4$ ft/s. Determine the x and y components of velocity when the particle is at $y = 5$ ft.

Velocity : Taking the first derivative of the path $y = e^{2x}$, we have

$$\dot{y} = 2e^{2x}\dot{x} \qquad [1]$$

However, $\dot{x} = v_x$ and $\dot{y} = v_y$. Thus, Eq. [1] becomes

$$v_y = 2e^{2x}v_x \qquad [2]$$

Here, $v = 4$ ft/s. Then

$$v^2 = v_x^2 + v_y^2$$

$$v_x^2 + v_y^2 = 16 \qquad [3]$$

Solving Eqs. [2] and [3] yields

$$v_x = 4\sqrt{\frac{1}{1+4e^{4x}}} \quad \text{and} \quad v_y = 8\sqrt{\frac{e^{4x}}{1+4e^{4x}}}$$

At $y = 5$ ft, $5 = e^{2x}$, $x = 0.8047$ ft. Thus,

$$v_x = 4\sqrt{\frac{1}{1+4e^{4(0.8047)}}} = 0.398 \text{ ft/s} \qquad \textbf{Ans}$$

$$v_y = 8\sqrt{\frac{e^{4(0.8047)}}{1+4e^{4(0.8047)}}} = 3.98 \text{ ft/s} \qquad \textbf{Ans}$$

12-75. The path of a particle is defined by $y^2 = 4kx$, and the component of velocity along the y axis is $v_y = ct$, where both k and c are constants. Determine the x and y components of acceleration.

$y^2 = 4kx$

$2yv_y = 4kv_x$

$2v_y^2 + 2ya_y = 4ka_x$

$v_y = ct$

$a_y = c$ **Ans**

$2(ct)^2 + 2yc = 4ka_x$

$a_x = \frac{c}{2k}\left(y + ct^2\right)$ **Ans**

***12-76.** A particle is moving along the curve $y = x - (x^2/400)$, where x and y are in ft. If the velocity component in the x direction is $v_x = 2$ ft/s and remains *constant*, determine the magnitudes of the velocity and acceleration when $x = 20$ ft.

Velocity **:** Taking the first derivative of the path $y = x - \frac{x^2}{400}$, we have

$$\dot{y} = \dot{x} - \frac{1}{400}(2x\dot{x})$$

$$\dot{y} = \dot{x} - \frac{x}{200}\dot{x} \qquad [1]$$

However, $\dot{x} = v_x$ and $\dot{y} = v_y$. Thus, Eq.[1] becomes

$$v_y = v_x - \frac{x}{200}v_x \qquad [2]$$

Here, $v_x = 2$ ft/s at $x = 20$ ft. Then, From Eq.[2]

$$v_y = 2 - \frac{20}{200}(2) = 1.80 \text{ ft/s}$$

Also,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{2^2 + 1.80^2} = 2.69 \text{ ft/s} \qquad \textbf{Ans}$$

Acceleration **:** Taking the second derivative of the path $y = x - \frac{x^2}{400}$, we have

$$\ddot{y} = \ddot{x} - \frac{1}{200}\left(\dot{x}^2 + x\ddot{x}\right) \qquad [3]$$

However, $\ddot{x} = a_x$ and $\ddot{x} = a_y$. Thus, Eq.[3] becomes

$$a_y = a_x - \frac{1}{200}\left(v_x^2 + xa_x\right) \qquad [4]$$

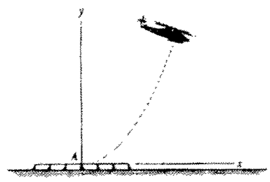
Since $v_x = 2$ ft/s is constant, hence $a_x = 0$ at $x = 20$ ft. Then, From Eq.[4]

$$a_y = 0 - \frac{1}{200}\left[2^2 + 20(0)\right] = -0.020 \text{ ft/s}$$

Also,

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{0^2 + (-0.020)^2} = 0.0200 \text{ ft/s}^2 \qquad \textbf{Ans}$$

12-77. The flight path of the helicopter as it takes off from A is defined by the parametric equations $x = (2t^2)$ m and $y = (0.04t^3)$ m, where t is the time in seconds after takeoff. Determine the distance the helicopter is from point A and the magnitudes of its velocity and acceleration when $t = 10$ s.



$x = 2t^2 \qquad y = 0.04t^3$

At $t = 10$ s, $\quad x = 200$ m $\quad y = 40$ m

$d = \sqrt{(200)^2 + (40)^2} = 204$ m $\qquad$ **Ans**

$v_x = \frac{dx}{dt} = 4t$

$a_x = \frac{dv_x}{dt} = 4$

$v_y = \frac{dy}{dt} = 0.12t^2$

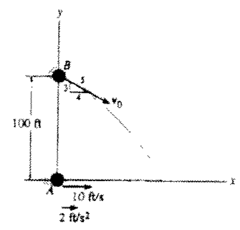
$a_y = \frac{dv_y}{dt} = 0.24t$

At $t = 10$ s,

$v = \sqrt{(40)^2 + (12)^2} = 41.8$ m/s $\qquad$ **Ans**

$a = \sqrt{(4)^2 + (2.4)^2} = 4.66$ m/s^2 $\qquad$ **Ans**

12-78. At the instant shown particle A is traveling to the right at 10 ft/s and has an acceleration of 2 ft/s^2. Determine the initial speed v_0 of particle B so that when it is fired at the same instant from the angle shown it strikes A. Also, at what speed does it strike A?



Particle A :

$$a_x = 2\ \text{ft/s}^2$$

$$v_x = v_0 + a_c t = 10 + 2t$$

$$x = x_0 + (v_x)_0 t + \frac{1}{2} a_c t^2 = 0 + 10t + t^2$$

Particle B :

$$v_x = (v_x)_0 = \frac{4}{5} v_0$$

$$x = (v_x)_0 t = \frac{4}{5} v_0 t$$

$$v_y = (v_y)_0 + a_c t = -\frac{3}{5} v_0 - 32.2t$$

$$y = y_0 + (v_y)_0 t + \frac{1}{2} a_c t^2 = 100 - \frac{3}{5} v_0 t - \frac{1}{2}(32.2)t^2$$

Require :

$$x_A = x_B \qquad 10t + t^2 = \frac{4}{5} v_0 t$$

$$v_0 = 1.25(10 + t)$$

$$y_A = y_B \qquad 0 = 100 - \frac{3}{5} v_0 t - 16.1t^2$$

$$0 = 100 - \frac{3}{5}(1.25)(10 + t)t - 16.1t^2$$

$$0 = 16.85t^2 + 7.5t - 100$$

$$t = 2.224\ \text{s}$$

$$v_0 = 1.25(10 + 2.224) = 15.28 = 15.3\ \text{ft/s}$$ **Ans**

$$v_{B_x} = \frac{4}{5}(15.28) = 12.224$$

$$v_{B_y} = -\frac{3}{5}(15.28) - 32.2(2.224) = -80.77$$

$$v_B = \sqrt{(12.224)^2 + (-80.77)^2} = 81.7\ \text{ft/s}$$ **Ans**

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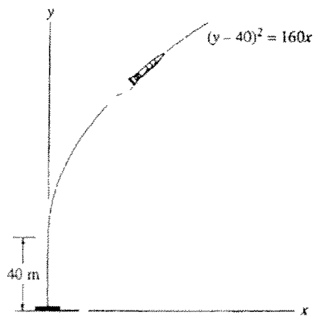
12-79. When a rocket reaches an altitude of 40 m it begins to travel along the parabolic path $(y - 40)^2 = 160x$, where the coordinates are measured in meters. If the component of velocity in the vertical direction is constant at $v_y = 180$ m/s, determine the magnitudes of the rocket's velocity and acceleration when it reaches an altitude of 80 m.



$$v_y = 180\ \text{m/s}$$

$$(y - 40)^2 = 160\,x$$

$$2(y - 40)v_y = 160\,v_x \qquad (1)$$

$$2(80 - 40)(180) = 160\,v_x$$

$$v_x = 90\ \text{m/s}$$

$$v = \sqrt{90^2 + 180^2} = 201\ \text{m/s}$$ **Ans**

$$a_y = \frac{d\,v_y}{dt} = 0$$

From Eq. 1,

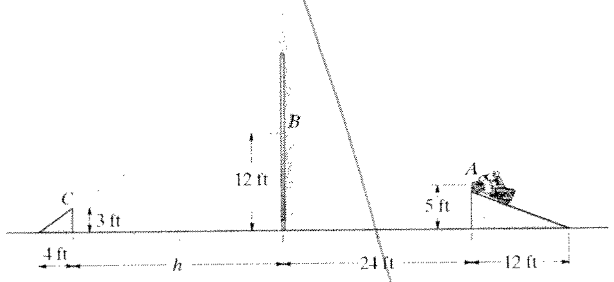
$$2\,v_y^2 + 2(y - 40)a_y = 160\,a_x$$

$$2(180)^2 + 0 = 160\,a_x$$

$$a_x = 405\ \text{m/s}^2$$

$$a = 405\ \text{m/s}^2$$ **Ans**

***12–80.** Determine the minimum speed of the stunt rider, so that when he leaves the ramp at A he passes through the center of the hoop at B. Also, how far h should the landing ramp be from the hoop so that he lands on it safely at C? Neglect the size of the motorcycle and rider.



$(\overset{+}{\leftarrow})\ s = s_0 + v_0 t$

$24 = 0 + v_A(\frac{12}{13})t$

$v_A t = 26 \qquad (1)$

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$12 = 5 + v_A(\frac{5}{13})t + \frac{1}{2}(-32.2)(t^2)$

Using Eq.(1)

$7 = 26(\frac{5}{13}) + \frac{1}{2}(-32.2)(t^2)$

$t = 0.43167$ s

$v_A = 60.2$ ft/s **Ans**

$(\overset{+}{\leftarrow})\ s = s_0 + v_0 t$

$h = 0 + 60.2(\frac{12}{13})t$

$(+\downarrow)\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$(12-3) = 0 + 0 + \frac{1}{2}(32.2)(t^2)$

$t = 0.7477$ s

$h = 41.6$ ft **Ans**

12-81. Show that if a projectile is fired at an angle θ from the horizontal with an initial velocity $\mathbf{v}_0$, the *maximum* range the projectile can travel is given by $R_{max} = v_0^2/g$, where g is the acceleration of gravity. What is the angle θ for this condition?

$(v_0)_x = v_0 \cos\theta$

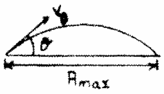
$(v_0)_y = v_0 \sin\theta$

After time t,

$(\overset{+}{\rightarrow})$ $\quad s_x = (v_0)_x t; \quad x = (v_0\cos\theta)t \quad (1)$

$(+\uparrow)$ $\quad s_y = (v_0)_y t + \frac{1}{2}a_c t^2; \quad y = (v_0\sin\theta)t - \frac{1}{2}gt^2 \quad (2)$

Substituting Eq. (1) into Eq. (2), $\quad y = x\tan\theta - \dfrac{gx^2}{2v_0^2\cos^2\theta}$



Set $y = 0$ to determine the range, $x = R$:

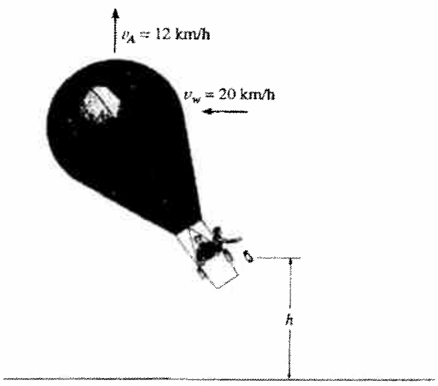
$$R = \frac{2v_0^2\sin\theta\cos\theta}{g} = \frac{v_0^2\sin 2\theta}{g}$$

R_{max} occurs when $\sin 2\theta = 1$ or,

$\theta = 45°$ **Ans**

This gives : $\quad R_{max} = \dfrac{v_0^2}{g}$ **Q.E.D**

12-82. The balloon A is ascending at the rate $v_A = 12$ km/h and is being carried horizontally by the wind at $v_w = 20$ km/h. If a ballast bag is dropped from the balloon at the instant $h = 50$ m, determine the time needed for it to strike the ground. Assume that the bag was released from the balloon with the same velocity as the balloon. Also, with what speed does the bag strike the ground?



$(+\uparrow)\ v^2 = v_0^2 + 2a_c(s - s_0)$

$v_y^2 = (3.33)^2 + 2(-9.81)(-50 - 0)$

$v_y = 31.50$ m/s

$(+\uparrow)\ v = v_0 + a_c t$

$-31.50 = 3.33 - 9.81t$

$t = 3.55$ s **Ans**

$v_x = 20$ km/h $= 5.556$ m/s

$v = \sqrt{(31.50)^2 + (5.556)^2} = 32.0$ m/s **Ans**

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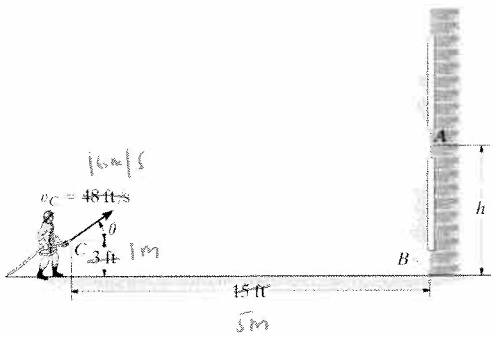
12–83. Determine the height on the wall to which the firefighter can project water from the hose, if $\theta = 40°$ and the speed of the water at the nozzle is v_C = ~~48 ft/s~~. 16 m/s



$(\xrightarrow{+})\; s = s_0 + v_0 t$

$15 = 0 + 48(\cos 40°)(t)$ 5, 16

$48 \sin\theta = 32.2 \dfrac{30}{48\cos\theta}$ 16, 9.81

$t = 0.40794$ s

$(+\uparrow)\; s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$h - 3 = 0 + 48 \sin 40° \;(0.40794) + \frac{1}{2}(-32.2)(0.40794)^2$ 1, 16, 9.81

h = ~~12.9 ft~~ **Ans**

4.38m

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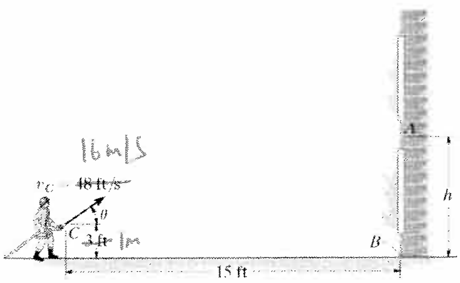
■***12-84.** Determine the smallest angle θ, measured above the horizontal, that the hose should be directed so that the water stream strikes the bottom of the wall at B. The speed of the water at the nozzle is v_C = ~~48 ft/s~~. 16 m/s



$(\xrightarrow{+})\, s = s_0 + v_0 t$

$15 = 0 + 48 \cos\theta\; t$ 5, 16

$t = \dfrac{15}{48\cos\theta} = \dfrac{5}{16\cos\theta}$

$(+\uparrow)\; s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$0 = 3 + 48 \sin\theta\; t + \frac{1}{2}(-32.2) t^2$ 1, 16, 9.81

$0 = 3 + \dfrac{48 \sin\theta\,(15)}{48\cos\theta} - 16.1\left(\dfrac{15}{48\cos\theta}\right)^2$ 1, 16, 5, 16, 4.905, 5, 16

~~$0 = 3\cos^2\theta + 15\sin\theta\cos\theta - 1.5723$~~

~~$3\cos^2\theta + 7.5\sin 2\theta = 1.5723$~~ $\cos^2\theta + 5\sin\theta\cos\theta = 0.479$

Solving

θ = ~~84.1°~~ 84.6° **Ans**

~~84.3~~

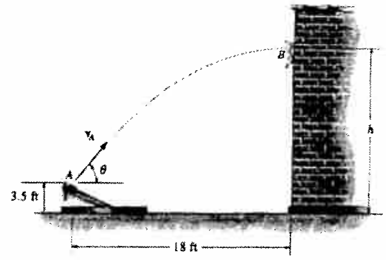
$3(48\cos\theta)^2 + 48\sin\theta\cos\theta \cdot 15 - \dfrac{16.1 \times 15 \times 15}{48 \times 48}$

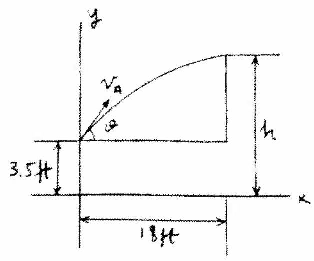
~~$3\cos^2\theta + 15\sin\theta\cos\theta$~~ –

$1(16\cos\theta)^2 + 16 \cdot \sin\theta\cos\theta \cdot 5 - \dfrac{4.905 \times 5 \times 5}{16 \times 16}$

$\cos\theta^2 + 5\sin\theta\cos\theta - 0.479$

12-85. The catapult is used to launch a ball such that it strikes the wall of the building at the maximum height of its trajectory. If it takes 1.5 s to travel from A to B, determine the velocity $\mathbf{v}_A$ at which it was launched, the angle of release θ, and the height h.





$(\xrightarrow{+})\quad s = v_0 t$

$18 = v_A \cos\theta(1.5) \quad (1)$

$(+\uparrow)\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$0 = (v_A \sin\theta)^2 + 2(-32.2)(h - 3.5)$

$(+\uparrow)\quad v = v_0 + a_c t$

$0 = v_A \sin\theta - 32.2(1.5) \quad (2)$

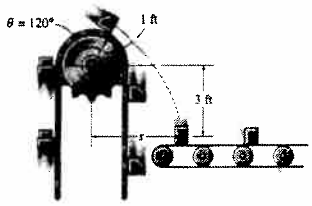
To solve, first divide Eq. (2) by Eq. (1), to get θ. Then

$\theta = 76.0^\circ$ **Ans**

$v_A = 49.8$ ft/s **Ans**

$h = 39.7$ ft **Ans**

12-86. The buckets on the conveyor travel with a speed of 15 ft/s. Each bucket contains a block which falls out of the bucket when $\theta = 120^\circ$. Determine the distance s to where the block strikes the conveyor. Neglect the size of the block.



Vertical Motion :

$(+\downarrow)\quad s_y = (s_0)_y + v_y t + \frac{1}{2}a_c t^2$

$3 + 1\cos 30^\circ = 0 + 15\sin 30^\circ t + \frac{1}{2}(32.2)t^2$

Take the positive root $\quad t = 0.3096$ s

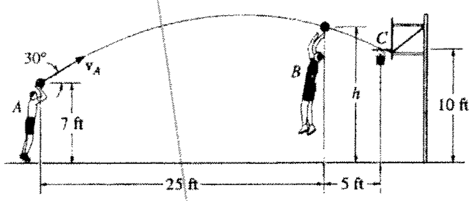
Horizontal Motion :

$(\xrightarrow{+})\quad s_x = (s_0)_x + v_x t$

$s - 1\sin 30^\circ = 0 + 15\cos 30^\circ(0.3096)$

$s = 4.52$ ft **Ans**

12-87. Measurements of a shot recorded on a videotape during a basketball game are shown. The ball passed through the hoop even though it barely cleared the hands of the player B who attempted to block it. Neglecting the size of the ball, determine the magnitude v_A of its initial velocity and the height h of the ball when it passes over player B.



$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$$30 = 0 + v_A \cos 30^\circ\, t_{AC}$$

$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$10 = 7 + v_A \sin 30^\circ\, t_{AC} - \frac{1}{2}(32.2)(t_{AC}^2)$$

Solving

$v_A = 36.73 = 36.7$ ft/s **Ans**

$t_{AC} = 0.943$ s

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$$25 = 0 + 36.73 \cos 30^\circ\, t_{AB}$$

$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

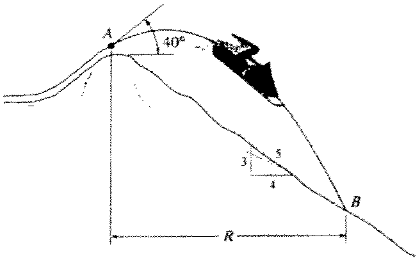
$$h = 7 + 36.73 \sin 30^\circ\, t_{AB} - \frac{1}{2}(32.2)(t_{AB}^2)$$

Solving

$t_{AB} = 0.786$ s

$h = 11.5$ ft **Ans**

***12-88.** The snowmobile is traveling at 10 m/s when it leaves the embankment at A. Determine the time of flight from A to B and the range R of the trajectory.



$(\overset{+}{\rightarrow})\quad s_B = s_A + v_A t$

$$R = 0 + 10 \cos 40^\circ\, t$$

$(+\uparrow)\quad s_B = s_A + v_A t + \frac{1}{2}a_c t^2$

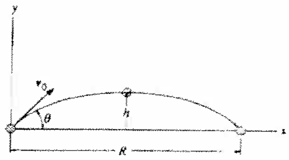
$$-R(\frac{3}{4}) = 0 + 10 \sin 40^\circ\, t - \frac{1}{2}(9.81)t^2$$

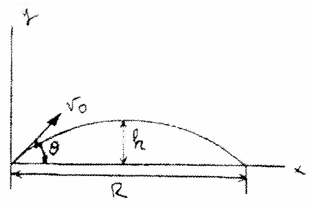
Solving :

$R = 19.0$ m **Ans**

$t = 2.48$ s **Ans**

12-89. The projectile is launched with a velocity $\mathbf{v}_0$. Determine the range R, the maximum height h attained, and the time of flight. Express the results in terms of the angle θ and v_0. The acceleration due to gravity is g.





$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$R = 0 + (v_0 \cos\theta)t$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$0 = 0 + (v_0 \sin\theta)t + \frac{1}{2}(-g)t^2$

$0 = v_0 \sin\theta - \frac{1}{2}(g)\left(\frac{R}{v_0 \cos\theta}\right)$

$R = \frac{v_0^2}{g}\sin 2\theta$ **Ans**

$t = \frac{R}{v_0 \cos\theta} = \frac{v_0^2(2\sin\theta\cos\theta)}{v_0 g \cos\theta}$

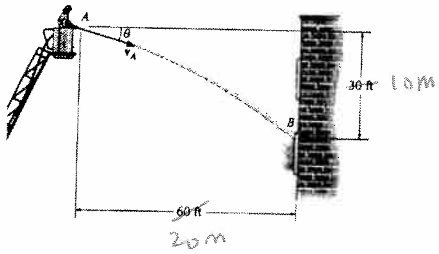
$= \frac{2v_0}{g}\sin\theta$ **Ans**

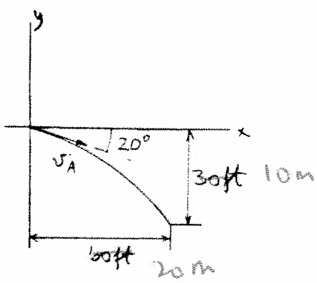
$(+\uparrow)$ $\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$0 = (v_0 \sin\theta)^2 + 2(-g)(h - 0)$

$h = \frac{v_0^2}{2g}\sin^2\theta$ **Ans**

12-90. The fireman standing on the ladder directs the flow of water from his hose to the fire at B. Determine the velocity of the water at A if it is observed that the hose is held at $\theta = 20°$.





$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$60 = 0 + (v_A \cos 20°)t$ 20

$t = \frac{63.851}{v_A}$ $\frac{18.794}{v_a}$ $\frac{21.28}{v_a}$

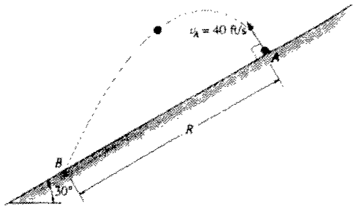
$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$-30 = 0 - v_A \sin 20°\left(\frac{63.851}{v_A}\right) + \frac{1}{2}(-32.2)\left(\frac{63.851}{v_A}\right)^2$ 10 21.28 9.81 21.28

$v_A = 89.7$ ft/s **Ans** 14.02 m/s 28.57 m/s

$-10V_A^2 = -21.28 V_a^2 \times \sin 20° - 2221.2$

12-91. A ball bounces on the 30° inclined plane such that it rebounds perpendicular to the incline with a velocity of $v_A = 40$ ft/s. Determine the distance R to where it strikes the plane at B.



$(v_A)_x = 40 \sin 30° = 20$ ft/s

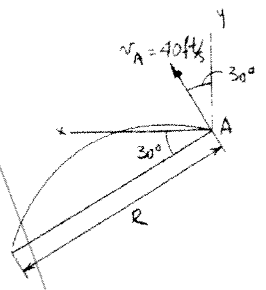
$(v_A)_y = 40 \cos 30° = 34.64$ ft/s

$(\overset{+}{\leftarrow})$ $\quad s = s_0 + v_0 t$

$R \cos 30° = 0 + 20t$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$-R \sin 30° = 0 + 34.64t + \frac{1}{2}(-32.2)t^2$



Combining the equations,

$20t \tan 30° = -34.64t + 16.1t^2$

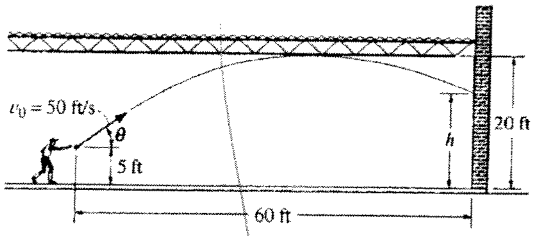
$46.19 = 16.1t$

$t = 2.87$ s

Thus,

$R = \frac{20(2.87)}{\cos 30°} = 66.3$ ft **Ans**

***12-92.** The man stands 60 ft from the wall and throws a ball at it with a speed $v_0 = 50$ ft/s. Determine the angle θ at which he should release the ball so that it strikes the wall at the highest point possible. What is this height? The room has a ceiling height of 20 ft.



$$v_x = 50\cos\theta$$

$$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$$

$$x = 0 + 50\cos\theta\ t \qquad (1)$$

$$(+\uparrow)\ v = v_0 + a_c t$$

$$v_y = 50\sin\theta - 32.2\,t \qquad (2)$$

$$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$y = 0 + 50\sin\theta\ t - 16.1\,t^2 \qquad (3)$$

$$(+\uparrow)\ v^2 = v_0^2 + 2a_c(s - s_0)$$

$$v_y^2 = (50\sin\theta)^2 + 2(-32.2)(s - 0)$$

$$v_y^2 = 2500\sin^2\theta - 64.4\,s \qquad (4)$$

Require $v_y = 0$ at $s = 20 - 5 = 15$ ft

$$0 = 2500\sin^2\theta - 64.4(15)$$

$$\theta = 38.433° = 38.4° \qquad \textbf{Ans}$$

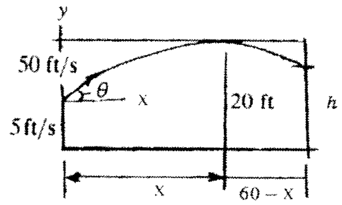
From Eq. (2)

$$0 = 50\sin 38.433° - 32.2\,t$$

$$t = 0.9652\ \text{s}$$

From Eq.(1)

$$x = 50\cos 38.433°(0.9652) = 37.8\ \text{ft}$$



Time for ball to hit wall

From Eq. (1),

$$60 = 50(\cos 38.433°)t$$

$$t = 1.53193\ \text{s}$$

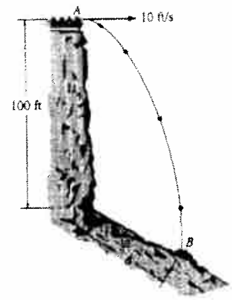
From Eq. (3)

$$y = 50\sin 38.433°(1.53193) - 16.1(1.53193)^2$$

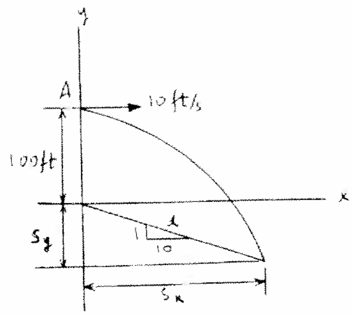
$$y = 9.830\ \text{ft}$$

$$h = 9.830 + 5 = 14.8\ \text{ft} \qquad \textbf{Ans}$$

12-93. The stones are thrown off the conveyor with a horizontal velocity of 10 ft/s as shown. Determine the distance d down the slope to where the stones hit the ground at B.



Place origin at A.



$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$s_x = 0 + 10t$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$-s_y = 100 + 0 + \frac{1}{2}(-32.2)t^2$

$s_y = 16.1t^2 - 100$

$\frac{s_y}{s_x} = \frac{1}{10}, \quad s_x = 10 s_y$

$s_y = \frac{10t}{10} = t$

$t = 16.1t^2 - 100$

$16.1t^2 - 1t - 100 = 0$

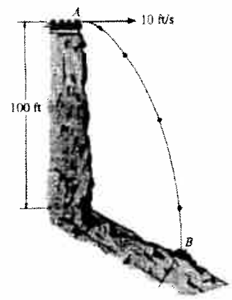
Solving for the positive root, $\quad t = 2.5235$ s

$s_x = 10(2.5235) = 25.235$ ft

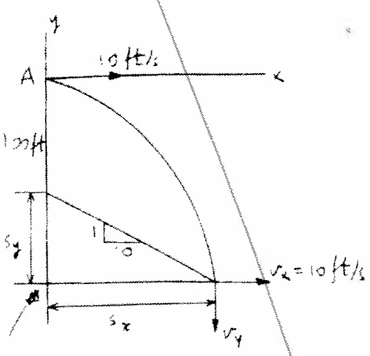
$s_y = 16.1(2.5235)^2 - 100 = 2.5235$ ft

$d = \sqrt{(25.235)^2 + (2.5235)^2} = 25.4$ ft **Ans**

12-94. The stones are thrown off the conveyor with a horizontal velocity of 10 ft/s as shown. Determine the speed at which the stones hit the ground at B.



Place origin at A.



$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$s_x = 0 + 10t$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$-100 - s_y = 0 + \frac{1}{2}(-32.2)t^2$

$\frac{s_y}{s_x} = \frac{1}{10}, \quad s_x = 10 s_y$

$10t = 10 s_y, \quad s_y = t$

$-100 - t = -16.1t^2$

$16.1t^2 - 1t - 100 = 0$

Solving for the positive root, $\quad t = 2.5235$ s

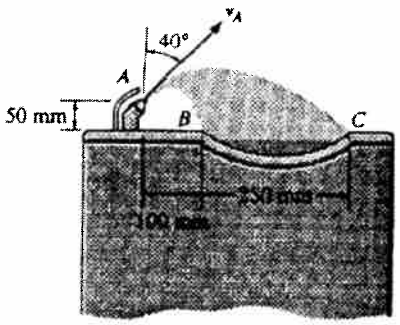
$(+\uparrow)$ $\quad v_y = (v_0)_y + a_c t$

$v_y = 0 - 32.2(2.5235) = -81.256$ ft/s

$v_x = 10$ ft/s

$v = \sqrt{(10)^2 + (-81.256)^2} = 81.9$ ft/s **Ans**

12-95. The drinking fountain is designed such that the nozzle is located from the edge of the basin as shown. Determine the maximum and minimum speed at which water can be ejected from the nozzle so that it does not splash over the sides of the basin at B and C.



Horizontal Motion :

$$(\xrightarrow{+}) \qquad s = v_0 t$$

$$R = v_A \sin 40^\circ t \qquad t = \frac{R}{v_A \sin 40^\circ} \qquad (1)$$

Vertical Motion :

$$(+\uparrow) \qquad s = s_0 + v_0 t + \tfrac{1}{2} a_c t^2$$

$$-0.05 = 0 + v_A \cos 40^\circ t + \tfrac{1}{2}(-9.81)t^2 \qquad (2)$$

Substituting Eq. (1) into (2) yields :

$$-0.05 = v_A \cos 40^\circ \left(\frac{R}{v_A \sin 40^\circ}\right) + \frac{1}{2}(-9.81)\left(\frac{R}{v_A \sin 40^\circ}\right)^2$$

$$v_A = \sqrt{\frac{4.905R^2}{\sin 40^\circ (R \cos 40^\circ + 0.05 \sin 40^\circ)}}$$

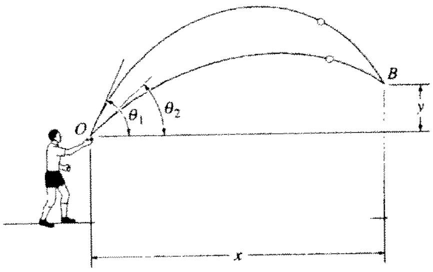
At point B, $R = 0.1$ m.

$$v_{min} = v_A = \sqrt{\frac{4.905(0.1)^2}{\sin 40^\circ (0.1 \cos 40^\circ + 0.05 \sin 40^\circ)}} = 0.838 \text{ m/s} \qquad \textbf{Ans}$$

At point C, $R = 0.35$ m.

$$v_{max} = v_A = \sqrt{\frac{4.905(0.35)^2}{\sin 40^\circ (0.35 \cos 40^\circ + 0.05 \sin 40^\circ)}} = 1.76 \text{ m/s} \qquad \textbf{Ans}$$

***12-96.** A boy at O throws a ball in the air with a speed v_O at an angle θ_1. If he then throws another ball at the same speed v_O at an angle $\theta_2 < \theta_1$, determine the time between the throws so the balls collide in mid air at B.



Time of flight

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$$x_1 = 0 + v_0 \cos\theta_1\ t_1 \qquad (1)$$

$$x_2 = 0 + v_0 \cos\theta_2\ t_2 \qquad (2)$$

Thus,

$x_1 = x_2$

$$\Delta t = t_1 - t_2 = \frac{x_1}{v_0}\left(\frac{\cos\theta_2 - \cos\theta_1}{\cos\theta_1 \cos\theta_2}\right) \qquad (3)$$

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$y = 0 + (v_0 \sin\theta_1)(t_1) - \frac{1}{2} g\, t_1^2 \qquad (4)$$

Use Eq. (1)

$$y = x_1 \tan\theta_1 - \frac{1}{2} g \frac{x_1^2}{v_0^2 \cos^2\theta_1}$$

In the same way :

$$y = x_2 \tan\theta_2 - \frac{1}{2} g \frac{x_2^2}{v_0^2 \cos^2\theta_2}$$

Equating and solving for $x_1 = x_2 = x$

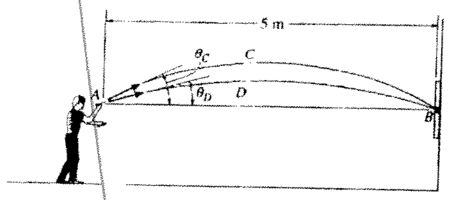
$$x = \frac{2v_0^2}{g}\left[\frac{(\cos^2\theta_1 \cos^2\theta_2)(\tan\theta_1 - \tan\theta_2)}{(\cos^2\theta_2 - \cos^2\theta_1)}\right]$$

Substituting into Eq. (3) yields

$$\Delta t = \frac{2v_0}{g}\left[\frac{(\cos\theta_1 \cos\theta_2)(\tan\theta_1 - \tan\theta_2)}{(\cos\theta_2 + \cos\theta_1)}\right]$$

$$\Delta t = \frac{2v_0}{g}\left[\frac{\sin(\theta_1 - \theta_2)}{\cos\theta_2 + \cos\theta_1}\right] \qquad \textbf{Ans}$$

12-97. The man at A wishes to throw two darts at the target at B so that they arrive at the *same time*. If each dart is thrown with a speed of 10 m/s, determine the angles θ_C and θ_D at which they should be thrown and the time between each throw. Note that the first dart must be thrown at $\theta_C (>\theta_D)$, then the second dart is thrown at θ_D.



$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$5 = 0 + (10 \cos \theta)\, t \qquad (1)$

$(+\uparrow)\ v = v_0 + a_c t$

$-10 \sin\theta = 10 \sin\theta - 9.81\, t$

$t = \dfrac{2(10 \sin\theta)}{9.81} = 2.039 \sin\theta$

From Eq. (1),

$5 = 20.39 \sin\theta \cos\theta$

Since $\quad \sin 2\theta = 2 \sin\theta \cos\theta$

$\sin 2\theta = 0.4905$

The two roots are $\theta_D = 14.7°$ **Ans**

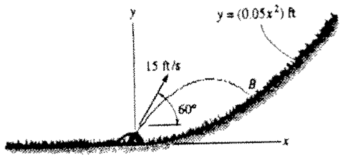
$\theta_C = 75.3°$ **Ans**

From Eq.(1) : $\quad t_D = 0.517$ s

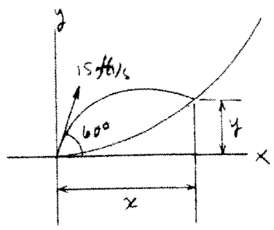
$t_C = 1.97$ s

So that $\qquad \Delta t = t_C - t_D = 1.45$ s **Ans**

12-98. The water sprinkler, positioned at the base of a hill, releases a stream of water with a velocity of 15 ft/s as shown. Determine the point $B(x, y)$ where the water strikes the ground on the hill. Assume that the hill is defined by the equation $y = (0.05x^2)$ ft and neglect the size of the sprinkler.



$v_x = 15\cos 60° = 7.5$ ft/s $\quad v_y = 15\sin 60° = 12.99$ ft/s



$(\overset{+}{\rightarrow})\quad s = v_0 t$

$$x = 7.5t$$

$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$y = 0 + 12.99t + \frac{1}{2}(-32.2)t^2$$

$$y = 1.732x - 0.286x^2$$

Since $y = 0.05x^2$,

$0.05x^2 = 1.732x - 0.286x^2$

$x(0.336x - 1.732) = 0$

$x = 5.15$ ft **Ans**

$y = 0.05(5.15)^2 = 1.33$ ft **Ans**

Also,

$(\overset{+}{\rightarrow})\quad s = v_0 t$

$$x = 15\cos 60° t$$

$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$y = 0 + 15\sin 60° t + \frac{1}{2}(-32.2)t^2$$

Since $y = 0.05x^2$

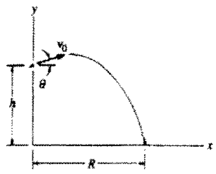
$12.99t - 16.1t^2 = 2.8125t^2 \qquad t = 0.6869$ s

So that,

$x = 15\cos 60°(0.6868) = 5.15$ ft **Ans**

$y = 0.05(5.15)^2 = 1.33$ ft **Ans**

12-99. The projectile is launched from a height h with a velocity $\mathbf{v}_0$. Determine the range R.



$(\xrightarrow{+})\quad s = s_0 + v_0 t$

$$R = 0 + (v_0 \cos\theta)t$$

$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$0 = h + (v_0 \sin\theta)t + \frac{1}{2}(-g)t^2$$

Thus,

$$0 = h + v_0 \sin\theta\left(\frac{R}{v_0\cos\theta}\right) - \frac{1}{2}g\left(\frac{R^2}{v_0^2\cos^2\theta}\right)$$

$$0 = 2v_0^2 h\cos^2\theta + Rv_0^2\sin 2\theta - gR^2$$

$$R^2 - R\left(\frac{v_0^2\sin 2\theta}{g}\right) - \left(\frac{2v_0^2 h\cos^2\theta}{g}\right) = 0$$

For positive R;

$$R = \frac{v_0^2\sin 2\theta}{2g} + \frac{1}{2}\sqrt{\left(\frac{v_0^2\sin 2\theta}{g}\right)^2 + \left(\frac{8v_0^2 h\cos^2\theta}{g}\right)}$$

$$R = \left(\frac{v_0^2\sin 2\theta}{2g}\right) + \frac{v_0}{2g}\sqrt{v_0^2\sin^2 2\theta + 8gh\cos^2\theta} \qquad \textbf{Ans}$$

Note: For $h = 0$, $R = \left(\frac{v_0^2\sin 2\theta}{g}\right)$, and for $\theta = 90°$, $R = 0$.

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*12-100. A car is traveling along a circular curve that has a radius of 50 m. If its speed is 16 m/s and is increasing uniformly at 8 m/s², determine the magnitude of its acceleration at this instant.

$$v = 16 \text{ m/s}$$

$$a_t = 8 \text{ m/s}^2$$

$$r = 50 \text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{(16)^2}{50} = 5.12 \text{ m/s}^2$$

$$a = \sqrt{(8)^2 + (5.12)^2} = 9.50 \text{ m/s}^2 \qquad \textbf{Ans}$$

12-101. A car moves along a circular track of radius 250 ft such that its speed for a short period of time $0 \le t \le 4$ s, is $v = 3(t + t^2)$ ft/s, where t is in seconds. Determine the magnitude of its acceleration when $t = 3$ s. How far has it traveled in $t = 3$ s?

$v = 3(t + t^2)$

$a_t = \frac{dv}{dt} = 3 + 6t$

When $t = 3$ s, $a_t = 3 + 6(3) = 21$ ft/s²

$a_n = \frac{[3(3+3^2)]^2}{250} = 5.18$ ft/s²

$a = \sqrt{(21)^2 + (5.18)^2} = 21.6$ ft/s² **Ans**

$\int ds = \int_0^3 3(t + t^2)\, dt$

$\Delta s = \frac{3}{2}t^2 + t^3\Big|_0^3$

$\Delta s = 40.5$ ft **Ans**

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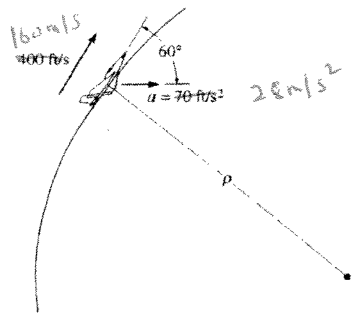
12-102. At a given instant the jet plane has a speed of ~~400 ft/s~~ 160 m/s and an acceleration of ~~70 ft/s²~~ 28 m/s² acting in the direction shown. Determine the rate of increase in the plane's speed and the radius of curvature ρ of the path.



$a_t = \cancel{70}\ 28 \cos 60° = \cancel{35.0 \text{ ft/s}^2}$ 14.0 m/s² **Ans**

$a_n = \frac{(\cancel{400}\ 160)^2}{\rho} = \cancel{70}\ 28 \sin 60°$

$\rho = \cancel{2.64(10^3) \text{ ft}}$ 1056 m **Ans**

12-103. A particle is moving along a curved path at a constant speed of 60 ft/s. The radii of curvature of the path at points P and P' are 20 and 50 ft, respectively. If it takes the particle 20 s to go from P to P', determine the acceleration of the particle at P and P'.

$a_t = 0$

$a_P = (a_n)_P = \frac{v^2}{\rho_P} = \frac{60^2}{20} = 180$ ft/s² **Ans**

$a_{P'} = (a_n)_{P'} = \frac{v^2}{\rho_{P'}} = \frac{60^2}{50} = 72$ ft/s² **Ans**

***12-104.** A boat is traveling along a circular path having a radius of 20 m. Determine the magnitude of the boat's acceleration if at a given instant the boat's speed is $v = 5$ m/s and the rate of increase in the speed is $\dot{v} = 2\ \text{m/s}^2$.

$$a_t = 2\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{5^2}{20} = 1.25\ \text{m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2^2 + 1.25^2} = 2.36\ \text{m/s}^2 \qquad \textbf{Ans}$$

•12-105. Starting from rest, a bicyclist travels around a horizontal circular path, $\rho = 10$ m, at a speed of $v = (0.09t^2 + 0.1t)$ m/s, where t is in seconds. Determine the magnitudes of his velocity and acceleration when he has traveled $s = 3$ m.

$$\int_0^s ds = \int_0^t (0.09t^2 + 0.1t)\,dt$$

$$s = 0.03t^3 + 0.05t^2$$

When $s = 3$ m, $\quad 3 = 0.03t^3 + 0.05t^2$

Solving,

$$t = 4.147\ \text{s}$$

$$v = \frac{ds}{dt} = 0.09t^2 + 0.1t$$

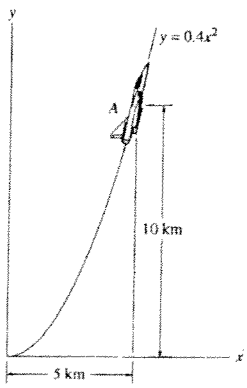
$$v = 0.09(4.147)^2 + 0.1(4.147) = 1.96\ \text{m/s} \qquad \textbf{Ans}$$

$$a_t = \frac{dv}{dt} = 0.18t + 0.1\Big|_{t=4.147\ \text{s}} = 0.8465\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{1.96^2}{10} = 0.3852\ \text{m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(0.8465)^2 + (0.3852)^2} = 0.930\ \text{m/s}^2 \qquad \textbf{Ans}$$

12-106. The jet plane travels along the vertical parabolic path. When it is at point A it has a speed of 200 m/s, which is increasing at the rate of $0.8\ \text{m/s}^2$. Determine the magnitude of acceleration of the plane when it is at point A.



$$y = 0.4x^2$$

$$\frac{dy}{dx} = 0.8x\Big|_{x=5\ \text{km}} = 4$$

$$\frac{d^2y}{dx^2} = 0.8$$

$$\rho = \frac{[1 + (4)^2]^{3/2}}{0.8} = 87.62\ \text{km}$$

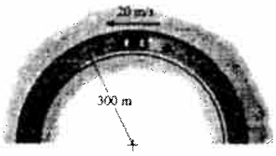
$$a_t = 0.8\ \text{m/s}^2$$

$$a_n = \frac{(0.200)^2}{87.62} = 0.457(10^{-3})\ \text{km/s}^2$$

$$a_n = 0.457\ \text{m/s}^2$$

$$a = \sqrt{(0.8)^2 + (0.457)^2} = 0.921\ \text{m/s}^2 \qquad \textbf{Ans}$$

12-107. The car travels along the curve having a radius of 300 m. If its speed is uniformly increased from 15 m/s to 27 m/s in 3 s, determine the magnitude of its acceleration at the instant its speed is 20 m/s.



$$a_n = \frac{v^2}{\rho} = \frac{(20)^2}{300} = 1.33 \text{ m/s}^2$$

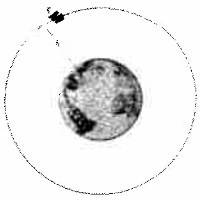
$$v = v_0 + a_c t$$

$$27 = 15 + a_t(3)$$

$$a_t = 4 \text{ m/s}^2$$

$$a = \sqrt{(1.33)^2 + 4^2} = 4.22 \text{ m/s}^2 \quad \textbf{Ans}$$

***12-108.** The satellite S travels around the earth in a circular path with a constant speed of 20 Mm/h. If the acceleration is 2.5 m/s², determine the altitude h. Assume the earth's diameter to be 12 713 km.



$$v = 20 \text{ Mm/h} = \frac{20(10^6)}{3600} = 5.56(10^3) \text{ m/s}$$

Since $a_t = \frac{dv}{dt} = 0$, then,

$$a = a_n = 2.5 = \frac{v^2}{\rho}$$

$$\rho = \frac{(5.56(10^3))^2}{2.5} = 12.35(10^6) \text{ m}$$

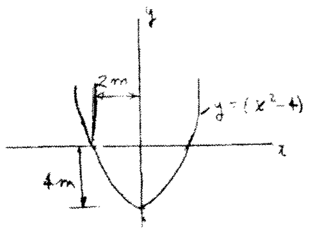
The radius of the earth is

$$\frac{12\,713(10^3)}{2} = 6.36(10^6) \text{ m}$$

Hence,

$$h = 12.35(10^6) - 6.36(10^6) = 5.99(10^6) \text{ m} = 5.99 \text{ Mm} \quad \textbf{Ans}$$

12-109. A particle P moves along the curve $y = (x^2 - 4)$ m with a constant speed of 5 m/s. Determine the point on the curve where the maximum magnitude of acceleration occurs and compute its value.



$y = (x^2 - 4)$

$a_t = \frac{dv}{dt} = 0,$

To obtain maximum $a = a_n$, ρ must be a minimum.
This occurs at :

$x = 0, \quad y = -4$ m **Ans**

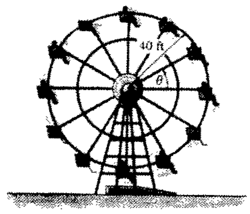
Hence,

$\left.\frac{dy}{dx}\right|_{x=0} = 2x = 0; \quad \frac{d^2y}{dx^2} = 2$

$$\rho_{min} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1+0]^{\frac{3}{2}}}{|2|} = \frac{1}{2}$$

$$(a)_{max} = (a_n)_{max} = \frac{v^2}{\rho_{min}} = \frac{5^2}{\frac{1}{2}} = 50 \text{ m/s}^2$$ **Ans**

12-110. The Ferris wheel turns such that the speed of the passengers is increased by $\dot{v} = (4t)$ ft/s², where t is in seconds. If the wheel starts from rest when $\theta = 0°$, determine the magnitudes of the velocity and acceleration of the passengers when the wheel turns $\theta = 30°$.



$\int_0^v dv = \int_0^t 4t\,dt$

$v = 2t^2$

$\int_0^s ds = \int_0^t 2t^2 dt$

$s = \frac{2}{3}t^3$

When $s = \frac{\pi}{6}(40)$ ft, $\quad \frac{\pi}{6}(40) = \frac{2}{3}t^3 \quad t = 3.1554$ s

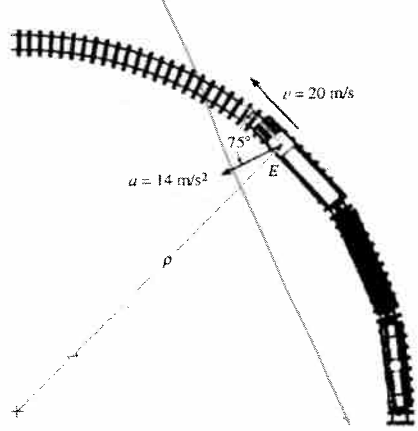
$v = 2(3.1554)^2 = 19.91$ ft/s $= 19.9$ ft/s **Ans**

$a_t = \dot{v} = 4t|_{t=3.1554\,s} = 12.62$ ft/s²

$a_n = \frac{v^2}{\rho} = \frac{19.91^2}{40} = 9.91$ ft/s²

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{12.62^2 + 9.91^2} = 16.0$ ft/s² **Ans**

12-111. At a given instant the train engine at E has a speed of 20 m/s and an acceleration of 14 m/s² acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature ρ of the path.



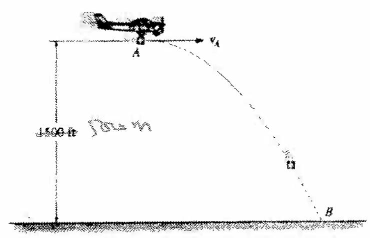
$a_t = 14\cos 75° = 3.62 \text{ m/s}^2$ **Ans**

$a_n = 14\sin 75°$

$a_n = \dfrac{(20)^2}{\rho}$

$\rho = 29.6 \text{ m}$ **Ans**

***12-112.** A package is dropped from the plane which is flying with a constant horizontal velocity of v_A = ~~150 ft/s~~ 50 m/s. Determine the normal and tangential components of acceleration and the radius of curvature of the path of motion (a) at the moment the package is released at A, where it has a horizontal velocity of v_A = ~~150 ft/s~~ 50 m/s, and (b) *just before* it strikes the ground at B.



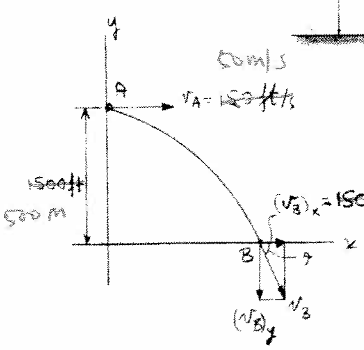
Initially (Point A):

$(a_n)_A = g =$ ~~32.2 ft/s²~~ 9.81 m/s² **Ans**

$(a_t)_A = 0$ **Ans**

$(a_n)_A = \dfrac{v_A^2}{\rho_A};\quad$ ~~32.2~~ 9.81 $= \dfrac{(\text{~~150~~ } 50)^2}{\rho_A}$

$\rho_A =$ ~~698.8 ft~~ 254.8 m **Ans**



$(v_B)_x = (v_A)_x =$ ~~150 ft/s~~ 50 m/s

$(+\downarrow)\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$(v_B)_y^2 = 0 + 2($~~32.2~~ 9.81$)($~~1500~~ 500 $- 0)$

$(v_B)_y =$ ~~340.8 ft/s~~ 99.05 m/s

$v_B = \sqrt{(\text{~~150~~ } 50)^2 + (\text{~~340.8~~ } 99.05)^2} =$ ~~345.1 ft/s~~ 111 m/s

$\theta = \tan^{-1}\left(\dfrac{v_{B_y}}{v_{B_x}}\right) = \tan^{-1}\left(\dfrac{\text{~~340.8~~ } 99.05}{\text{~~150~~ } 50}\right) =$ ~~64.23°~~ 63.2°

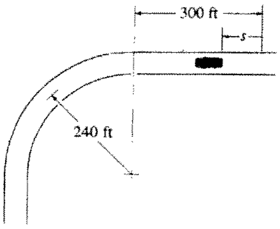
$(a_n)_B = g\cos\theta =$ ~~32.2~~ 9.81 $\cos$ ~~64.24°~~ 63.2° $=$ ~~14.0 ft/s²~~ 4.42 m/s² **Ans**

$(a_t)_B = g\sin\theta =$ ~~32.2~~ 9.81 $\sin$ ~~64.24°~~ 63.2° $=$ ~~29.0 ft/s²~~ 8.76 m/s² **Ans**

$(a_n)_B = \dfrac{v_B^2}{\rho_B};\quad$ ~~14.0~~ 4.42 $= \dfrac{(\text{~~345.1~~ } 111)^2}{\rho_B}$

$\rho_B =$ ~~8509.8 ft = 8.51(10³) ft~~ = 2788 m **Ans**

12-113. The automobile is originally at rest at $s = 0$. If its speed is increased by $\dot{v} = (0.05t^2)$ ft/s^2, where t is in seconds, determine the magnitudes of its velocity and acceleration when $t = 18$ s.



$a_t = 0.05\,t^2$

$$\int_0^v dv = \int_0^t 0.05\,t^2\,dt$$

$v = 0.0167\,t^3$

$$\int_0^s ds = \int_0^t 0.0167\,t^3\,dt$$

$s = 4.167(10^{-3})\,t^4$

When $t = 18$ s, $\quad s = 437.4$ ft

Therefore the car is on a curved path.

$v = 0.0167(18)^3 = 97.2$ ft/s **Ans**

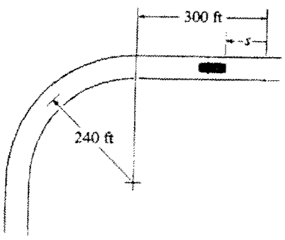
$$a_n = \frac{(97.2)^2}{240} = 39.37 \text{ ft/s}^2$$

$a_t = 0.05(18)^2 = 16.2$ ft/s^2

$a = \sqrt{(39.37)^2 + (16.2)^2}$

$a = 42.6$ ft/s^2 **Ans**

12-114. The automobile is originally at rest $s = 0$. If it then starts to increase its speed at $\dot{v} = (0.05t^2)$ ft/s^2, where t is in seconds, determine the magnitudes of its velocity and acceleration at $s = 550$ ft.



The car is on the curved path.

$a_t = 0.05\,t^2$

$$\int_0^v dv = \int_0^t 0.05\,t^2\,dt$$

$v = 0.0167\,t^3$

$$\int_0^s ds = \int_0^t 0.0167\,t^3\,dt$$

$s = 4.167(10^{-3})\,t^4$

$550 = 4.167(10^{-3})\,t^4$

$t = 19.06$ s

So that

$v = 0.0167\,(19.06)^3 = 115.4$

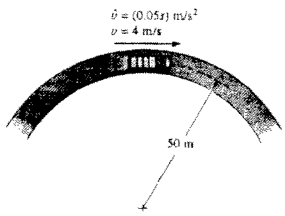
$v = 115$ ft/s **Ans**

$$a_n = \frac{(115.4)^2}{240} = 55.48 \text{ ft/s}^2$$

$a_t = 0.05(19.06)^2 = 18.16$ ft/s^2

$a = \sqrt{(55.48)^2 + (18.16)^2} = 58.4$ ft/s^2 **Ans**

12-115. The truck travels in a circular path having a radius of 50 m at a speed of 4 m/s. For a short distance from $s = 0$, its speed is increased by $\dot{v} = (0.05s)\ \text{m/s}^2$, where s is in meters. Determine its speed and the magnitude of its acceleration when it has moved $s = 10$ m.



$$v\,dv = a_t\,ds$$

$$\int_4^v v\,dv = \int_0^{10} 0.05s\,ds$$

$$0.5v^2 - 8 = \frac{0.05}{2}(10)^2$$

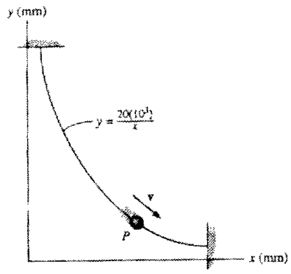
$$v = 4.583 = 4.58\ \text{m/s} \qquad \textbf{Ans}$$

$$a_n = \frac{v^2}{\rho} = \frac{(4.583)^2}{50} = 0.420\ \text{m/s}^2$$

$$a_t = 0.05(10) = 0.5\ \text{m/s}^2$$

$$a = \sqrt{(0.420)^2 + (0.5)^2} = 0.653\ \text{m/s}^2 \qquad \textbf{Ans}$$

***12-116.** The particle travels with a constant speed of 300 mm/s along the curve. Determine the particle's acceleration when it is located at point (200 mm, 100 mm) and sketch this vector on the curve.



$$v = 300\ \text{mm/s}$$

$$a_t = \frac{dv}{dt} = 0$$

$$y = \frac{20(10^3)}{x}$$

$$\left.\frac{dy}{dx}\right|_{x=200} = -\frac{20(10^3)}{x^2} = -0.5$$

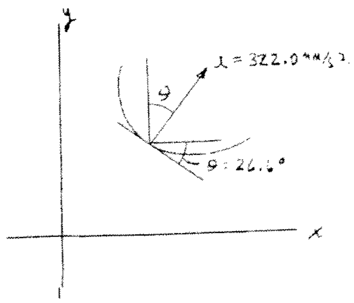
$$\left.\frac{d^2y}{dx^2}\right|_{x=200} = \frac{40(10^3)}{x^3} = 5(10^{-3})$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (-0.5)^2\right]^{\frac{3}{2}}}{|5(10^{-3})|} = 279.5\ \text{mm}$$

$$a_n = \frac{v^2}{\rho} = \frac{(300)^2}{279.5} = 322\ \text{mm/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2}$$

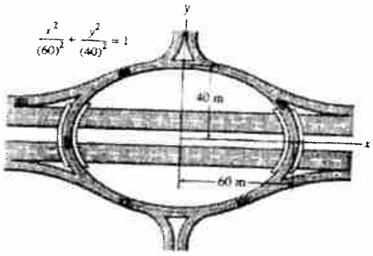
$$= \sqrt{(0)^2 + (322)^2} = 322\ \text{mm/s}^2 \qquad \textbf{Ans}$$



Since $\frac{dy}{dx} = -0.5$,

$$\theta = \tan^{-1}(-0.5) = -26.6^\circ$$

12-117. Cars move around the "traffic circle" which is in the shape of an ellipse. If the speed limit is posted at 60 km/h, determine the maximum acceleration experienced by the passengers.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$b^2(2x) + a^2(2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{b^2 x}{a^2 y}$$

$$\frac{dy}{dx}y = \frac{-b^2 x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \left(\frac{-b^2 x}{a^2 y}\right)^2$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{-b^2x}{a^2y}\right)^2\right]^{3/2}}{\left|\frac{-b^4}{a^2y^3}\right|}$$

At $x = a$, $y = 0$,

$$\rho = \frac{b^2}{a}$$

Then

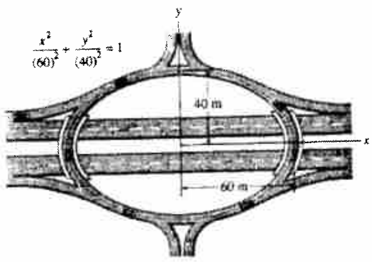
$$a_t = 0$$

$$a_{max} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{b^2}{a}} = \frac{v^2 a}{b^2}$$

Set $a = 60$ m, $b = 40$ m, $v = \frac{60(10^3)}{3600} = 16.67$ m/s

$$a_{max} = \frac{(16.67)^2(60)}{(40)^2} = 10.4 \text{ m/s}^2 \quad \textbf{Ans}$$

12-118. Cars move around the "traffic circle" which is in the shape of an ellipse. If the speed limit is posted at 60 km/h, determine the minimum acceleration experienced by the passengers.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$b^2(2x) + a^2(2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{dy}{dx}y = \frac{-b^2x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \left(\frac{-b^2x}{a^2y}\right)^2$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \left(\frac{b^4}{a^2y^2}\right)\left(\frac{x^2}{a^2}\right)$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \frac{b^4}{a^2y^2}\left(1 - \frac{y^2}{b^2}\right)$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \frac{b^4}{a^2y^2} + \frac{b^2}{a^2}$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{-b^2x}{a^2y}\right)^2\right]^{3/2}}{\left|\frac{-b^4}{a^2y^3}\right|}$$

At $x = 0$, $y = b$,

$$\rho = \frac{a^2}{b}$$

Thus

$$a_t = 0$$

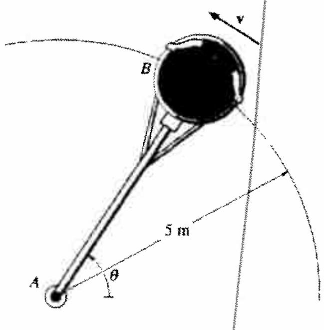
$$a_{min} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{a^2}{b}} = \frac{v^2b}{a^2}$$

Set $a = 60$ m, $b = 40$ m,

$$v = \frac{60(10)^3}{3600} = 16.67 \text{ m/s}$$

$$a_{min} = \frac{(16.67)^2(40)}{(60)^2} = 3.09 \text{ m/s}^2 \quad \textbf{Ans}$$

■12-119. The car B turns such that its speed is increased by $\dot{v}_B = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. If the car starts from rest when $\theta = 0°$, determine the magnitudes of its velocity and acceleration when the arm AB rotates $\theta = 30°$. Neglect the size of the car.



$$\frac{dv_B}{dt} = 0.5\,e^t$$

$$\int_0^{v_B} dv_B = \int_0^t 0.5\,e^t\,dt$$

$$v_B = 0.5(e^t - 1)$$

$$\int_0^{s_B} ds_B = \int_0^t 0.5(e^t - 1)\,dt$$

$$s_B = 0.5\,(e^t - t)|_0^t = 0.5(e^t - t - 1)$$

At $\theta = 30°$,

$$s_B = \left(\frac{30°}{180°}\pi\right)(5) = 2.618\text{ m}$$

Thus,

$$6.236 = (e^t - t)$$

Solving by trial and error,

$$t = 2.123\text{ s}$$

Thus,

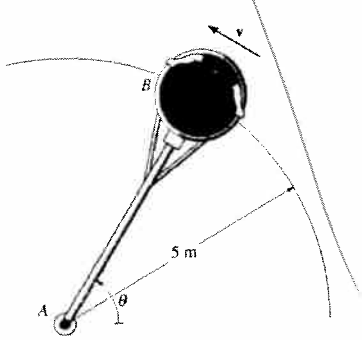
$$v_B = 0.5(e^{2.123} - 1) = 3.678 = 3.68\text{ m/s}$$ **A**

$$(a_B)_t = \dot{v}_B = 0.5(e^{2.123}) = 4.178\text{ m/s}^2$$

$$(a_B)_n = \frac{v_B^2}{5} = \frac{(3.678)^2}{5} = 2.706\text{ m/s}^2$$

$$a_B = \sqrt{(4.178)^2 + (2.706)^2} = 4.98\text{ m/s}^2$$ **Ans**

***12-120.** The car B turns such that its speed is increased by $\dot{v}_B = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. If the car starts from rest when $\theta = 0°$, determine the magnitudes of its velocity and acceleration when $t = 2$ s. Neglect the size of the car. Also, through what angle θ has it traveled?



$$\int_0^v dv = \int_0^t 0.5\,e^t\,dt$$

$$v = 0.5\,e^t\Big|_0^t = 0.5\,(e^t - 1)$$

$t = 2$ s,

$$v = 0.5(e^2 - 1) = 3.1945 = 3.19\ \text{m/s} \qquad \textbf{Ans}$$

$$a_t = 0.5\,e^2 = 3.6945\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(3.1945)^2}{5} = 2.041\ \text{m/s}^2$$

$$a = \sqrt{(3.6945)^2 + (2.041)^2} = 4.22\ \text{m/s}^2 \qquad \textbf{Ans}$$

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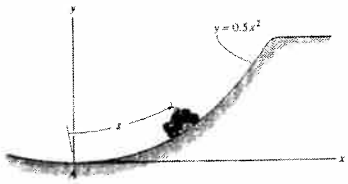
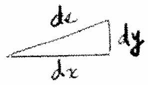
■12-121. The motorcycle is traveling at 1 m/s when it is at A. If the speed is then increased at $\dot{v} = 0.1\ \text{m/s}^2$, determine its speed and acceleration at the instant $t = 5$ s.





$$a_t = \dot{v} = 0.1$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s = 0 + 1(5) + \frac{1}{2}(0.1)(5)^2 = 6.25\ \text{m}$$

$$\int_0^{6.25} ds = \int_0^x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$y = 0.5x^2$$

$$\frac{dy}{dx} = x$$

$$\frac{d^2y}{dx^2} = 1$$

$$6.25 = \int_0^x \sqrt{1 + x^2}\, dx$$

$$6.25 = \frac{1}{2}\Big[x\sqrt{1+x^2} + \ln\left(x + \sqrt{1+x^2}\right)\Big]_0^x$$

$$x\sqrt{1+x^2} + \ln\left(x + \sqrt{1+x^2}\right) = 12.5$$

Solving,

$$x = 3.184\ \text{m}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + x^2\right]^{\frac{3}{2}}}{|1|}\Bigg|_{x=3.184} = 37.17\ \text{m}$$

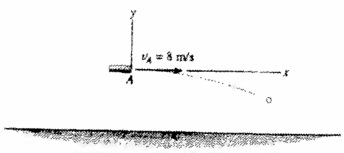
$$v = v_0 + a_c t$$

$$= 1 + 0.1(5) = 1.5\ \text{m/s} \qquad \textbf{Ans}$$

$$a_n = \frac{v^2}{\rho} = \frac{(1.5)^2}{37.17} = 0.0605\ \text{m/s}^2$$

$$a = \sqrt{(0.1)^2 + (0.0605)^2} = 0.117\ \text{m/s}^2 \qquad \textbf{Ans}$$

12-122. The ball is ejected horizontally from the tube with a speed of 8 m/s. Find the equation of the path, $y = f(x)$, and then find the ball's velocity and the normal and tangential components of acceleration when $t = 0.25$ s.



$v_x = 8$ m/s

$(\overset{+}{\rightarrow})$ $\quad s = v_0 t$

$x = 8t$

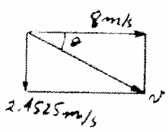
$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

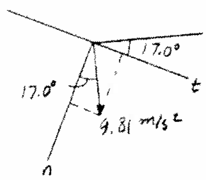
$y = 0 + 0 + \frac{1}{2}(-9.81)t^2$

$y = -4.905t^2$

$y = -4.905\left(\frac{x}{8}\right)^2$

$y = -0.0766x^2$ (Parabola) **Ans**





$v = v_0 + a_c t$

$v_y = 0 - 9.81t$

When $t = 0.25$ s,

$v_y = -2.4525$ m/s

$v = \sqrt{(8)^2 + (2.4525)^2} = 8.37$ m/s **Ans**

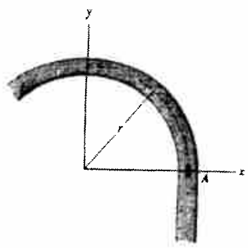
$\theta = \tan^{-1}\left(\frac{2.4525}{8}\right) = 17.04°$

$a_x = 0 \quad a_y = 9.81$ m/s^2

$a_n = 9.81 \cos 17.04° = 9.38$ m/s^2 **Ans**

$a_t = 9.81 \sin 17.04° = 2.88$ m/s^2 **Ans**

•12-123. The car travels around the circular track having a radius of $r = 300$ m such that when it is at point A it has a velocity of 5 m/s, which is increasing at the rate of $\dot{v} = (0.06t)$ m/s^2, where t is in seconds. Determine the magnitudes of its velocity and acceleration when it has traveled one-third the way around the track.



$a_t = \dot{v} = 0.06t$

$dv = a_t\, dt$

$\int_5^v dv = \int_0^t 0.06t\, dt$

$v = 0.03t^2 + 5$

$ds = v\, dt$

$\int_0^s ds = \int_0^t \left(0.03t^2 + 5\right) dt$

$s = 0.01t^3 + 5t$

$s = \frac{1}{3}(2\pi(300)) = 628.3185$

$0.01t^3 + 5t - 628.3185 = 0$

Solve for the positive root,

$t = 35.58$ s

$v = 0.03(35.58)^2 + 5 = 42.978$ m/s $= 43.0$ m/s **Ans**

$a_n = \frac{v^2}{\rho} = \frac{(42.978)^2}{300} = 6.157$ m/s^2

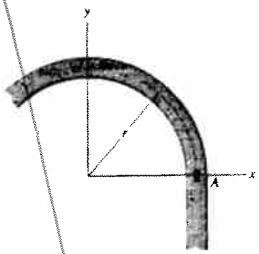
$a_t = 0.06(35.58) = 2.135$ m/s^2

$a = \sqrt{(6.157)^2 + (2.135)^2} = 6.52$ m/s^2 **Ans**

***12-124.** The car travels around the portion of a circular track having a radius of $r = 500$ ft such that when it is at point A it has a velocity of 2 ft/s, which is increasing at the rate of $\dot{v} = (0.002s)$ ft/s^2, where t is in seconds. Determine the magnitudes of its velocity and acceleration when it has traveled three-fourths the way around the track.

300 m

5 m/s



$a_t = 0.002\,s$

$a_t\,ds = v\,dv$

$\int_0^s 0.002s\,ds = \int_2^v v\,dv$

$0.001s^2 = \frac{1}{2}v^2 - \frac{1}{2}(2)^2$

$v^2 = 0.002s^2 + 4$

$s = \frac{3}{4}[2\pi(500)] = 2356.194$ ft

$v^2 = 0.002(2356.194)^2 + 4$

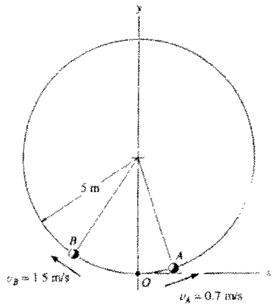
$v = 105.39$ ft/s $= 105$ ft/s **Ans**

$a_n = \frac{v^2}{\rho} = \frac{(105.39)^2}{500} = 22.21$ ft/s^2

$a_t = 0.002(2356.194) = 4.712$ ft/s^2

$a = \sqrt{(22.21)^2 + (4.712)^2} = 22.7$ ft/s^2 **Ans**

12-125. The two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds $v_A = 0.7$ m/s and $v_B = 1.5$ m/s, respectively. Determine in $t = 2$ s, (a) the displacement along the path of each particle, (b) the position vector to each particle, and (c) the shortest distance between the particles.

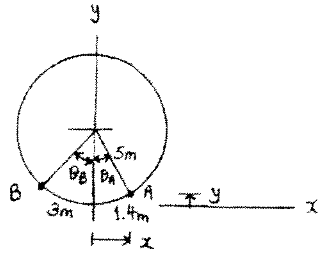


(a) $s_A = 0.7(2) = 1.40$ m **Ans**

$s_B = 1.5(2) = 3$ m **Ans**

(b) $\theta_A = \dfrac{1.40}{5} = 0.280$ rad. $= 16.04°$

$\theta_B = \dfrac{3}{5} = 0.600$ rad. $= 34.38°$



For A

$x = 5\sin 16.04° = 1.382 = 1.38$ m

$y = 5(1 - \cos 16.04°) = 0.1947 = 0.195$ m

$\mathbf{r}_A = \{1.38\,\mathbf{i} + 0.195\,\mathbf{j}\}$ m **Ans**

For B

$x = -5\sin 34.38° = -2.823 = -2.82$ m

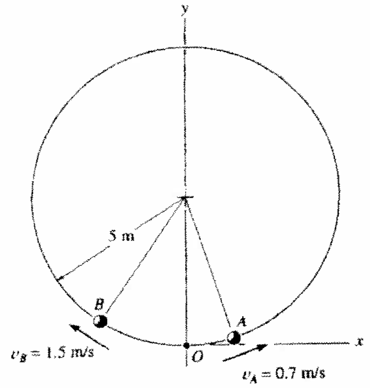
$y = 5(1 - \cos 34.38°) = 0.8734 = 0.873$ m

$\mathbf{r}_B = \{-2.82\,\mathbf{i} + 0.873\,\mathbf{j}\}$ m **Ans**

(c) $\Delta\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A = \{-4.20\,\mathbf{i} + 0.678\,\mathbf{j}\}$ m

$\Delta r = \sqrt{(-4.20)^2 + (0.678)^2} = 4.26$ m **Ans**

12-126. The two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds $v_A = 0.7$ m/s and $v_B = 1.5$ m/s, respectively. Determine the time when they collide and the magnitude of the acceleration of B just before this happens.



$$s_t = 2\,\pi(5) = 31.4159 \text{ m}$$

$$s_A = 0.7\,t$$

$$s_B = 1.5\,t$$

Require

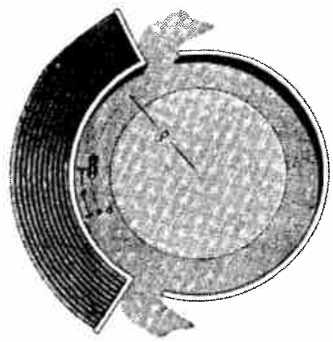
$$s_A + s_B = 31.4159$$

$$0.7\,t + 1.5\,t = 31.4159$$

$$t = 14.28 \text{ s} = 14.3 \text{ s}$$ **Ans**

$$a_B = \frac{v_B^2}{\rho} = \frac{(1.5)^2}{5} = 0.45 \text{ m/s}^2$$ **Ans**

12-127. The race car has an initial speed $v_A = 15$ m/s at A. If it increases its speed along the circular track at the rate $a_t = (0.4s)$ m/s^2, where s is in meters, determine the time needed for the car to travel 20 m. Take $\rho = 150$ m.



$$a_t = 0.4s = \frac{v\,dv}{ds}$$

$$a\,ds = v\,dv$$

$$\int_0^s 0.4s\,ds = \int_{15}^v v\,dv$$

$$\frac{0.4s^2}{2}\Big|_0^s = \frac{v^2}{2}\Big|_{15}^v$$

$$\frac{0.4s^2}{2} = \frac{v^2}{2} - \frac{225}{2}$$

$$v^2 = 0.4s^2 + 225$$

$$v = \frac{ds}{dt} = \sqrt{0.4s^2 + 225}$$

$$\int_0^s \frac{ds}{\sqrt{0.4s^2 + 225}} = \int_0^t dt$$

$$\int_0^s \frac{ds}{\sqrt{s^2 + 562.5}} = 0.632\,456t$$

$$\ln\left(s + \sqrt{s^2 + 562.5}\right)\Big|_0^s = 0.632\,456t$$

$$\ln\left(s + \sqrt{s^2 + 562.5}\right) - 3.166\,196 = 0.632\,456t$$

At $s = 20$ m,

$$t = 1.21 \text{ s}$$ **Ans**

***12-128.** A boy sits on a merry-go-round so that he is always located at $r = 8$ ft from the center of rotation. The merry-go-round is originally at rest, and then due to rotation the boy's speed is increased at 2 ft/s^2. Determine the time needed for his acceleration to become 4 ft/s^2.

$$a = \sqrt{a_n^2 + a_t^2}$$

$$a_t = 2$$

$$v = v_0 + a_c t$$

$$v = 0 + 2t$$

$$a_n = \frac{v^2}{\rho} = \frac{(2t)^2}{8}$$

$$4 = \sqrt{(2)^2 + (\frac{(2t)^2}{8})^2}$$

$$16 = 4 + \frac{16\,t^4}{64}$$

$t = 2.63$ s **Ans**

12-129. A particle moves along the curve $y = \sin x$ with a constant speed $v = 2$ m/s. Determine the normal and tangential components of its velocity and acceleration at any instant.

$$y = \sin x$$

$$\frac{dy}{dx} = \cos x$$

$$\frac{d^2y}{dx^2} = -\sin x$$

$v_t = v = 2$ m/s **Ans**

$v_n = 0$ **Ans**

$a_t = \frac{dv}{dt} = 0$ **Ans**

$$a_n = \frac{v^2}{\rho} = \frac{2^2}{\rho}$$

$$\rho = \frac{[1 + (\frac{dy}{dx})^2]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{(1 + \cos^2 x)^{3/2}}{|-\sin x|}$$

$a_n = \frac{4 \sin x}{(1 + \cos^2 x)^{3/2}}$ **Ans**

12-130. The motion of a particle along a fixed path is defined by the parametric equations $r = 8$ ft, $\theta = (4t)$ rad, and $z = (6t^2)$ ft, where t is in seconds. Determine the unit vector that specifies the direction of the binormal axis to the osculating plane with respect to a set of fixed x, y, z coordinate axes when $t = 2$ s. *Hint:* Formulate the particle's velocity $\mathbf{v}_P$ and acceleration $\mathbf{a}_P$ in terms of their $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ components. Note that $x = r\cos\theta$ and $y = r\sin\theta$. The binormal is parallel to $\mathbf{v}_P \times \mathbf{a}_P$. Why?

$r = 8$ ft $\quad \theta = 4t \quad z = 6t^2$

$x = r\cos\theta = 8\cos 4t \quad y = r\sin\theta = 8\sin 4t \quad z = 6t^2$

Hence,

$$\mathbf{r}_P = 8\cos(4t)\mathbf{i} + 8\sin(4t)\mathbf{j} + 6t^2\mathbf{k}$$

$$\mathbf{v}_P = -32\sin(4t)\mathbf{i} + 32\cos(4t)\mathbf{j} + 12t\mathbf{k}$$

$$\mathbf{a}_P = -128\cos(4t)\mathbf{i} - 128\sin(4t)\mathbf{j} + 12\mathbf{k}$$

When $t = 2$ s,

$$\mathbf{v}_P = -32\sin(8\text{ rad})\mathbf{i} + 32\cos(8\text{ rad})\mathbf{j} + 24\mathbf{k} = -31.659\mathbf{i} - 4.6560\mathbf{j} + 24\mathbf{k}$$

$$\mathbf{a}_P = -128\cos(8\text{ rad})\mathbf{i} - 128\sin(8\text{ rad})\mathbf{j} + 12\mathbf{k} = 18.624\mathbf{i} - 126.64\mathbf{j} + 12\mathbf{k}$$

Since $\mathbf{a}_P$ and $\mathbf{v}_P$ are in the $n-t$ plane, and the binormal axis is perpendicular to this plane, then by definition of the vector cross product, we have

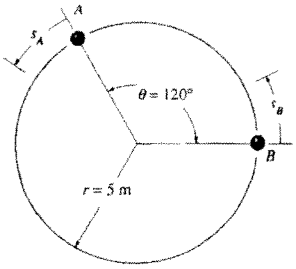
$$\mathbf{b} = \mathbf{v}_P \times \mathbf{a}_P = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -31.659 & -4.6560 & 24 \\ 18.624 & -126.64 & 12 \end{vmatrix} = 2983.44\mathbf{i} + 826.89\mathbf{j} + 4096\mathbf{k}$$

$$b = \sqrt{(2983.44)^2 + (826.89)^2 + (4096)^2} = 5134.38$$

$$\mathbf{u}_b = \frac{\mathbf{b}}{b} = 0.581\mathbf{i} + 0.161\mathbf{j} + 0.798\mathbf{k} \qquad \textbf{Ans}$$

Note : It is also possible to define the binormal axis using $\mathbf{a}_P \times \mathbf{v}_P$ for the calculation. For this case, $\mathbf{u}_{b'} = -\mathbf{u}_b = -0.581\mathbf{i} - 0.161\mathbf{j} - 0.798\mathbf{k}$.

•12-131. Particles A and B are traveling counterclockwise around a circular track at a constant speed of 8 m/s. If at the instant shown the speed of A is increased by $\dot{v}_A = (4s_A)$ m/s², where s_A is in meters, determine the distance measured counterclockwise along the track from B to A when $t = 1$ s. What is the magnitude of the acceleration of each particle at this instant?



Distance Traveled : Initially the distance between two particles is $d_0 = \rho\theta = 5\left(\frac{120°}{180°}\pi\right) = 10.47$ m. When $t = 1$ s, particle B travels a distance of $s_B = 8(1) = 8$ m. The distance traveled by particle A can be obtained as follows

$$v_A\,dv_A = a_A\,ds_A$$

$$\int_{8\,\text{m/s}}^{v_A} v_A\,dv_A = \int_0^{s_A} 4s_A\,ds_A$$

$$v_A = 2\sqrt{s_A^2 + 16} \qquad [1]$$

$$dt = \frac{ds_A}{v_A}$$

$$\int_0^{1\,s} dt = \int_0^{s_A} \frac{ds_A}{2\sqrt{s_A^2+16}}$$

$$1 = \frac{1}{2}\sinh^{-1}\left(\frac{s_A}{4}\right)$$

$$s_A = 14.51\text{ m}$$

Thus, the distance between two particles after $t = 1$ s is

$$d = d_0 + s_A - s_B = 10.47 + 14.51 - 8 = 17.0\text{ m} \qquad \textbf{Ans}$$

Acceleration : The tangential acceleration for particle A and B when $t = 1$ s is $(a_t)_A = 4s_A = 4(14.51) = 58.03$ m/s² and $(a_t)_B = 0$ (particle B travels at constant speed), respectively. When $t = 1$ s, from Eq. [1], $v_A = 2\sqrt{14.51^2 + 16} = 30.10$ m/s. To determine the normal acceleration, apply Eq. 12 – 20.

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{30.10^2}{5} = 181.17\text{ m/s}^2$$

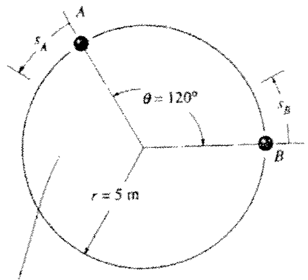
$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{8^2}{5} = 12.80\text{ m/s}^2$$

The magnitudes of the acceleration for particles A and B are

$$a_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{58.03^2 + 181.17^2} = 190\text{ m/s}^2 \qquad \textbf{Ans}$$

$$a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{0^2 + 12.80^2} = 12.8\text{ m/s}^2 \qquad \textbf{Ans}$$

***12-132.** Particles A and B are traveling around a circular track at a speed of 8 m/s at the instant shown. If the speed of B is increased by $\dot{v}_B = 4\ \text{m/s}^2$, and at the same instant A has an increase in speed $\dot{v}_A = 0.8t\ \text{m/s}^2$, determine how long it takes for a collision to occur. What is the magnitude of the acceleration of each particle just before the collision occurs?



Distance Traveled : Initially the distance between the two particles is $d_0 = \rho\theta = 5\left(\frac{120°}{180°}\pi\right) = 10.47$ m. Since particle B travels with a constant acceleration, distance can be obtained by applying equation

$$s_B = (s_0)_B + (v_0)_B t + \frac{1}{2}a_c t^2$$

$$s_B = 0 + 8t + \frac{1}{2}(4)t^2 = \left(8t + 2t^2\right)\ \text{m} \qquad [1]$$

The distance traveled by particle A can be obtained as follows.

$$dv_A = a_A\,dt$$

$$\int_{8\,\text{m/s}}^{v_A} dv_A = \int_0^t 0.8t\,dt$$

$$v_A = \left(0.4t^2 + 8\right)\ \text{m/s} \qquad [2]$$

$$ds_A = v_A\,dt$$

$$\int_0^{s_A} ds_A = \int_0^t \left(0.4t^2 + 8\right) dt$$

$$s_A = 0.1333t^3 + 8t$$

In order for the collision to occur

$$s_A + d_0 = s_B$$

$$0.1333t^3 + 8t + 10.47 = 8t + 2t^2$$

Solving by trial and error $\quad t = 2.5074\ \text{s} = 2.51\ \text{s}$ **Ans**

Note : If particle A strikes B then, $s_A = 5\left(\frac{240°}{180°}\pi\right) + s_B$. This equation will result in $t = 14.6\ \text{s} > 2.51\ \text{s}$.

Acceleration : The tangential acceleration for particles A and B When $t = 2.5074$ are $(a_t)_A = 0.8t = 0.8(2.5074) = 2.006\ \text{m/s}^2$ and $(a_t)_B = 4\ \text{m/s}^2$, respectively. When $t = 2.5074$ s, from Eq. [1], $v_A = 0.4\left(2.5074^2\right) + 8 = 10.51$ m/s and $v_B = (v_0)_B + a_c t = 8 + 4(2.5074) = 18.03$ m/s. To determine the normal acceleration, apply Eq. 12 – 20.

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{10.51^2}{5} = 22.11\ \text{m/s}^2$$

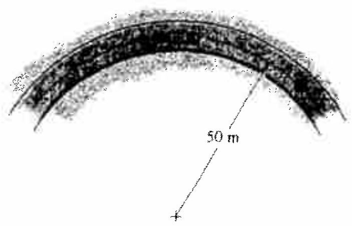
$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{18.03^2}{5} = 65.01\ \text{m/s}^2$$

…e magnitude of the acceleration for particles A and B just before collision are

$$a_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{2.006^2 + 22.11^2} = 22.2\ \text{m/s}^2 \qquad \textbf{Ans}$$

$$a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{4^2 + 65.01^2} = 65.1\ \text{m/s}^2 \qquad \textbf{Ans}$$

12-133. The truck travels at a speed of 4 m/s along a circular road that has a radius of 50 m. For a short distance from $s = 0$, its speed is then increased by $\dot{v} = (0.05s)$ m/s^2, where s is in meters. Determine its speed and the magnitude of its acceleration when it has moved $s = 10$ m.



Velocity : The speed v in terms of position s can be obtained by applying $v\,dv = a\,ds$.

$$v\,dv = a\,ds$$

$$\int_{4\,m/s}^{v} v\,dv = \int_0^s 0.05s\,ds$$

$$v = \left(\sqrt{0.05s^2 + 16}\right)\ m/s$$

At $s = 10$ m, $v = \sqrt{0.05(10^2) + 16} = 4.583\ m/s = 4.58\ m/s$ **Ans**

Acceleration : The tangential acceleration of the truck at $s = 10$ m is $a_t = 0.05(10) = 0.500\ m/s^2$. To determine the normal acceleration, apply Eq. 12 – 20.

$$a_n = \frac{v^2}{\rho} = \frac{4.583^2}{50} = 0.420\ m/s^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.500^2 + 0.420^2} = 0.653\ m/s^2$$ **Ans**

■**12-134.** A go-cart moves along a circular track of radius 100 ft such that its speed for a short period of time, $0 \le t \le 4$ s, is $v = 60(1 - e^{-t^2})$ ft/s. Determine the magnitude of its acceleration when $t = 2$ s. How far has it traveled in $t = 2$ s? Use Simpson's rule with $n = 50$ to evaluate the integral.

$$v = 60(1 - e^{-t^2})$$

$$a_t = \frac{dv}{dt} = 60(-e^{-t^2})(-2t) = 120\,t\,e^{-t^2}$$

$$a_t\big|_{t=2} = 120(2)e^{-4} = 4.3958$$

$$v\big|_{t=2} = 60(1 - e^{-4}) = 58.9011$$

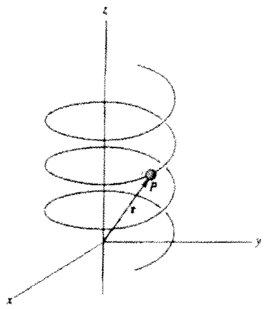
$$a_n = \frac{(58.9011)^2}{100} = 34.693$$

$$a = \sqrt{(4.3958)^2 + (34.693)^2} = 35.0\ m/s^2$$ **Ans**

$$\int_0^s ds = \int_0^2 60(1 - e^{-t^2})\,dt$$

$s = 67.1$ ft **Ans**

12-135. A particle P travels along an elliptical spiral path such that its position vector $\mathbf{r}$ is defined by $\mathbf{r} = \{2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + (2t)\mathbf{k}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. When $t = 8$ s, determine the coordinate direction angles α, β, and γ, which the binormal axis to the osculating plane makes with the x, y, and z axes. *Hint:* Solve for the velocity $\mathbf{v}_P$ and acceleration $\mathbf{a}_P$ of the particle in terms of their $\mathbf{i}, \mathbf{j}, \mathbf{k}$ components. The binormal is parallel to $\mathbf{v}_P \times \mathbf{a}_P$. Why?



$\mathbf{r}_P = 2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + 2t\mathbf{k}$

$\mathbf{v}_P = \dot{\mathbf{r}} = -0.2\sin(0.1t)\mathbf{i} + 0.15\cos(0.1t)\mathbf{j} + 2\mathbf{k}$

$\mathbf{a}_P = \ddot{\mathbf{r}} = -0.02\cos(0.1t)\mathbf{i} - 0.015\sin(0.1t)\mathbf{j}$

When $t = 8$ s,

$\mathbf{v}_P = -0.2\sin(0.8\text{rad})\mathbf{i} + 0.15\cos(0.8\text{rad})\mathbf{j} + 2\mathbf{k} = -0.143\,47\mathbf{i} + 0.104\,51\mathbf{j} + 2\mathbf{k}$

$\mathbf{a}_P = -0.02\cos(0.8\text{rad})\mathbf{i} - 0.015\sin(0.8\text{rad})\mathbf{j} = -0.013\,934\mathbf{i} - 0.010\,76\mathbf{j}$

Since the binormal vector is perpendicular to the plane containing the $n-t$ axis, and $\mathbf{a}_P$ and $\mathbf{v}_P$ are in this plane, then by the definition of the cross product,

$$\mathbf{b} = \mathbf{v}_P \times \mathbf{a}_P = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.143\,47 & 0.104\,51 & 2 \\ -0.013\,934 & -0.010\,76 & 0 \end{vmatrix} = 0.021\,52\mathbf{i} - 0.027\,868\mathbf{j} + 0.003\mathbf{k}$$

$b = \sqrt{(0.02152)^2 + (-0.027868)^2 + (0.003)^2} = 0.035\,338$

$\mathbf{u}_b = 0.608\,99\mathbf{i} - 0.788\,62\mathbf{j} + 0.085\mathbf{k}$

$\alpha = \cos^{-1}(0.608\,99) = 52.5^\circ$ **Ans**

$\beta = \cos^{-1}(-0.788\,62) = 142^\circ$ **Ans**

$\gamma = \cos^{-1}(0.085) = 85.1^\circ$ **Ans**

Note : The direction of the binormal axis may also be specified by the unit vector $\mathbf{u}_{b'} = -\mathbf{u}_b$, which is obtained from $\mathbf{b}' = \mathbf{a}_P \times \mathbf{v}_P$.
For this case, $\alpha = 128^\circ$, $\beta = 37.9^\circ$, $\gamma = 94.9^\circ$ **Ans**

***12-136.** The time rate of change of acceleration is referred to as the *jerk*, which is often used as a means of measuring passenger discomfort. Calculate this vector, $\dot{\mathbf{a}}$, in terms of its cylindrical components, using Eq. 12–32.

$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{u}_\theta + \ddot{z}\mathbf{u}_z$$

$$\dot{\mathbf{a}} = \left(\dddot{r} - \dot{r}\dot{\theta}^2 - 2r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(\ddot{r} - r\dot{\theta}^2\right)\dot{\mathbf{u}}_r + \left(\dot{r}\ddot{\theta} + r\dddot{\theta} + 2\ddot{r}\dot{\theta} + 2\dot{r}\ddot{\theta}\right)\mathbf{u}_\theta + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\dot{\mathbf{u}}_\theta + \dddot{z}\mathbf{u}_z + \ddot{z}\dot{\mathbf{u}}_z$$

But, $\dot{\mathbf{u}}_r = \dot{\theta}\mathbf{u}_\theta$ $\quad \dot{\mathbf{u}}_\theta = -\dot{\theta}\mathbf{u}_r$ $\quad \dot{\mathbf{u}}_z = 0$

Substituting and combining terms yields

$$\dot{\mathbf{a}} = \left(\dddot{r} - 3\dot{r}\dot{\theta}^2 - 3r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(3\dot{r}\ddot{\theta} + r\dddot{\theta} + 3\ddot{r}\dot{\theta} - r\dot{\theta}^3\right)\mathbf{u}_\theta + \left(\dddot{z}\right)\mathbf{u}_z \qquad \textbf{Ans}$$

12-137. If a particle's position is described by the polar coordinates $r = 4(1 + \sin t)$ m and $\theta = (2e^{-t})$ rad, where t is in seconds and the argument for the sine is in radians, determine the radial and tangential components of the particle's velocity and acceleration when $t = 2$ s.

When $t = 2$ s,

$r = 4(1 + \sin t) = 7.637$

$\dot{r} = 4\cos t = -1.66459$

$\ddot{r} = -4\sin t = -3.6372$

$\theta = 2e^{-t}$

$\dot{\theta} = -2e^{-t} = -0.27067$

$\ddot{\theta} = 2e^{-t} = 0.270665$

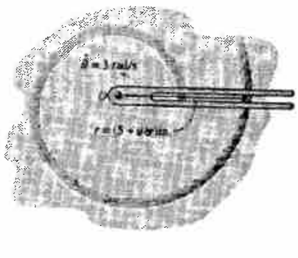
$v_r = \dot{r} = -1.66$ m/s **Ans**

$v_\theta = r\dot{\theta} = 7.637(-0.27067) = -2.07$ m/s **Ans**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -3.6372 - 7.637(-0.27067)^2 = -4.20\ \text{m/s}^2$ **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 7.637(0.270665) + 2(-1.66459)(-0.27067) = 2.97\ \text{m/s}^2$ **Ans**

12-138. The slotted fork is rotating about O at a constant rate of $\dot{\theta} = 3$ rad/s. Determine the radial and transverse components of the velocity and acceleration of the pin A at the instant $\theta = 360°$. The path is defined by the spiral groove $r = (5 + \theta/\pi)$ in., where θ is in radians.



$r = 5 + \left.\dfrac{\theta}{\pi}\right|_{\theta = 2\pi} = 7 \qquad \dot{r} = \dfrac{\dot{\theta}}{\pi} \qquad \ddot{r} = \dfrac{\ddot{\theta}}{\pi}$

$\theta = 360° = 2\pi \qquad \dot{\theta} = 3 \qquad \ddot{\theta} = 0$

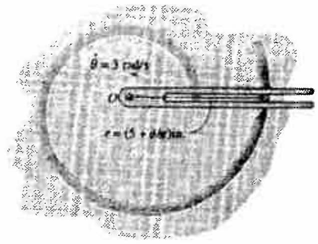
$v_r = \dot{r} = \dfrac{3}{\pi} = 0.955$ in./s **Ans**

$v_\theta = r\dot{\theta} = 7(3) = 21$ in./s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 7(3)^2 = -63\ \text{in./s}^2$ **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2\left(\dfrac{3}{\pi}\right)(3) = 5.73\ \text{in./s}^2$ **Ans**

12-139. The slotted fork is rotating about O at $\dot{\theta} = 3\text{ rad/s}$, which is increasing at $\ddot{\theta} = 2\text{ rad/s}^2$ when $\theta = 360°$. Determine the radial and transverse components of the velocity and acceleration of the pin A at this instant. The path is defined by the spiral groove $r = (5 + \theta/\pi)$ in., where θ is in radians.



$$r = 5 + \frac{\theta}{\pi}\bigg|_{\theta = 2\pi} = 7 \qquad \dot{r} = \frac{\dot{\theta}}{\pi} \qquad \ddot{r} = \frac{\ddot{\theta}}{\pi}$$

$$\theta = 360° = 2\pi \qquad \dot{\theta} = 3 \qquad \ddot{\theta} = 2$$

$$v_r = \dot{r} = \frac{3}{\pi} = 0.955\text{ in./s} \qquad \textbf{Ans}$$

$$v_\theta = r\dot{\theta} = 7(3) = 21\text{ in./s} \qquad \textbf{Ans}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = \frac{2}{\pi} - 7(3)^2 = -62.4\text{ in./s}^2 \qquad \textbf{Ans}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 7(2) + 2\left(\frac{3}{\pi}\right)(3) = 19.7\text{ in./s}^2 \qquad \textbf{Ans}$$

***12-140.** If a particle moves along a path such that $r = (2\cos t)$ ft and $\theta = (t/2)$ rad, where t is in seconds, plot the path $r = f(\theta)$ and determine the particle's radial and transverse components of velocity and acceleration.

$$r = 2\cos t \qquad \dot{r} = -2\sin t \qquad \ddot{r} = -2\cos t$$

$$\theta = \frac{t}{2} \qquad \dot{\theta} = \frac{1}{2} \qquad \ddot{\theta} = 0$$

$$v_r = \dot{r} = -2\sin t \qquad \textbf{Ans}$$

$$v_\theta = r\dot{\theta} = 2\cos t\left(\frac{1}{2}\right) = \cos t \qquad \textbf{Ans}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2\cos t - 2\cos t\left(\frac{1}{2}\right)^2 = -\frac{5}{2}\cos t \qquad \textbf{Ans}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2\cos t(0) + 2(-2\sin t)\left(\frac{1}{2}\right) = -2\sin t \qquad \textbf{Ans}$$

12-141. If a particle's position is described by the polar coordinates $r = (2\sin 2\theta)$ m and $\theta = (4t)$ rad, where t is in seconds, determine the radial and tangential components of its velocity and acceleration when $t = 1$ s.

When $t = 1$ s,

$\theta = 4t = 4$

$\dot{\theta} = 4$

$\ddot{\theta} = 0$

$r = 2\sin 2\theta = 1.9787$

$\dot{r} = 4\cos 2\theta\,\dot{\theta} = -2.3280$

$\ddot{r} = -8\sin 2\theta(\dot{\theta})^2 + 8\cos 2\theta\,\ddot{\theta} = -126.638$

$v_r = \dot{r} = -2.33$ m/s **Ans**

$v_\theta = r\dot{\theta} = 1.9787(4) = 7.91$ m/s **Ans**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -126.638 - (1.9787)(4)^2 = -158$ m/s^2 **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.9787(0) + 2(-2.3280)(4) = -18.6$ m/s^2 **Ans**

12-142. A particle is moving along a circular path having a 400-mm radius. Its position as a function of time is given by $\theta = (2t^2)$ rad, where t is in seconds. Determine the magnitude of the particle's acceleration when $\theta = 30°$. The particle starts from rest when $\theta = 0°$.

$r = 400$ mm $\quad \dot{r} = 0 \quad \ddot{r} = 0$

$\theta = 2t^2 \quad \dot{\theta} = 4t \quad \ddot{\theta} = 4$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 400(4t)^2 = -6400t^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 400(4) + 0 = 1600$

When $\theta = 30° = 30°\left(\frac{\pi}{180°}\right) = 0.5236$ rad, then,

$0.5236 = 2t^2, \quad t = 0.5117$ s

Hence,

$a = \sqrt{(-6400(0.5117)^2)^2 + (1600)^2} = 2316.76$ mm/s^2 $= 2.32$ m/s^2 **Ans**

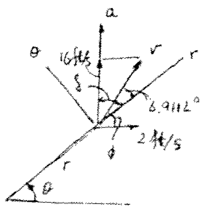
12-143. A particle moves in the x–y plane such that its position is defined by $\mathbf{r} = \{2t\mathbf{i} + 4t^2\mathbf{j}\}$ ft, where t is in seconds. Determine the radial and tangential components of the particle's velocity and acceleration when $t = 2$ s.

$\mathbf{r} = 2t\,\mathbf{i} + 4t^2\,\mathbf{j}|_{t=2} = 4\,\mathbf{i} + 16\,\mathbf{j}$

$\mathbf{v} = 2\,\mathbf{i} + 8t\,\mathbf{j}|_{t=2} = 2\,\mathbf{i} + 16\,\mathbf{j}$

$\mathbf{a} = 8\,\mathbf{j}$

$\theta = \tan^{-1}(\frac{16}{4}) = 75.964°$



$v = \sqrt{(2)^2 + (16)^2} = 16.1245$ ft/s

$\phi = \tan^{-1}(\frac{16}{2}) = 82.875°$

$a = 8$ ft/s^2

$\phi - \theta = 6.9112°$

$v_r = 16.1245\cos 6.9112° = 16.0$ ft/s **Ans**

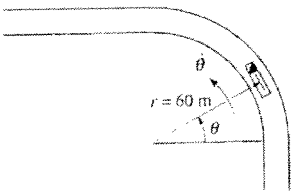
$v_\theta = 16.1245\sin 6.9112° = 1.94$ ft/s **Ans**

$\delta = 90° - \theta = 14.036°$

$a_r = 8\cos 14.036° = 7.76$ ft/s^2 **Ans**

$a_\theta = 8\sin 14.036° = 1.94$ ft/s^2 **Ans**

•12-144. A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a constant speed $v = 20$ m/s. Determine the angular rate of rotation $\dot{\theta}$ of the radial line r and the magnitude of the truck's acceleration.



$r = 60$

$\dot{r} = 0$

$\ddot{r} = 0$

$v = 20$

$v_r = \dot{r} = 0$

$v_\theta = r\dot{\theta} = 60\,\dot{\theta}$

$v = \sqrt{(v_r)^2 + (v_\theta)^2}$

$20 = 60\,\dot{\theta}$

$\dot{\theta} = 0.333$ rad/s **Ans**

$a_r = \ddot{r} - r(\dot{\theta})^2$

$= 0 - 60(0.333)^2$

$= -6.67$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$= 60\,\ddot{\theta}$

Since

$v = r\dot{\theta}$

$\dot{v} = \dot{r}\dot{\theta} + r\ddot{\theta}$

$0 = 0 + 60\,\ddot{\theta}$

$\ddot{\theta} = 0$

Thus,

$a_\theta = 0$

$a = |a_r| = 6.67$ m/s^2 **Ans**

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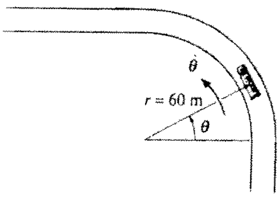
12-145. A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a speed of 20 m/s which is increasing at 3 m/s^2. Determine the truck's radial and transverse components of acceleration.



$r = 60$

$a_t = 3 \text{ m/s}^2$

$$a_n = \frac{v^2}{r} = \frac{(20)^2}{60} = 6.67 \text{ m/s}^2$$

$a_r = -a_n = -6.67 \text{ m/s}^2$ **Ans**

$a_\theta = a_t = 3 \text{ m/s}^2$ **Ans**

12-146. A particle is moving along a circular path having a radius of 6 in. such that its position as a function of time is given by $\theta = \sin 3t$, where θ is in radians, the argument for the sine is in degrees, and t is in seconds. Determine the acceleration of the particle at $\theta = 30°$. The particle starts from rest at $\theta = 0°$.

$r = 6$ in., $\dot{r} = 0$, $\ddot{r} = 0$

$\theta = \sin 3t$

$\dot{\theta} = 3\cos 3t$

$\ddot{\theta} = -9\sin 3t$

At $\theta = 30°$,

$$\frac{30°}{180°}\pi = \sin 3t$$

$t = 10.525$ s

Thus,

$\dot{\theta} = 2.5559$ rad/s

$\ddot{\theta} = -4.7124$ rad/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 6(2.5559)^2 = -39.196$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 6(-4.7124) + 0 = -28.274$

$a = \sqrt{(-39.196)^2 + (-28.274)^2} = 48.3 \text{ in./s}^2$ **Ans**

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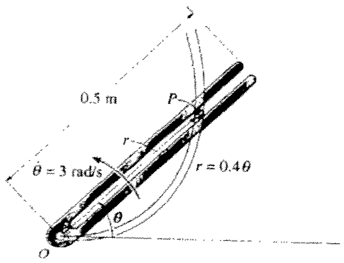
12-147. The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3$ rad/s it drives the peg P for a short distance along the spiral guide $r = (0.4\,\theta)$ m, where θ is in radians. Determine the radial and transverse components of the velocity and acceleration of P at the instant $\theta = \pi/3$ rad.



$\dot{\theta} = 3$ rad/s $\quad r = 0.4\,\theta$

$\dot{r} = 0.4\,\dot{\theta}$

$\ddot{r} = 0.4\,\ddot{\theta}$

At $\theta = \frac{\pi}{3}$, $\quad r = 0.4189$

$\dot{r} = 0.4(3) = 1.20$

$\ddot{r} = 0.4(0) = 0$

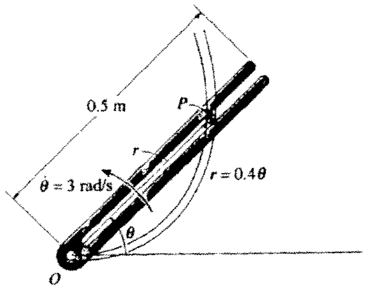
$v_r = \dot{r} = 1.20$ m/s **Ans**

$v_\theta = r\dot{\theta} = 0.4189(3) = 1.26$ m/s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.4189(3)^2 = -3.77 \text{ m/s}^2$ **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1.20)(3) = 7.20 \text{ m/s}^2$ **Ans**

•12-148. Solve Prob. 12-147 if the slotted link has an angular acceleration $\ddot{\theta} = 8\ \text{rad/s}^2$ when $\dot{\theta} = 3\ \text{rad/s}$ at $\theta = \pi/3$ rad.



$\dot{\theta} = 3$ rad/s $\quad r = 0.4\,\theta$

$\dot{r} = 0.4\,\dot{\theta}$

$\ddot{r} = 0.4\,\ddot{\theta}$

$\theta = \frac{\pi}{3}$

$\dot{\theta} = 3$

$\ddot{\theta} = 8$

$r = 0.4189$

$\dot{r} = 1.20$

$\ddot{r} = 0.4(8) = 3.20$

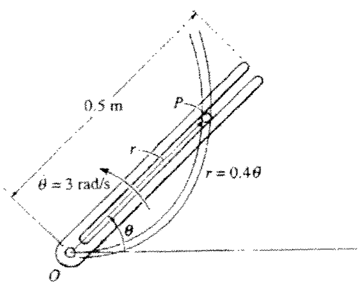
$v_r = \dot{r} = 1.20$ m/s **Ans**

$v_\theta = r\,\dot{\theta} = 0.4189(3) = 1.26$ m/s **Ans**

$a_r = \ddot{r} - r\,\dot{\theta}^2 = 3.20 - 0.4189(3)^2 = -0.570\ \text{m/s}^2$ **Ans**

$a_\theta = r\,\ddot{\theta} + 2\,\dot{r}\,\dot{\theta} = 0.4189(8) + 2(1.20)(3) = 10.6\ \text{m/s}^2$ **Ans**

12-149. The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3$ rad/s it drives the peg P for a short distance along the spiral guide $r = (0.4\,\theta)$ m, where θ is in radians. Determine the velocity and acceleration of the particle at the instant it leaves the slot in the link, i.e., when $r = 0.5$ m.



$r = 0.4\,\theta$

$\dot{r} = 0.4\,\dot{\theta}$

$\ddot{r} = 0.4\,\ddot{\theta}$

$\dot{\theta} = 3$

$\ddot{\theta} = 0$

At $r = 0.5$ m,

$\theta = \frac{0.5}{0.4} = 1.25$ rad

$\dot{r} = 1.20$

$\ddot{r} = 0$

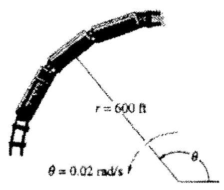
$v_r = \dot{r} = 1.20$ m/s **Ans**

$v_\theta = r\,\dot{\theta} = 0.5(3) = 1.50$ m/s **Ans**

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 0.5(3)^2 = -4.50\ \text{m/s}^2$ **Ans**

$a_\theta = r\,\ddot{\theta} + 2\,\dot{r}\,\dot{\theta} = 0 + 2(1.20)(3) = 7.20\ \text{m/s}^2$ **Ans**

12-150. A train is traveling along the circular curve of radius $r = 600$ ft. At the instant shown, its angular rate of rotation is $\dot{\theta} = 0.02$ rad/s, which is decreasing at $\ddot{\theta} = -0.001$ rad/s². Determine the magnitudes of the train's velocity and acceleration at this instant.



$r = 600 \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

$v_r = \dot{r} = 0$

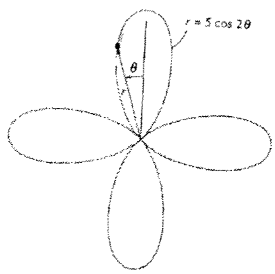
$v = v_\theta = r\dot{\theta} = 600(0.02) = 12$ ft/s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 600(0.02)^2 = -0.24$ ft/s²

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 600(-0.001) + 0 = -0.6$ ft/s²

$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-0.24)^2 + (-0.6)^2} = 0.646$ ft/s² **Ans**

12-151. A particle travels along a portion of the "four-leaf rose" defined by the equation $r = (5 \cos 2\theta)$ m. If the angular velocity of the radial coordinate line is $\dot{\theta} = (3t^2)$ rad/s, where t is in seconds, determine the radial and transverse components of the particle's velocity and acceleration at the instant $\theta = 30°$. When $t = 0$, $\theta = 0°$.



$\dot{\theta} = 3t^2$

$\ddot{\theta} = 6t$

$\int_0^\theta d\theta = \int_0^t 3t^2\, dt$

$\theta = t^3$

At $\theta = 30° = \dfrac{\pi}{6}$

$t = (\pi/6)^{1/3} = 0.806$

$\dot{\theta} = 1.95$

$\ddot{\theta} = 4.84$

$r = 5 \cos 2\theta$

$\dot{r} = -10 \sin 2\theta\, \dot{\theta}$

$\ddot{r} = -10(2\cos 2\theta (\dot{\theta})^2 + \sin 2\theta\, \ddot{\theta})$

At $\theta = 30°$

$r = 2.5$

$\dot{r} = -16.88$ m/s

$\ddot{r} = -79.86$ m/s²

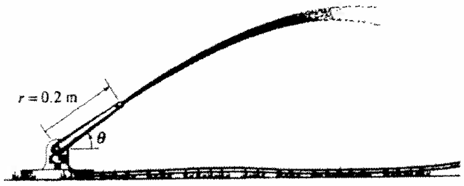
$v_r = \dot{r} = -16.9$ m/s **Ans**

$v_\theta = r\dot{\theta} = 2.5(1.95) = 4.87$ m/s **Ans**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -79.86 - 2.5(1.95)^2 = -89.4$ m/s² **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.5(4.84) + 2(-16.88)(1.95) = -53.7$ m/s² **Ans**

***12-152.** At the instant shown, the watersprinkler is rotating with an angular speed $\dot{\theta} = 2$ rad/s and an angular acceleration $\ddot{\theta} = 3$ rad/s^2. If the nozzle lies in the vertical plane and water is flowing through it at a constant rate of 3 m/s, determine the magnitudes of the velocity and acceleration of a water particle as it exits the open end, $r = 0.2$ m.



$r = 0.2$

$\dot{r} = 3 \qquad \dot{\theta} = 2$

$\ddot{r} = 0 \qquad \ddot{\theta} = 3$

$v_r = 3$

$v_\theta = 0.2(2) = 0.4$

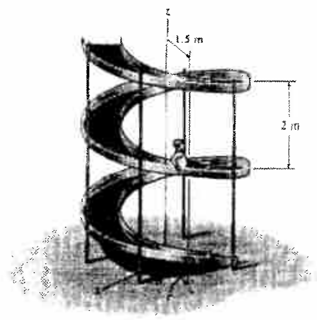
$v = \sqrt{(3)^2 + (0.4)^2} = 3.03$ m/s **Ans**

$a_r = 0 - (0.2)(2)^2 = -0.80$ m/s^2

$a_\theta = 0.2(3) + 2(3)(2) = 12.6$ m/s^2

$a = \sqrt{(-0.80)^2 + (12.6)^2} = 12.6$ m/s^2 **Ans**

12-153. The boy slides down the slide at a constant speed of 2 m/s. If the slide is in the form of a helix, defined by the equations $r = 1.5$ m and $z = -\theta/\pi$, determine the boy's angular velocity about the z axis, $\dot{\theta}$, and the magnitude of his acceleration.



$z = -\dfrac{\theta}{\pi} \qquad \dot{z} = -\dfrac{\dot{\theta}}{\pi} \qquad \ddot{z} = -\dfrac{\ddot{\theta}}{\pi}$

$r = 1.5 \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

$v_r = \dot{r} = 0$

$v_\theta = r\dot{\theta} = 1.5\dot{\theta}$

$v_z = \dot{z} = -\dfrac{\dot{\theta}}{\pi}$

$2 = \sqrt{(0)^2 + \left(1.5\dot{\theta}\right)^2 + \left(-\dot{\theta}/\pi\right)^2}$

$4 = 2.25\dot{\theta}^2 + 0.1013\dot{\theta}^2$

$\dot{\theta} = 1.3043 = 1.30$ rad/s **Ans**

$\ddot{\theta} = 0$

$a_r = \ddot{r} - r\dot{\theta}^2$

$= 0 - 1.5(1.3043)^2 = -2.552$

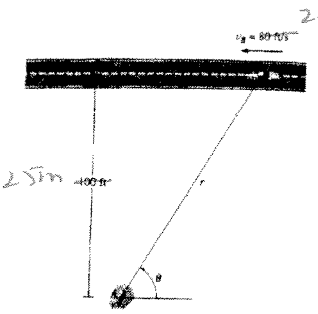
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$= 1.5(0) + 0 = 0$

$a_z = \ddot{z} = -\dfrac{(0)}{\pi} = 0$

$a = \sqrt{(-2.552)^2 + (0)^2 + (0)^2} = 2.55$ m/s^2 **Ans**

12-154. A cameraman standing at A is following the movement of a race car, B, which is traveling along a straight track at a constant speed of 80 ft/s. Determine the angular rate at which he must turn in order to keep the camera directed on the car at the instant $\theta = 60°$.



$$r = \frac{100}{\sin\theta} = 100\csc\theta$$

$$\dot{r} = -100\csc\theta\cot\theta\dot{\theta}$$

$$v^2 = (\dot{r})^2 + (r\dot{\theta})^2$$

$$(80)^2 = (-100)^2\csc^2\theta\cot^2\theta\dot{\theta}^2 + (100)^2\csc^2\theta\dot{\theta}^2$$

$$\frac{(80)^2}{(100)^2} = \csc^2\theta\dot{\theta}^2\left(1 + \cot^2\theta\right)$$

Since $1 + \cot^2\theta = \csc^2\theta$,

$$\frac{(80)^2}{(100)^2} = \csc^4\theta\dot{\theta}^2$$

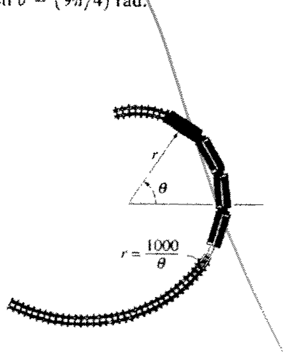
$$\dot{\theta}^2 = \frac{(80)^2}{(100)^2}\sin^4\theta$$

$$\dot{\theta} = \left(\frac{80}{100}\right)\sin^2\theta$$

At $\theta = 60°$,

$$\dot{\theta} = \left(\frac{8}{10}\right)\sin^2 60° = 0.6 \text{ rad/s} \qquad \textbf{Ans}$$

12-155. For a short distance the train travels along a track having the shape of a spiral, $r = (1000/\theta)$ m, where θ is in radians. If it maintains a constant speed $v = 20$ m/s, determine the radial and transverse components of its velocity when $\theta = (9\pi/4)$ rad.



$$r = \frac{1000}{\theta}$$

$$\dot{r} = -\frac{1000}{\theta^2}\dot{\theta}$$

Since

$$v^2 = (\dot{r})^2 + (r\dot{\theta})^2$$

$$(20)^2 = \frac{(1000)^2}{\theta^4}(\dot{\theta})^2 + \frac{(1000)^2}{\theta^2}(\dot{\theta})^2$$

$$(20)^2 = \frac{(1000)^2}{\theta^4}(1 + \theta^2)(\dot{\theta})^2$$

Thus,

$$\dot{\theta} = \frac{0.02\theta^2}{\sqrt{1 + \theta^2}}$$

At $\theta = \frac{9\pi}{4}$

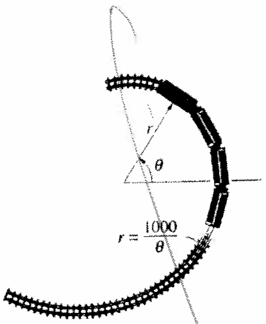
$$\dot{\theta} = 0.140$$

$$\dot{r} = \frac{-1000}{(9\pi/4)^2}(0.140) = -2.80$$

$$v_r = \dot{r} = -2.80 \text{ m/s} \qquad \textbf{Ans}$$

$$v_\theta = r\dot{\theta} = \frac{1000}{(9\pi/4)}(0.140) = 19.8 \text{ m/s} \qquad \textbf{Ans}$$

•12-156. For a short distance the train travels along a track having the shape of a spiral, $r = (1000/\theta)$ m, where θ is in radians. If the angular rate is constant, $\dot{\theta} = 0.2$ rad/s, determine the radial and transverse components of its velocity and acceleration when $\theta = (9\pi/4)$ rad.



$\dot{\theta} = 0.2$

$\ddot{\theta} = 0$

$r = \dfrac{1000}{\theta}$

$\dot{r} = -1000(\theta^{-2})\dot{\theta}$

$\ddot{r} = 2000(\theta^{-3})(\dot{\theta})^2 - 1000(\theta^{-2})\ddot{\theta}$

When $\theta = \dfrac{9\pi}{4}$

$r = 141.477$

$\dot{r} = -4.002812$

$\ddot{r} = 0.226513$

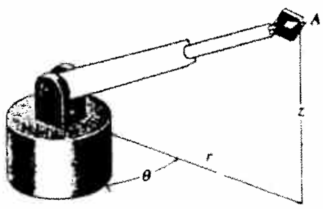
$v_r = \dot{r} = -4.00$ m/s **Ans**

$v_\theta = r\dot{\theta} = 141.477(0.2) = 28.3$ m/s **Ans**

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0.226513 - 141.477(0.2)^2 = -5.43$ m/s^2 **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-4.002812)(0.2) = -1.60$ m/s^2 **Ans**

12-157. The arm of the robot has a fixed length so that $r = 3$ ft and its grip A moves along the path $z = (3 \sin 4\theta)$ ft, where θ is in radians. If $\theta = (0.5t)$ rad, where t is in seconds, determine the magnitudes of the grip's velocity and acceleration when $t = 3$ s.



$\theta = 0.5\,t$ $\quad r = 3$ $\quad z = 3\sin 2t$

$\dot{\theta} = 0.5$ $\quad \dot{r} = 0$ $\quad \dot{z} = 6\cos 2t$

$\ddot{\theta} = 0$ $\quad \ddot{r} = 0$ $\quad \ddot{z} = -12\sin 2t$

At $t = 3$ s,

$z = -0.8382$

$\dot{z} = 5.761$

$\ddot{z} = 3.353$

$v_r = 0$

$v_\theta = 3(0.5) = 1.5$

$v_z = 5.761$

$v = \sqrt{(0)^2 + (1.5)^2 + (5.761)^2} = 5.95$ ft/s **Ans**

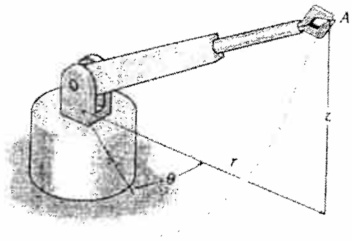
$a_r = 0 - 3(0.5)^2 = -0.75$

$a_\theta = 0 + 0 = 0$

$a_z = 3.353$

$a = \sqrt{(-0.75)^2 + (0)^2 + (3.353)^2} = 3.44$ ft/s^2 **Ans**

12-158. For a short time the arm of the robot is extending at a constant rate such that $\dot{r} = 1.5$ ft/s when $r = 3$ ft, $z = (4t^2)$ ft, and $\theta = 0.5t$ rad, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the grip A when $t = 3$ s.



$\theta = 0.5\,t$ rad $\qquad r = 3$ ft $\qquad z = 4\,t^2$ ft

$\dot{\theta} = 0.5$ rad/s $\qquad \dot{r} = 1.5$ ft/s $\qquad \dot{z} = 8\,t$ ft/s

$\ddot{\theta} = 0 \qquad \ddot{r} = 0 \qquad \ddot{z} = 8$ ft/s^2

At $t = 3$ s,

$\theta = 1.5 \qquad r = 3 \qquad z = 36$

$\dot{\theta} = 0.5 \qquad \dot{r} = 1.5 \qquad \dot{z} = 24$

$\ddot{\theta} = 0 \qquad \ddot{r} = 0 \qquad \ddot{z} = 8$

$v_r = 1.5$

$v_\theta = 3(0.5) = 1.5$

$v_z = 24$

$v = \sqrt{(1.5)^2 + (1.5)^2 + (24)^2} = 24.1$ ft/s **Ans**

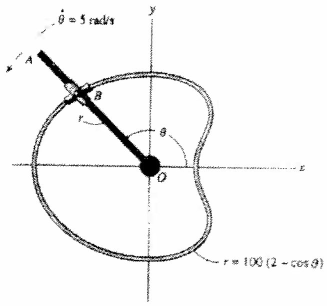
$a_r = 0 - 3(0.5)^2 = -0.75$

$a_\theta = 0 + 2(1.5)(0.5) = 1.5$

$a_z = 8$

$a = \sqrt{(-0.75)^2 + (1.5)^2 + (8)^2} = 8.17$ ft/s^2 **Ans**

12-159. The rod OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 5$ rad/s. Two pin-connected slider blocks, located at B, move freely on OA and the curved rod whose shape is a limaçon described by the equation $r = 100(2 - \cos\theta)$ mm. Determine the speed of the slider blocks at the instant $\theta = 120°$.



$\dot{\theta} = 5$

$r = 100(2 - \cos\theta)$

$\dot{r} = 100\sin\theta\dot{\theta} = 500\sin\theta$

$\ddot{r} = 500\cos\theta\dot{\theta} = 2500\cos\theta$

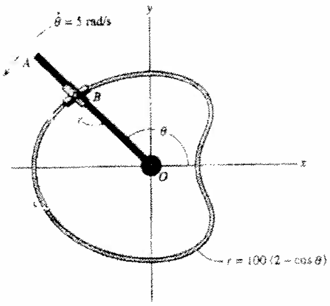
At $\theta = 120°$,

$v_r = \dot{r} = 500\sin 120° = 433.013$

$v_\theta = r\dot{\theta} = 100(2 - \cos 120°)(5) = 1250$

$v = \sqrt{(433.013)^2 + (1250)^2} = 1322.9$ mm/s $= 1.32$ m/s **Ans**

***12-160.** Determine the magnitude of the acceleration of the slider blocks in Prob. 12-159 when $\theta = 120°$.



$\dot{\theta} = 5$

$\ddot{\theta} = 0$

$r = 100(2 - \cos\theta)$

$\dot{r} = 100\sin\theta\dot{\theta} = 500\sin\theta$

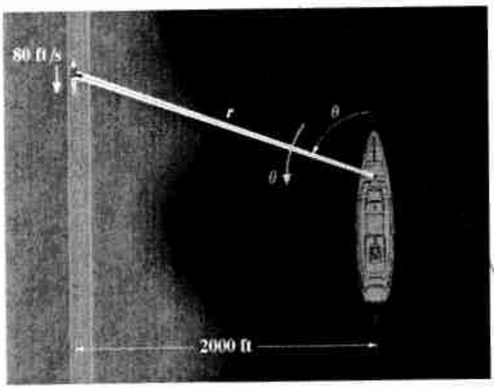
$\ddot{r} = 500\cos\theta\dot{\theta} = 2500\cos\theta$

$a_r = \ddot{r} - r\dot{\theta}^2 = 2500\cos\theta - 100(2 - \cos\theta)(5)^2 = 5000(\cos 120° - 1) = -7500 \text{ mm/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(500\sin\theta)(5) = 5000\sin 120° = 4330.1 \text{ mm/s}^2$

$a = \sqrt{(-7500)^2 + (4330.1)^2} = 8660.3 \text{ mm/s}^2 = 8.66 \text{ m/s}^2$ **Ans**

12–161. The searchlight on the boat anchored 2000 ft from shore is turned on the automobile, which is traveling along the straight road at a constant speed of 80 ft/s. Determine the angular rate of rotation of the light when the automobile is $r = 3000$ ft from the boat.



$r = 2000 \csc\theta$

$\dot{r} = -2000 \csc\theta \text{ctn}\theta \, \dot{\theta}$

At $r = 3000$ ft, $\theta = 41.8103°$

$\dot{r} = -3354.102 \, \dot{\theta}$

$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$

$(80)^2 = [(-3354.102)^2 + (3000)^2](\dot{\theta})^2$

$\dot{\theta} = 0.0177778 = 0.0178 \text{ rad/s}$ **Ans**

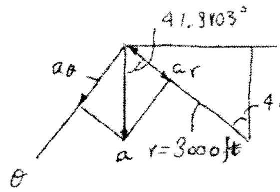
12-162. If the car in Prob. 12-161 is accelerating at 15 ft/s^2 at the instant $r = 3000$ ft, determine the required angular acceleration $\ddot{\theta}$ of the light at this instant.

$r = 2000 \csc\theta$

$\dot{r} = -2000 \csc\theta \operatorname{ctn}\theta\ \dot{\theta}$

At $r = 3000$ ft, $\theta = 41.8103°$

$\dot{r} = -3354.102\ \dot{\theta}$



$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

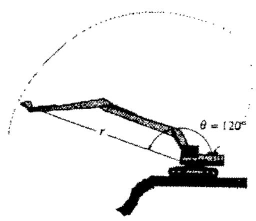
$a_\theta = 3000\ddot{\theta} + 2(-3354.102)(0.0177778)^2$

Since $a_\theta = 15 \sin 41.8103° = 10$ m/s

Then,

$\ddot{\theta} = 0.00404$ rad/s^2 **Ans**

12-163. For a short time the bucket of the backhoe traces the path of the cardioid $r = 25(1 - \cos\theta)$ ft. Determine the magnitudes of the velocity and acceleration of the bucket when $\theta = 120°$ if the boom is rotating with an angular velocity of $\dot{\theta} = 2$ rad/s and an angular acceleration of $\ddot{\theta} = 0.2$ rad/s^2 at the instant shown.



$r = 25(1 - \cos\theta) = 25(1 - \cos 120°) = 37.5$ ft

$\dot{r} = 25\sin\theta\dot{\theta} = 25\sin 120°(2) = 43.30$ ft/s

$\ddot{r} = 25\left[\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta}\right] = 25\left[\cos 120°(2)^2 + \sin 120°(0.2)\right] = -45.67$ ft/s^2

$v_r = \dot{r} = 43.30$ ft/s

$v_\theta = r\dot{\theta} = 37.5(2) = 75$ ft/s

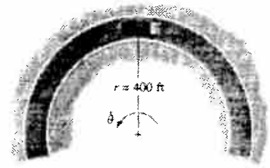
$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{43.30^2 + 75^2} = 86.6$ ft/s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = -45.67 - 37.5(2)^2 = -195.67$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 37.5(0.2) + 2(43.30)(2) = 180.71$ ft/s^2

$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-195.67)^2 + 180.71^2} = 266$ ft/s^2 **Ans**

***12-164.** A car is traveling along the circular curve having a radius $r = 400$ ft. At the instant shown, its angular rate of rotation is $\dot{\theta} = 0.025$ rad/s, which is decreasing at the rate $\ddot{\theta} = -0.008$ rad/s^2. Determine the radial and transverse components of the car's velocity and acceleration at this instant and sketch these components on the curve.



$r = 400 \quad \dot{r} = 0 \quad \ddot{r} = 0$

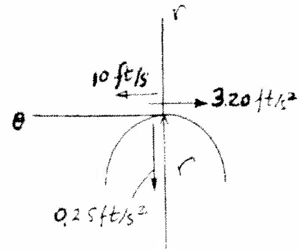
$\dot{\theta} = 0.025 \quad \ddot{\theta} = -0.008$

$v_r = \dot{r} = 0$ **Ans**

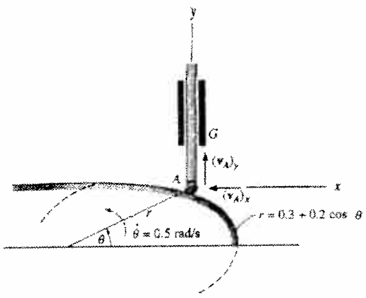
$v_\theta = r\dot{\theta} = 400(0.025) = 10$ ft/s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 400(0.025)^2 = -0.25$ ft/s^2 **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 400(-0.008) + 0 = -3.20$ ft/s^2 **Ans**



12-165. The mechanism of a machine is constructed so that for a short time the roller at A follows the surface of the cam described by the equation $r = (0.3 + 0.2\cos\theta)$ m. If $\dot{\theta} = 0.5$ rad/s and $\ddot{\theta} = 0$, determine the magnitudes of the roller's velocity and acceleration at the instant $\theta = 30°$. Neglect the size of the roller. Also determine the velocity components $(\mathbf{v}_A)_x$ and $(\mathbf{v}_A)_y$ of the roller at this instant. The rod to which the roller is attached remains vertical and can slide up or down along the guides while the guides move horizontally to the left.



$\dot{\theta} = 0.5$

$\ddot{\theta} = 0$

$r = (0.3 + 0.2\cos\theta)$

$\dot{r} = -0.2\sin\theta\dot{\theta}$

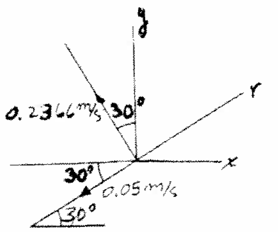
$\ddot{r} = -0.2\left(\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta}\right)$

At $\theta = 30°$,

$r = 0.4732$

$\dot{r} = -0.05$

$\ddot{r} = -0.04330$



$v_r = \dot{r} = -0.05$

$v_\theta = r\dot{\theta} = 0.473(0.5) = 0.2366$

$v = \sqrt{(-0.05)^2 + (0.2366)^2} = 0.242$ m/s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.04330 - 0.4732(0.5)^2 = -0.1616$ m/s^2

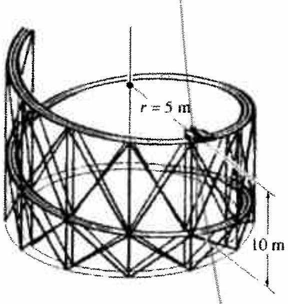
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-0.05)(0.5) = -0.05$ m/s^2

$a = \sqrt{(-0.1616)^2 + (-0.05)^2} = 0.169$ m/s^2 **Ans**

$(\overset{+}{\rightarrow})$ $(v_A)_x = -0.05\cos 30° - 0.2366\sin 30° = -0.162$ m/s **Ans**

$(+\uparrow)$ $(v_A)_y = -0.05\sin 30° + 0.2366\cos 30° = 0.180$ m/s **Ans**

12-166. The roller coaster is traveling down along the spiral ramp with a constant speed $v = 6$ m/s. If the track descends a distance of 10 m for every full revolution, $\theta = 2\pi$ rad, determine the magnitude of the roller coaster's acceleration as it moves along the track, $r = 5$ m. *Hint:* For part of the solution, note that the tangent to the ramp at any point is at an angle $\phi = \tan^{-1}[10/2\pi(5)] = 17.66°$ from the horizontal. Use this to determine the velocity components v_θ and v_z, which in turn are used to determine $\dot{\theta}$ and $\dot{z}$.



$$\phi = 17.66°$$

$$v = 6 \text{ m/s}$$

$$v_z = -6 \sin 17.66° = -1.820 \text{ m/s}$$

$$v_\theta = 6 \cos 17.66° = 5.717 \text{ m/s}$$

Since $r = 5$

$$\dot{r} = 0$$

$$\ddot{r} = 0$$

$$r\dot{\theta} = v_\theta = 5.717$$

$$\dot{\theta} = \frac{5.717}{5} = 1.143$$

$$v^2 = (\dot{r})^2 + (r\,\dot{\theta})^2$$

$$0 = 2\,\dot{r}\,\ddot{r} + 2(r\,\dot{\theta})(\dot{r}\,\dot{\theta} + r\,\ddot{\theta})$$

$$\ddot{\theta} = 0$$

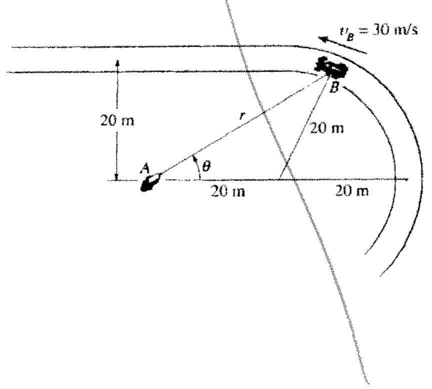
$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 5(1.143)^2 = -6.537 \text{ m/s}^2$$

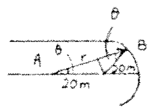
$$a_\theta = r\,\ddot{\theta} + 2\,\dot{r}\dot{\theta} = 0$$

$$a_z = \ddot{z} = \dot{v}_z = 0$$

$$a = 6.54 \text{ m/s}^2$$ **Ans**

12-167. A cameraman standing at A is following the movement of a race car, B, which is traveling around a curved track at a constant speed of 30 m/s. Determine the angular rate $\dot{\theta}$ at which the man must turn in order to keep the camera directed on the car at the instant $\theta = 30°$.





$$r = 2(20\cos\theta) = 40\cos\theta$$

$$\dot{r} = -40\sin\theta\,\dot{\theta}$$

$$\mathbf{v} = \dot{r}\mathbf{u}_r + r\dot{\theta}\,\mathbf{u}_\theta$$

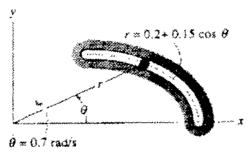
$$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$$

$$(30)^2 = (-40\sin\theta)^2(\dot{\theta})^2 + (40\cos\theta)^2(\dot{\theta})^2$$

$$(30)^2 = (40)^2[\sin^2\theta + \cos^2\theta](\dot{\theta})^2$$

$$\dot{\theta} = \frac{30}{40} = 0.75 \text{ rad/s} \qquad \textbf{Ans}$$

***12-168.** The pin follows the path described by the equation $r = (0.2 + 0.15\cos\theta)$ m. At the instant $\theta = 30°$, $\dot{\theta} = 0.7$ rad/s and $\ddot{\theta} = 0.5$ rad/s^2. Determine the magnitudes of the pin's velocity and acceleration at this instant. Neglect the size of the pin.



$$r = 0.2 + 0.15\cos\theta = 0.2 + 0.15\cos 30° = 0.3299 \text{ m}$$

$$\dot{r} = -0.15\sin\theta\dot{\theta} = -0.15\sin 30°(0.7) = -0.0525 \text{ m/s}$$

$$\ddot{r} = -0.15\left[\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta}\right] = -0.15\left[\cos 30°(0.7)^2 + \sin 30°(0.5)\right] = -0.10115 \text{ m/s}^2$$

$$v_r = \dot{r} = -0.0525 \text{ m/s}$$

$$v_\theta = r\dot{\theta} = 0.3299(0.7) = 0.2309 \text{ m/s}$$

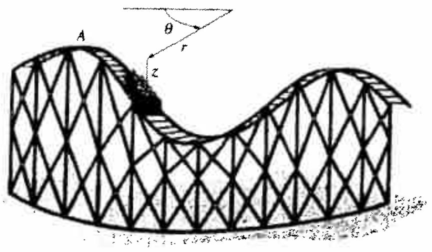
$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-0.0525)^2 + (0.2309)^2} = 0.237 \text{ m/s} \qquad \textbf{Ans}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -0.10115 - 0.3299(0.7)^2 = -0.2628 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.3299(0.5) + 2(-0.0525)(0.7) = 0.09145 \text{ m/s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-0.2628)^2 + (0.09145)^2} = 0.278 \text{ m/s}^2 \qquad \textbf{Ans}$$

12-169. For a short time the position of the roller-coaster car along its path is defined by the equations $r = 25$ m, $\theta = (0.3t)$ rad, and $z = (-8\cos\theta)$ m, where t is in seconds. Determine the magnitude of the car's velocity and acceleration when $t = 4$ s.



$r = 25$ m $\quad \theta = 0.3t|_{t=4\,s} = 1.2$ rad

$\dot{r} = 0 \quad \dot{\theta} = 0.3$ rad/s

$\ddot{r} = 0 \quad \ddot{\theta} = 0$

$z = -8\cos\theta \quad \dot{z} = 8\sin\theta\dot{\theta}|_{\theta=1.2\,\text{rad}} = 2.2369$ m/s

$\ddot{z} = 8\left[\cos\theta\dot{\theta}^2 + \sin\theta\,\ddot{\theta}\right]|_{\theta=1.2\,\text{rad}} = 0.2609$ m/s^2

$v_r = \dot{r} = 0 \quad v_\theta = r\dot{\theta} = 25(0.3) = 7.5$ m/s $\quad v_z = \dot{z} = 2.2369$ m/s

$v = \sqrt{v_r^2 + v_\theta^2 + v_z^2} = \sqrt{0^2 + 7.5^2 + 2.2369^2} = 7.83$ m/s **Ans**

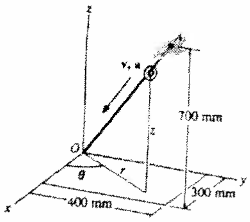
$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 25(0.3)^2 = -2.25$ m/s^2

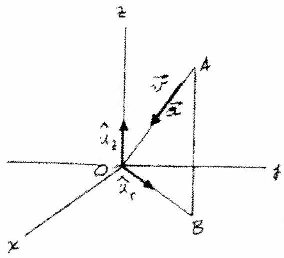
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 25(0) + 2(0)(0.3) = 0$

$a_z = \ddot{z} = 0.2609$ m/s^2

$a = \sqrt{a_r^2 + a_\theta^2 + a_z^2} = \sqrt{(-2.25)^2 + 0^2 + 0.2609^2} = 2.27$ m/s^2 **Ans**

12-170. The small washer is sliding down the cord OA. When it is halfway down the cord, its speed is 200 mm/s and its acceleration is 10 mm/s^2. Express the velocity and acceleration of the washer at this point in terms of its cylindrical components.





$OA = \sqrt{(400)^2 + (300)^2 + (700)^2} = 860.23$ mm

$OB = \sqrt{(400)^2 + (300)^2} = 500$ mm

$v_r = (200)\left(\frac{500}{860.23}\right) = 116$ mm/s

$v_\theta = 0$

$v_z = (200)\left(\frac{700}{860.23}\right) = 163$ mm/s

Thus, $\mathbf{v} = \{-116\mathbf{u}_r - 163\mathbf{u}_z\}$ mm/s **Ans**

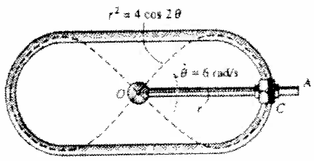
$a_r = 10\left(\frac{500}{860.23}\right) = 5.81$

$a_\theta = 0$

$a_z = 10\left(\frac{700}{860.23}\right) = 8.14$

Thus, $\mathbf{a} = \{-5.81\mathbf{u}_r - 8.14\mathbf{u}_z\}$ mm/s^2 **Ans**

12-171. A double collar C is pin connected together such that one collar slides over a fixed rod and the other slides over a rotating rod. If the geometry of the fixed rod for a short distance can be defined by a lemniscate, $r^2 = (4\cos 2\theta)\ \text{ft}^2$, determine the collar's radial and transverse components of velocity and acceleration at the instant $\theta = 0°$ as shown. Rod OA is rotating at a constant rate of $\dot{\theta} = 6$ rad/s.



$r^2 = 4\cos 2\theta$

$r\dot{r} = -4\sin 2\theta\dot{\theta}$

$r\ddot{r} + \dot{r}^2 = -4\sin 2\theta\ddot{\theta} - 8\cos 2\theta\dot{\theta}^2$

When $\theta = 0$, $\dot{\theta} = 6$, $\ddot{\theta} = 0$

$r = 2$, $\dot{r} = 0$, $\ddot{r} = -144$

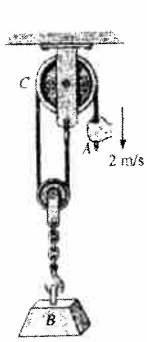
$v_r = \dot{r} = 0$ **Ans**

$v_\theta = r\dot{\theta} = 2(6) = 12$ ft/s **Ans**

$a_r = \ddot{r} - r\dot{\theta}^2 = -144 - 2(6)^2 = -216\ \text{ft/s}^2$ **Ans**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2(0) + 2(0)(6) = 0$ **Ans**

***12-172.** If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.

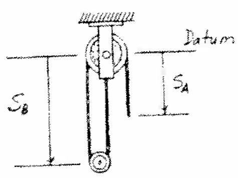


Position - Coordinate Equation : Datum is established at fixed pulley C. The position of point A and block B with respect to datum are s_A and s_B, respectively.

$$2s_B + s_A = l$$

Time Derivative : Taking the time derivative of the above equation yields

$$2v_B + v_A = 0 \qquad [1]$$



Since $v_A = 2$ m/s, from Eq. [1]

$(+\downarrow)$ $\quad 2v_B + 2 = 0$

$v_B = -1$ m/s $= 1$ m/s $\uparrow$ **Ans**

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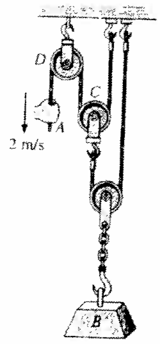
12-173. If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.



Position - Coordinate Equation : Datum is established at fixed pulley D. The position of point A, block B and pulley C with respect to datum are s_A, s_B and s_C respectively. Since the system consists of two cords, two position-coordinate equations can be derived.

$$(s_A - s_C) + (s_B - s_C) + s_B = l_1 \qquad [1]$$

$$s_B + s_C = l_2 \qquad [2]$$

Eliminating s_C from Eqs. [1] and [2] yields

$$s_A + 4s_B = l_1 + 2l_2$$

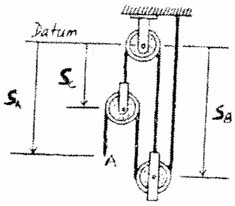
Time Derivative : Taking the time derivative of the above equation yields

$$v_A + 4v_B = 0 \qquad [3]$$

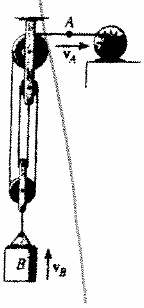
Since $v_A = 2$ m/s, from Eq. [3]

$$(+\downarrow) \qquad 2 + 4v_B = 0$$

$$v_B = -0.5 \text{ m/s} = 0.5 \text{ m/s} \uparrow \qquad \textbf{Ans}$$



12-174. Determine the constant speed at which the cable at A must be drawn in by the motor in order to hoist the load at B 15 ft in 5 s.



$$v_B = \frac{-15}{5} = -3 \text{ ft/s} = 3 \text{ ft/s} \uparrow$$

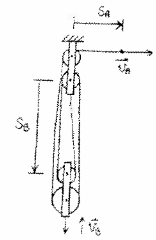
$$4s_B + s_A = l$$

$$4\Delta s_B = -\Delta s_A$$

$$4v_B = -v_A$$

$$4(-3) = -v_A$$

$$v_A = 12 \text{ ft/s} \rightarrow \qquad \textbf{Ans}$$



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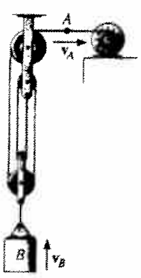
12-175. Determine the time needed for the load at B to attain a speed of 8 m/s, starting from rest, if the cable is drawn into the motor with an acceleration of 0.2 m/s^2.



$$4s_B + s_A = l$$

$$4v_B = -v_A$$

$$4a_B = -a_A$$

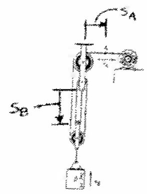
$$4a_B = -0.2$$

$$a_B = -0.05 \text{ m/s}^2$$

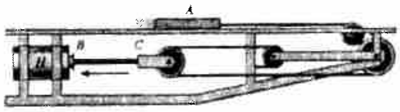
$$(+\downarrow) \qquad v_B = (v_B)_0 + a_B t$$

$$-8 = 0 - (0.05)(t)$$

$$t = 160 \text{ s} \qquad \textbf{Ans}$$



***12-176.** If the hydraulic cylinder at H draws rod BC in by 8 in., determine how far the slider at A moves.

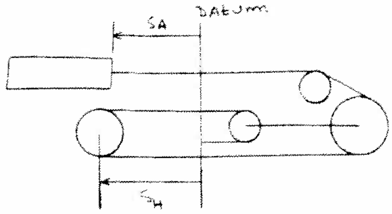


$$2s_H + s_A = l$$

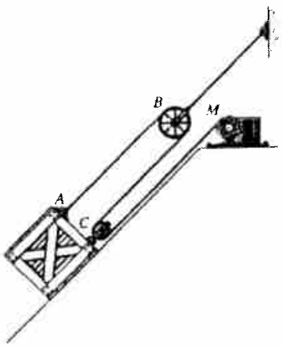
$$2\Delta s_H = -\Delta s_A$$

$$-\Delta s_A = 2(8)$$

$$\Delta s_A = -16 \text{ in.} = 16 \text{ in.} \rightarrow \quad \text{Ans}$$



12-~~177~~ 92. The crate is being lifted up the inclined plane using the motor M and the rope and pulley arrangement shown. Determine the speed at which the cable must be taken up by the motor in order to move the crate up the plane with a constant speed of ~~4 ft/s.~~ 1 m/s



Position - Coordinate Equation : Datum is established at fixed pulley B. The position of point P and crate A with respect to datum are s_P and s_A, respectively.

$$2s_A + (s_A - s_P) = l$$

$$3s_A - s_P = 0$$

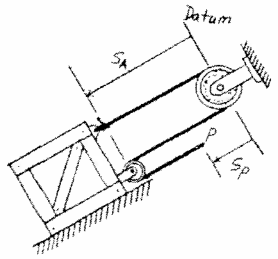
Time Derivative : Taking the time derivative of the above equation yields

$$3v_A - v_P = 0 \qquad [1]$$

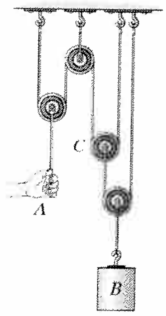
Since v_A = ~~4 ft/s~~ 1 m/s, from Eq.[1]

$$(+\ ) \qquad 3(\not{4}\ 1) - v_P = 0$$

$$v_P = \not{12 \text{ ft/s}}\ 3 \text{ m/s} \qquad \text{Ans}$$



12-178. Determine the displacement of the block at B if A is pulled down 4 ft.



$$2\,s_A + 2\,s_C = l_1$$

$$\Delta s_A = -\,\Delta s_C$$

$$s_B - s_C + s_B = l_2$$

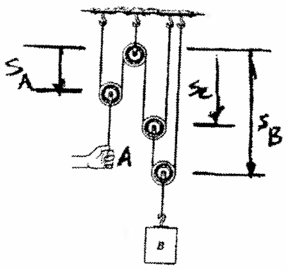
$$2\,\Delta s_B = \Delta s_C$$

Thus,

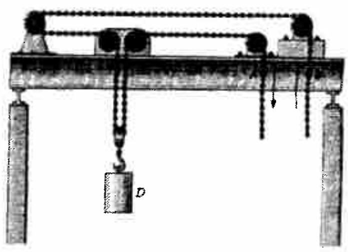
$$2\,\Delta s_B = -\,\Delta s_A$$

$$2\,\Delta s_B = -4$$

$$\Delta s_B = -\,2\text{ ft} = 2\text{ ft}\uparrow$$ **Ans**



12-179. The hoist is used to lift the load at D. If the end A of the chain is traveling downward at $v_A = 5$ ft/s and the end B is traveling upward at $v_B = 2$ ft/s, determine the velocity of the load at D.

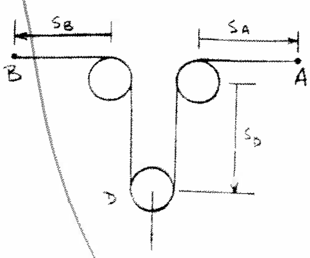


$$s_A + s_B + 2s_D = l$$

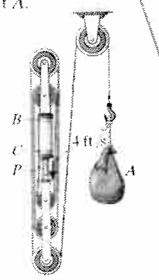
$$v_A + v_B + 2v_D = 0$$

$$5 - 2 + 2v_D = 0$$

$$v_D = -1.5\text{ ft/s} = 1.5\text{ ft/s}\uparrow$$ **Ans**



***12-180.** The pulley arrangement shown is designed for hoisting materials. If *BC remains fixed* while the plunger *P* is pushed downward with a speed of 4 ft/s, determine the speed of the load at *A*.



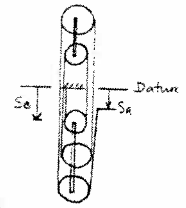
$$5s_B + (s_B - s_A) = l$$

$$6s_B - s_A = l$$

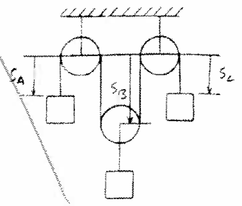
$$6v_B - v_A = 0$$

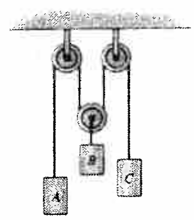
$$6(4) = v_A$$

$v_A = 24$ ft/s **Ans**



12-181. If block *A* is moving downward with a speed of 4 ft/s while *C* is moving up at 2 ft/s, determine the speed of block *B*.





$$s_A + 2s_B + s_C = l$$

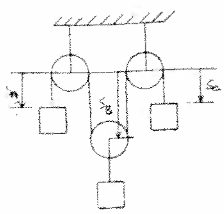
$$v_A + 2v_B + v_C = 0$$

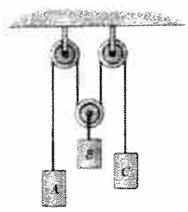
$$4 + 2v_B - 2 = 0$$

$v_B = -1$ ft/s $= 1$ ft/s ↑ **Ans**

9m 2m/s

12-182. If block *A* is moving downward at ~~6 ft/s~~ while block *C* is moving down at ~~18 ft/s~~, determine the relative velocity of block *B* with respect to *C*. 6m/s





$$s_A + 2s_B + s_C = l$$

$$v_A + 2v_B + v_C = 0$$

2 6

$$6 + 2v_B + 18 = 0$$

4 m/s

$v_B = -12$ ft/s $= 12$ ft/s ↑

4 m/s

$$v_B = v_C + v_{B/C}$$

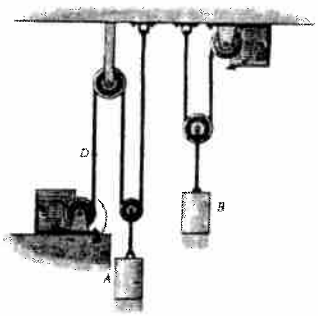
4 6

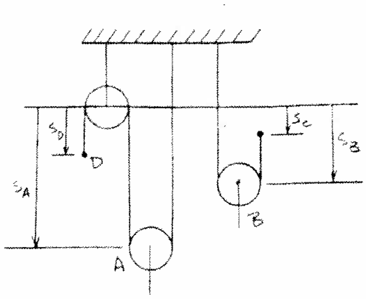
$$[12\uparrow] = [18\downarrow] + [v_{B/C}\uparrow]$$

$v_{B/C} = 30$ ft/s ↑ **Ans**

10 m/s

12-183. The motor draws in the cable at C with a constant velocity of $v_C = 4$ m/s. The motor draws in the cable at D with a constant acceleration of $a_D = 8$ m/s^2. If $v_D = 0$ when $t = 0$, determine (a) the time needed for block A to rise 3 m, and (b) the relative velocity of block A with respect to block B when this occurs.





(a) $a_D = 8$ m/s^2

$v_D = 8\,t$

$s_D = 4\,t^2$

$s_D + 2s_A = l$

$\Delta s_D = -2\Delta s_A$ (1)

$\Delta s_A = -2\,t^2$

$-3 = -2\,t^2$

$t = 1.2247 = 1.22$ s **Ans**

(b) $v_A = \dot{s}_A = -4\,t = -4(1.2247) = -4.90$ m/s $= 4.90$ m/s $\uparrow$

$s_B + (s_B - s_C) = l'$

$2v_B = v_C = -4$

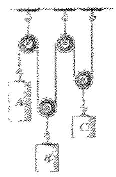
$v_B = -2$ m/s $= 2$ m/s $\uparrow$

$(+\downarrow)$ $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$-4.90 = -2 + v_{A/B}$

$v_{A/B} = -2.90$ m/s $= 2.90$ m/s $\uparrow$ **Ans**

***12-184.** If block A of the pulley system is moving downward with a speed of 4 ft/s while block C is moving up at 2 ft/s, determine the speed of block B.

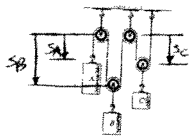


$$s_A + 2s_B + 2s_C = l$$

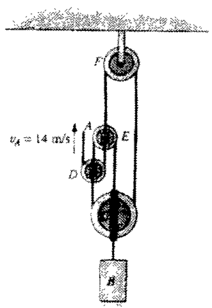
$$v_A + 2v_B + 2v_C = 0$$

$$4 + 2v_B + 2(-2) = 0$$

$$v_B = 0 \qquad \textbf{Ans}$$



12-185. If the end A on the cable is moving upwards at $v_A = 14$ m/s, determine the speed of block B.



$$s_E + s_C + (s_C - s_D) = l$$

$$(s_D - s_E) + (s_C - s_E) = l'$$

$$(s_D - s_E) + (s_D - s_A) = l''$$

$$v_E + 2v_C = v_D$$

$$2v_E - v_C = v_D$$

$$v_E + v_A = 2v_D$$

Thus

$$v_E + 2v_C = 2v_E - v_C$$

$$3v_C = v_E$$

$$2v_E - v_C = \frac{v_E}{2} + \frac{v_A}{2}$$

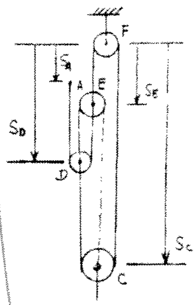
$$3v_E = 2v_C + v_A$$

$$3(3v_C) = 2v_C + v_A$$

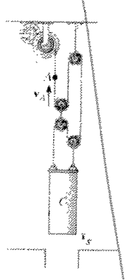
$$v_A = 7v_C$$

If $v_A = -14$ m/s,

$$v_B = v_C = -2 \text{ m/s} = 2 \text{ m/s} \uparrow \qquad \textbf{Ans}$$



12-186. The cylinder C is being lifted using the cable and pulley system shown. If point A on the cable is being drawn toward the drum with a speed of 2 m/s, determine the speed of the cylinder.

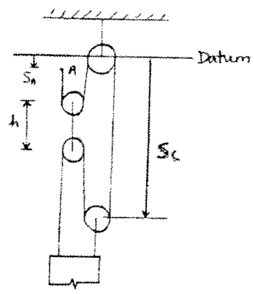


$$l = s_C + (s_C - h) + (s_C - h - s_A)$$

$$l = 3s_C - 2h - s_A$$

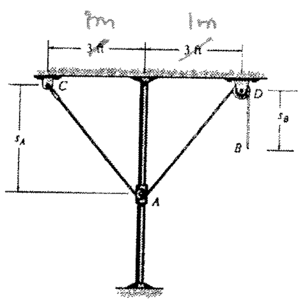
$$0 = 3v_C - v_A$$

$$v_C = \frac{v_A}{3} = \frac{-2}{3} = -0.667 \text{ m/s} = 0.667 \text{ m/s} \uparrow \qquad \textbf{Ans}$$



96

12-~~187~~. The cord is attached to the pin at C and passes over the two pulleys at A and D. The pulley at A is attached to the smooth collar that travels along the vertical rod. Determine the velocity and acceleration of the end of the cord at B if at the instant s_A = ~~4 ft~~ 1m the collar is moving upwards at ~~5 ft/s~~ 2m/s, which is decreasing at ~~2 ft/s²~~ 1m/s².



~~$2\sqrt{s_A^2 + 3^2} + s_B = l$~~ $\quad 2\sqrt{s_A^2 + 1^2} + s_B = l$

$$2\left(\frac{1}{2}\right)(s_A^2 + 9)^{-\frac{1}{2}}(2s_A\dot{s}_A) + \dot{s}_B = 0$$

$$\dot{s}_B = -\frac{2s_A\dot{s}_A}{(s_A^2 + 9)^{\frac{1}{2}}}$$

$$\ddot{s}_B = -2\dot{s}_A^2(s_A^2 + 9)^{-\frac{1}{2}} - (2s_A\ddot{s}_A)(s_A^2 + 9)^{-\frac{1}{2}} - (2s_A\dot{s}_A)\left[\left(-\frac{1}{2}\right)(s_A^2 + 9)^{-\frac{3}{2}}(2s_A\dot{s}_A)\right]$$

$$\ddot{s}_B = -\frac{2(\dot{s}_A^2 + s_A\ddot{s}_A)}{(s_A^2 + 9)^{\frac{1}{2}}} + \frac{2(s_A\dot{s}_A)^2}{(s_A^2 + 9)^{\frac{3}{2}}}$$

At s_A = ~~4 ft~~ 1 m

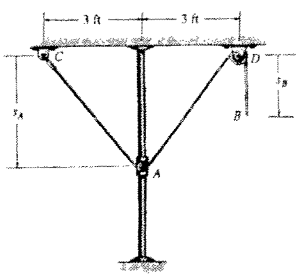
$$v_B = \dot{s}_B = -\frac{2(4)(-5)}{(4^2 + 9)^{\frac{1}{2}}} = 8 \text{ ft/s} \downarrow \qquad \textbf{Ans}$$

2.83 m/s

$$a_B = \ddot{s}_B = -\frac{2[(-5)^2 + (4)(2)]}{(4^2 + 9)^{\frac{1}{2}}} + \frac{2[(4)(-5)]^2}{(4^2 + 9)^{\frac{3}{2}}} = -6.80 \text{ ft/s}^2 = 6.80 \text{ ft/s}^2 \uparrow \qquad \textbf{Ans}$$

$1^2 + 1 \quad -0.72 \text{ m/s}^2 = 0.72 \text{ m/s}^2 \uparrow$

***12-188.** The 16-ft-long cord is attached to the pin at C and passes over the two pulleys at A and D. The pulley at A is attached to the smooth collar that travels along the vertical rod. When $s_B = 6$ ft, the end of the cord at B is pulled downwards with a velocity of 4 ft/s and is given an acceleration of 3 ft/s². Determine the velocity and acceleration of the collar at this instant.



$$2\sqrt{s_A^2 + 3^2} + s_B = l$$

$$2\left(\frac{1}{2}\right)\left(s_A^2 + 9\right)^{-\frac{1}{2}}\left(2s_A \dot{s}_A\right) + \dot{s}_B = 0$$

$$\dot{s}_B = -\frac{2s_A \dot{s}_A}{\left(s_A^2 + 9\right)^{\frac{1}{2}}}$$

$$\ddot{s}_B = -2\dot{s}_A^2\left(s_A^2 + 9\right)^{-\frac{1}{2}} - \left(2s_A \ddot{s}_A\right)\left(s_A^2 + 9\right)^{-\frac{1}{2}} - \left(2s_A \dot{s}_A\right)\left[\left(-\frac{1}{2}\right)\left(s_A^2 + 9\right)^{-\frac{3}{2}}\left(2s_A \dot{s}_A\right)\right]$$

$$\ddot{s}_B = -\frac{2\left(\dot{s}_A^2 + s_A \ddot{s}_A\right)}{\left(s_A^2 + 9\right)^{\frac{1}{2}}} + \frac{2\left(s_A \dot{s}_A\right)^2}{\left(s_A^2 + 9\right)^{\frac{3}{2}}}$$

At $s_B = 6$ ft, $\dot{s}_B = 4$ ft/s, $\ddot{s}_B = 3$ ft/s²

$$2\sqrt{s_A^2 + 3^2} + 6 = 16$$

$$s_A = 4 \text{ ft}$$

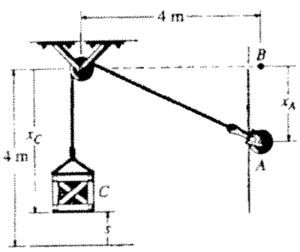
$$4 = -\frac{2(4)\left(\dot{s}_A\right)}{\left(4^2 + 9\right)^{\frac{1}{2}}}$$

$v_A = \dot{s}_A = -2.5$ ft/s $= 2.5$ ft/s ↑ **Ans**

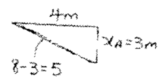
$$3 = -\frac{2\left[(-2.5)^2 + 4\left(\ddot{s}_A\right)\right]}{\left(4^2 + 9\right)^{\frac{1}{2}}} + \frac{2[4(-2.5)]^2}{\left(4^2 + 9\right)^{\frac{3}{2}}}$$

$a_A = \ddot{s}_A = -2.4375 = 2.44$ ft/s² ↑ **Ans**

12-189. The crate C is being lifted by moving the roller at A downward with a constant speed of $v_A = 2$ m/s along the guide. Determine the velocity and acceleration of the crate at the instant $s = 1$ m. When the roller is at B, the crate rests on the ground. Neglect the size of the pulley in the calculation. *Hint:* Relate the coordinates x_C and x_A using the problem geometry, then take the first and second time derivatives.



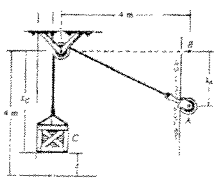
$$x_C + \sqrt{x_A^2 + (4)^2} = l$$



$$\dot{x}_C + \frac{1}{2}(x_A^2 + 16)^{-1/2}(2x_A)(\dot{x}_A) = 0$$

$$\ddot{x}_C - \frac{1}{2}(x_A^2 + 16)^{-3/2}(2x_A^2)(\dot{x}_A^2) + (x_A^2 + 16)^{-1/2}(\dot{x}_A)^2 + (x_A^2 + 16)^{-1/2}(x_A)(\ddot{x}_A) = 0$$

When $s = 1$ m, $l = 8$ m,



$x_C = 3$ m

$x_A = 3$ m

$v_A = \dot{x}_A = 2$ m/s

$a_A = \ddot{x}_A = 0$

Thus,

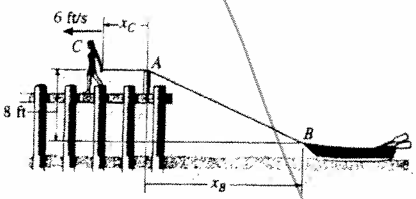
$$v_C + [(3)^2 + 16]^{-1/2}(3)(2) = 0$$

$v_C = -1.2$ m/s $= 1.2$ m/s ↑ **Ans**

$$a_C - [(3)^2 + 16]^{-3/2}(3)^2(2)^2 + [(3)^2 + 16]^{-1/2}(2)^2 + 0 = 0$$

$a_C = -0.512$ m/s² $= 0.512$ m/s² ↑ **Ans**

12-190. The girl at C stands near the edge of the pier and pulls in the rope *horizontally* at a constant speed of 6 ft/s. Determine how fast the boat approaches the pier at the instant the rope length AB is 50 ft.



The length l of cord is

$$\sqrt{(8)^2 + x_B^2} + x_C = l$$

Taking the time derivative:

$$\frac{1}{2}[(8)^2 + x_B^2]^{-1/2}\, 2x_B\dot{x}_B + \dot{x}_C = 0 \qquad (1)$$

$$\dot{x}_C = 6 \text{ ft/s}$$

When $AB = 50$ ft,

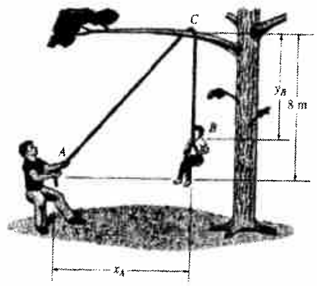
$$x_B = \sqrt{(50)^2 - (8)^2} = 49.356 \text{ ft}$$

From Eq. (1)

$$\frac{1}{2}[(8)^2 + (49.356)^2]^{-1/2}\, 2(49.356)(\dot{x}_B) + 6 = 0$$

$$\dot{x}_B = -6.0783 = 6.08 \text{ ft/s} \leftarrow \qquad \textbf{Ans}$$

12-191. The man pulls the boy up to the tree limb C by walking backward. If he starts from rest when $x_A = 0$ and moves backward with a constant acceleration $a_A = 0.2\ \text{m/s}^2$, determine the speed of the boy at the instant $y_B = 4$ m. Neglect the size of the limb. When $x_A = 0$, $y_B = 8$ m, so that A and B are coincident, i.e., the rope is 16 m long.



Position - Coordinate Equation : Using the Pythagorean theorem to determine l_{AC}, we have $l_{AC} = \sqrt{x_A^2 + 8^2}$. Thus,

$$l = l_{AC} + y_B$$
$$16 = \sqrt{x_A^2 + 8^2} + y_B$$
$$y_B = 16 - \sqrt{x_A^2 + 64} \qquad [1]$$

Time Derivative : Taking the time derivative of Eq. [1] where $v_A = \frac{dx_A}{dt}$ and $v_B = \frac{dy_B}{dt}$, we have

$$v_B = \frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 64}}\frac{dx_A}{dt}$$
$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 64}}v_A \qquad [2]$$

At the instant $y_B = 4$ m, from Eq. [1], $4 = 16 - \sqrt{x_A^2 + 64}$, $x_A = 8.944$ m. The velocity of the man at that instant can be obtained.

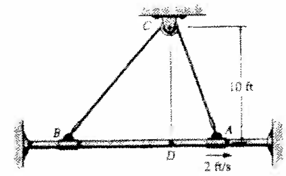
$$v_A^2 = (v_0)_A^2 + 2(a_c)_A[s_A - (s_0)_A]$$
$$v_A^2 = 0 + 2(0.2)(8.944 - 0)$$
$$v_A = 1.891 \text{ m/s}$$

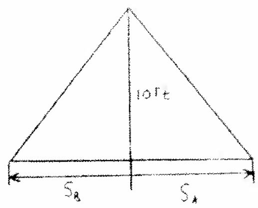
Substitute the above results into Eq. [2] yields

$$v_B = -\frac{8.944}{\sqrt{8.944^2 + 64}}(1.891) = -1.41 \text{ m/s} = 1.41 \text{ m/s} \uparrow \qquad \textbf{Ans}$$

Note : The negative sign indicates that velocity v_B is in the opposite direction to that of positive y_B.

***12-192.** Collars A and B are connected to the cord that passes over the small pulley at C. When A is located at D, B is 24 ft to the left of D. If A moves at a constant speed of 2 ft/s to the right, determine the speed of B when A is 4 ft to the right of D.





$$l = \sqrt{(24)^2 + (10)^2} + 10 = 36 \text{ ft}$$

$$\sqrt{(10)^2 + s_B^2} + \sqrt{(10)^2 + s_A^2} = 36$$

$$\frac{1}{2}\left(100 + s_B^2\right)^{-\frac{1}{2}}\left(2 s_B \dot{s}_B\right) + \frac{1}{2}\left(100 + s_A^2\right)^{-\frac{1}{2}}\left(2 s_A \dot{s}_A\right) = 0$$

$$\dot{s}_B = -\left(\frac{s_A \dot{s}_A}{s_B}\right)\left(\frac{100 + s_B^2}{100 + s_A^2}\right)^{\frac{1}{2}}$$

At $s_A = 4$,

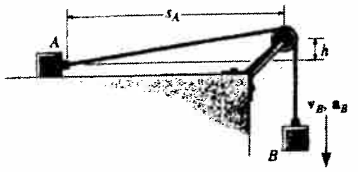
$$\sqrt{(10)^2 + s_B^2} + \sqrt{(10)^2 + (4)^2} = 36$$

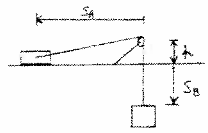
$$s_B = 23.163 \text{ ft}$$

Thus,

$$\dot{s}_B = -\left(\frac{4(2)}{23.163}\right)\left(\frac{100 + (23.163)^2}{100 + 4^2}\right)^{\frac{1}{2}} = -0.809 \text{ ft/s} = 0.809 \text{ ft/s} \rightarrow$$ **Ans**

12-193. If block B is moving down with a velocity v_B and has an acceleration a_B, determine the velocity and acceleration of block A in terms of the parameters shown.





$$l = s_B + \sqrt{s_A^2 + h^2}$$

$$0 = \dot{s}_B + \frac{1}{2}(s_A^2 + h^2)^{-1/2}\, 2 s_A \dot{s}_A$$

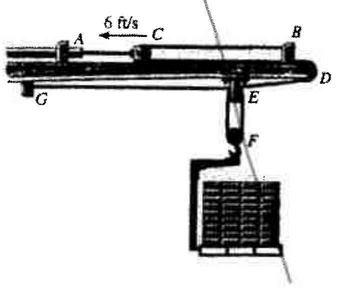
$$v_A = \dot{s}_A = \frac{-\dot{s}_B (s_A^2 + h^2)^{1/2}}{s_A}$$

$$v_A = -v_B\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{1/2}$$ **Ans**

$$a_A = \dot{v}_A = -\dot{v}_B\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{1/2} - v_B\left(\frac{1}{2}\right)\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{-1/2}(h^2)(-2)(s_A)^{-3}\dot{s}_A$$

$$a_A = -a_B\left(1 + \left(\frac{h}{s_B}\right)^2\right)^{1/2} + \frac{v_A v_B h^2}{s_A^3}\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{-1/2}$$ **Ans**

12-194. Vertical motion of the load is produced by movement of the piston at A on the boom. Determine the distance the piston or pulley at C must move to the left in order to lift the load 2 ft. The cable is attached at B, passes over the pulley at C, then D, E, F, and again around E, and is attached at G.

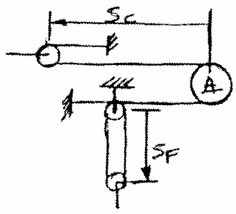


$$2s_C + 2s_F = l$$

$$2\Delta s_C = -2\Delta s_F$$

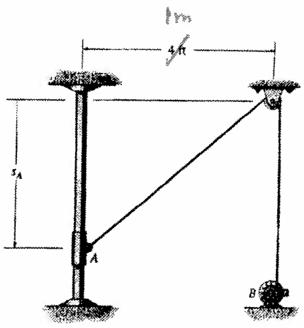
$$\Delta s_C = -\Delta s_F$$

$$\Delta s_C = -(-2\text{ ft}) = 2\text{ ft} \qquad \textbf{Ans}$$



100

12-195. The motion of the collar at A is controlled by a motor at B such that when the collar is at s_A = ~~3 ft~~ 0.75 m it is moving upwards at ~~2 ft/s~~ 0.5 m/s and slowing down at ~~1 ft/s²~~ 0.25 m/s². Determine the velocity and acceleration of the cable as it is drawn into the motor at B at this instant.



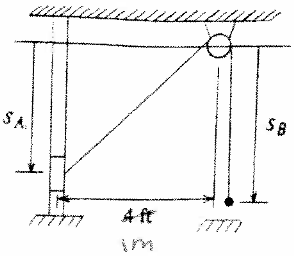
$$\sqrt{s_A^2 + 4^2} + s_B = l$$

$$\frac{1}{2}(s_A^2 + 16)^{-\frac{1}{2}}(2s_A)\dot{s}_A + \dot{s}_B = 0$$

$$\dot{s}_B = -s_A\dot{s}_A(s_A^2 + 16)^{-\frac{1}{2}}$$

$$\ddot{s}_B = -\left[(\dot{s}_A)^2(s_A^2 + 16)^{-\frac{1}{2}} + s_A\ddot{s}_A(s_A^2 + 16)^{-\frac{1}{2}} + s_A\dot{s}_A\left(-\frac{1}{2}\right)(s_A^2 + 16)^{-\frac{3}{2}}(2s_A\dot{s}_A)\right]$$

$$\ddot{s}_B = \frac{(s_A\dot{s}_A)^2}{(s_A^2 + 16)^{\frac{3}{2}}} - \frac{(\dot{s}_A)^2 + s_A\ddot{s}_A}{(s_A^2 + 16)^{\frac{1}{2}}}$$



Evaluating these equations :

$$\dot{s}_B = -3(-2)((3)^2 + 16)^{-\frac{1}{2}} = 1.20\text{ ft/s}\downarrow \qquad \textbf{Ans}$$

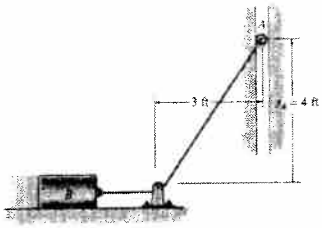
0.469 m/s ↓

$$\ddot{s}_B = \frac{((3)(-2))^2}{((3)^2 + 16)^{\frac{3}{2}}} - \frac{(-2)^2 + 3(1)}{((3)^2 + 16)^{\frac{1}{2}}} = -1.11\text{ ft/s}^2 = 1.11\text{ ft/s}^2\uparrow \qquad \textbf{Ans}$$

$$= \frac{((0.75)(-0.5))^2}{((0.75)^2 + 1)^{3/2}} - \frac{(-0.5)^2 + (0.75)(0.25)}{((0.75)^2 + 1)^{1/2}} = -0.278\text{ m/s}^2$$

$$= 0.278\text{ m/s}^2\uparrow$$

***12-196.** The roller at A is moving upward with a velocity of $v_A = 3$ ft/s and has an acceleration of $a_A = 4\ \text{ft/s}^2$ when $s_A = 4$ ft. Determine the velocity and acceleration of block B at this instant.



$$s_B + \sqrt{(s_A)^2 + 3^2} = l$$

$$\dot{s}_B + \frac{1}{2}\left[(s_A)^2 + 3^2\right]^{-\frac{1}{2}}(2s_A)\dot{s}_A = 0$$

$$\dot{s}_B + \left[s_A^2 + 9\right]^{-\frac{1}{2}}\left(s_A\dot{s}_A\right) = 0$$

$$\ddot{s}_B - \left[(s_A)^2 + 9\right]^{-\frac{3}{2}}\left(s_A^2\dot{s}_A^2\right) + \left[s_A^2 + 9\right]^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left[s_A^2 + 9\right]^{-\frac{1}{2}}\left(s_A\ddot{s}_A\right) = 0$$

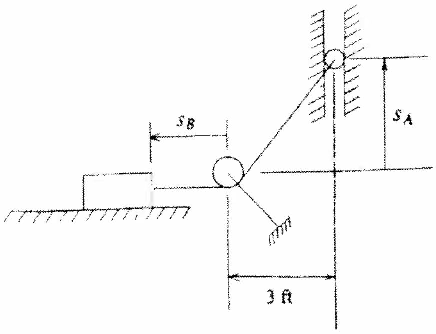
At $s_A = 4$ ft, $\dot{s}_A = 3$ ft/s, $\ddot{s}_A = 4\ \text{ft/s}^2$

$$\dot{s}_B + \left(\frac{1}{5}\right)(4)(3) = 0$$

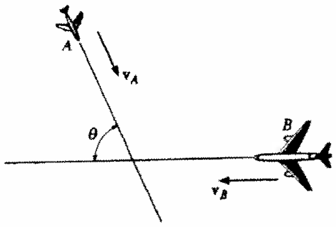
$v_B = -2.4$ ft/s $= 2.40$ ft/s $\rightarrow$ **Ans**

$$\ddot{s}_B - \left(\frac{1}{5}\right)^3(4)^2(3)^2 + \left(\frac{1}{5}\right)(3)^2 + \left(\frac{1}{5}\right)(4)(4) = 0$$

$a_B = -3.85\ \text{ft/s}^2 = 3.85\ \text{ft/s}^2 \rightarrow$ **Ans**



12-197. Two planes, A and B, are flying at the same altitude. If their velocities are $v_A = 600$ km/h and $v_B = 500$ km/h such that the angle between their straight-line courses is $\theta = 75°$, determine the velocity of plane B with respect to plane A.



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$[\,500 \leftarrow\,] = [\,600 \overset{75°}{\searrow}\,] + \mathbf{v}_{B/A}$$

$(\overset{+}{\leftarrow})$ $\quad 500 = -600\cos 75° + (v_{B/A})_x$

$(v_{B/A})_x = 655.29 \leftarrow$

$(+\uparrow)$ $\quad 0 = -600\sin 75° + (v_{B/A})_y$

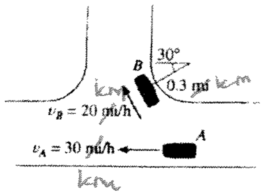
$(v_{B/A})_y = 579.56 \uparrow$

$$(v_{B/A}) = \sqrt{(655.29)^2 + (579.56)^2}$$

$v_{A/B} = 875$ km/h **Ans**

$\theta = \tan^{-1}\left(\dfrac{579.56}{655.29}\right) = 41.5°$ **Ans**

12-198. At the instant shown, cars A and B are traveling at speeds of 30 mi/h and 20 mi/h, respectively. If B is increasing its speed by 1200 mi/h², while A maintains a constant speed, determine the velocity and acceleration of B with respect to A.



$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

$20 \overset{30°}{\searrow} = 30 \leftarrow + (v_{B/A})_x \rightarrow + (v_{B/A})_y \uparrow$

$(\overset{+}{\rightarrow}) \quad -20\sin 30° = -30 + (v_{B/A})_x$

$(+\uparrow) \quad 20\cos 30° = (v_{B/A})_y$

Solving

$(v_{B/A})_x = 20 \rightarrow$

$(v_{B/A})_y = 17.32 \uparrow$

$v_{B/A} = \sqrt{(20)^2 + (17.32)^2} = 26.5$ mi/h **Ans**

$\theta = \tan^{-1}\left(\frac{17.32}{20}\right) = 40.9°$ **Ans**

$(a_B)_n = \frac{(20)^2}{0.3} = 1333.3$

$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

$1200 \overset{30°}{\searrow} + 1333.3 \underset{30°}{\angle\theta} = 0 + (a_{B/A})_x \rightarrow + (a_{B/A})_y \uparrow$

$(\overset{+}{\rightarrow}) \quad -1200\sin 30° + 1333.3\cos 30° = (a_{B/A})_x$

$(+\uparrow) \quad 1200\cos 30° + 1333.3\sin 30° = (a_{B/A})_y$

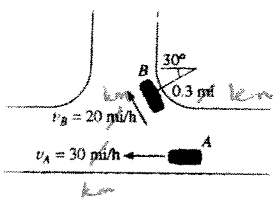
Solving

$(a_{B/A})_x = 554.7 \rightarrow \quad ; \quad (a_{B/A})_y = 1705.9 \uparrow$

$a_{B/A} = \sqrt{(554.7)^2 + (1705.9)^2} = 1.79(10^3)$ mi/h² **Ans**

$\theta = \tan^{-1}\left(\frac{1705.9}{554.7}\right) = 72.0°$ **Ans**

12-199. At the instant shown, cars A and B are traveling at speeds of 30 mi/h and 20 mi/h, respectively. If A is increasing its speed at 400 mi/h² whereas the speed of B is decreasing at 800 mi/h², determine the velocity and acceleration of B with respect to A.



$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

$\left[\frac{20^2}{0.3} = 1333.3\right]_{\angle\theta\ 30°} + [800]_{30°\nwarrow} = [400]_{\leftarrow} + [(a_{B/A})_x]_{\rightarrow} + [(a_{B/A})_y]_{\uparrow}$

$(\overset{+}{\rightarrow}) \quad 1333.3\cos 30° + 800\sin 30° = -400 + (a_{B/A})_x$

$(a_{B/A})_x = 1954.7 \rightarrow$

$(+\uparrow) \quad 1333.3\sin 30° - 800\cos 30° = (a_{B/A})_y$

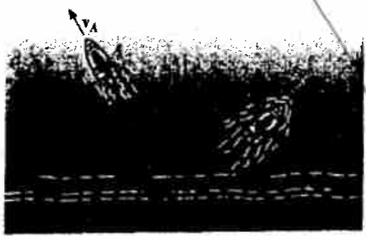
$(a_{B/A})_y = -26.154 = 26.154 \downarrow$

$(a_{B/A}) = \sqrt{(1954.7)^2 + (26.154)^2}$

$a_{B/A} = 1955$ mi/h² **Ans**

$\theta = \tan^{-1}\left(\frac{26.154}{1954.7}\right) = 0.767°$ **Ans**

•12-200. Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 20$ ft/s and $v_B = 15$ ft/s, determine the speed of boat A with respect to boat B. How long after leaving the shore will the boats be 800 ft apart?



$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-20\sin 30°\mathbf{i} + 20\cos 30°\mathbf{j} = 15\cos 45°\mathbf{i} + 15\sin 45°\mathbf{j} + \mathbf{v}_{A/B}$$

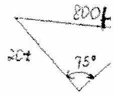
$$\mathbf{v}_{A/B} = \{-20.61\mathbf{i} - 6.714\mathbf{j}\} \text{ ft/s}$$

$$v_{A/B} = \sqrt{(-20.61)^2 + (-6.714)^2} = 21.7 \text{ ft/s} \quad \textbf{Ans}$$

$$\theta = \tan^{-1}\left(\frac{6.714}{20.61}\right) = 18.0° \quad \textbf{Ans}$$

$$(800)^2 = (20\,t)^2 + (15\,t)^2 - 2(20\,t)(15\,t)\cos 75°$$

$$t = 36.9 \text{ s} \quad \textbf{Ans}$$



Also

$$t = \frac{800}{v_{A/B}} = \frac{800}{21.68} = 36.9 \text{ s} \quad \textbf{Ans}$$

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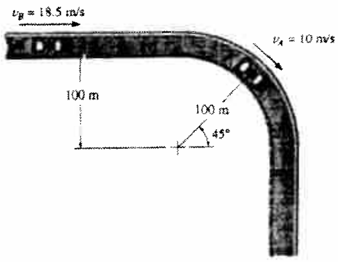
12-201. At the instant shown, the car at A is traveling at 10 m/s around the curve while increasing its speed at 5 m/s^2. The car at B is traveling at 18.5 m/s along the straightaway and increasing its speed at 2 m/s^2. Determine the relative velocity and relative acceleration of A with respect to B at this instant.



$$\mathbf{v}_A = 10\cos 45°\mathbf{i} - 10\sin 45°\mathbf{j} = \{7.071\mathbf{i} - 7.071\mathbf{j}\} \text{ m/s}$$

$$\mathbf{v}_B = \{18.5\mathbf{i}\} \text{ m/s}$$

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$= (7.071\mathbf{i} - 7.071\mathbf{j}) - 18.5\mathbf{i} = \{-11.429\mathbf{i} - 7.071\mathbf{j}\} \text{ m/s}$$

$$v_{A/B} = \sqrt{(-11.429)^2 + (-7.071)^2} = 13.4 \text{ m/s} \quad \textbf{Ans}$$

$$\theta = \tan^{-1}\frac{7.071}{11.429} = 31.7° \quad \textbf{Ans}$$

$$(a_A)_n = \frac{v_A^2}{\rho} = \frac{10^2}{100} = 1 \text{ m/s}^2 \qquad (a_A)_t = 5 \text{ m/s}^2$$

$$\mathbf{a}_A = (5\cos 45° - 1\cos 45°)\mathbf{i} + (-1\sin 45° - 5\sin 45°)\mathbf{j}$$

$$= \{2.828\mathbf{i} - 4.243\mathbf{j}\} \text{ m/s}^2$$

$$\mathbf{a}_B = \{2\mathbf{i}\} \text{ m/s}^2$$

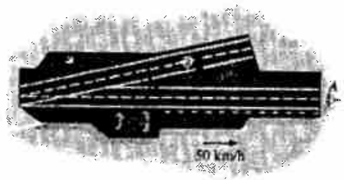
$$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B$$

$$= (2.828\mathbf{i} - 4.243\mathbf{j}) - 2\mathbf{i} = \{0.828\mathbf{i} - 4.24\mathbf{j}\} \text{ m/s}^2$$

$$a_{A/B} = \sqrt{0.828^2 + (-4.243)^2} = 4.32 \text{ m/s}^2 \quad \textbf{Ans}$$

$$\theta = \tan^{-1}\frac{4.243}{0.828} = 79.0° \quad \textbf{Ans}$$

12-202. An aircraft carrier is traveling forward with a velocity of 50 km/h. At the instant shown, the plane at A has just taken off and has attained a forward horizontal air speed of 200 km/h, measured from still water. If the plane at B is traveling along the runway of the carrier at 175 km/h in the direction shown, determine the velocity of A with respect to B.



$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$

$\mathbf{v}_B = 50\mathbf{i} + 175\cos 15°\mathbf{i} + 175\sin 15°\mathbf{j} = 219.04\mathbf{i} + 45.293\mathbf{j}$

$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$200\mathbf{i} = 219.04\mathbf{i} + 45.293\mathbf{j} + (v_{A/B})_x\mathbf{i} + (v_{A/B})_y\mathbf{j}$

$200 = 219.04 + (v_{A/B})_x$

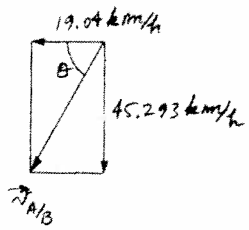
$0 = 45.293 + (v_{A/B})_y$

$(v_{A/B})_x = -19.04$

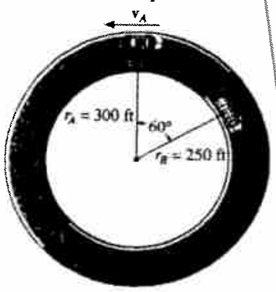
$(v_{A/B})_y = -45.293$

$v_{A/B} = \sqrt{(-19.04)^2 + (-45.293)^2} = 49.1$ km/h **Ans**

$\theta = \tan^{-1}\left(\frac{45.293}{19.04}\right) = 67.2°$ **Ans**



12-203. Cars A and B are traveling around the circular race track. At the instant shown, A has a speed of 90 ft/s and is increasing its speed at the rate of 15 ft/s^2, whereas B has a speed of 105 ft/s and is decreasing its speed at 25 ft/s^2. Determine the relative velocity and relative acceleration of car A with respect to car B at this instant.



$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$-90\mathbf{i} = -105\sin 30°\,\mathbf{i} + 105\cos 30°\mathbf{j} + \mathbf{v}_{A/B}$

$\mathbf{v}_{A/B} = \{-37.5\mathbf{i} - 90.93\mathbf{j}\}$ ft/s

$v_{A/B} = \sqrt{(-37.5)^2 + (-90.93)^2} = 98.4$ ft/s **Ans**

$\theta = \tan^{-1}\left(\frac{90.93}{37.5}\right) = 67.6°$ **Ans**

$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$

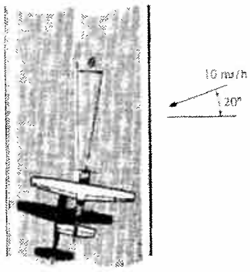
$-15\mathbf{i} - \frac{(90)^2}{300}\mathbf{j} = 25\cos 60°\mathbf{i} - 25\sin 60°\mathbf{j} - 44.1\sin 60°\mathbf{i} - 44.1\cos 60°\mathbf{j} + \mathbf{a}_{A/B}$

$\mathbf{a}_{A/B} = \{10.69\mathbf{i} + 16.70\mathbf{j}\}$ ft/s^2

$a_{A/B} = \sqrt{(10.69)^2 + (16.70)^2} = 19.8$ ft/s^2 **Ans**

$\theta = \tan^{-1}\left(\frac{16.70}{10.69}\right) = 57.4°$ **Ans**

***12-204.** The airplane has a speed relative to the wind of 100 mi/h. If the wind relative to the ground is 10 mi/h, determine the angle θ at which the plane must be directed in order to travel in the direction of the runway. Also, what is its speed relative to the runway?



$\mathbf{v}_P = \mathbf{v}_{P/W} + \mathbf{v}_W$

$v_P\mathbf{j} = 100\sin\theta\mathbf{i} + 100\cos\theta\mathbf{j} - 10\cos20°\mathbf{i} - 10\sin20°\mathbf{j}$

$0 = 100\sin\theta - 10\cos20°$

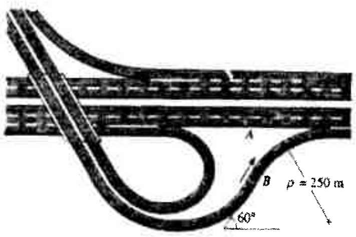
$v_P = 100\cos\theta - 10\sin20°$

Solving,

$v_P = 96.1$ mi/h **Ans**

$\theta = 5.39°$ **Ans**

12-205. At the instant shown car A is traveling with a velocity of 30 m/s and has an acceleration of 2 m/s² along the highway. At the same instant B is traveling on the trumpet interchange curve with a speed of 15 m/s, which is decreasing at 0.8 m/s². Determine the relative velocity and relative acceleration of B with respect to A at this instant.



$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

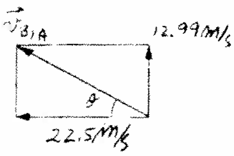
$15\cos60°\mathbf{i} + 15\sin60°\mathbf{j} = 30\mathbf{i} + (v_{B/A})_x\mathbf{i} + (v_{B/A})_y\mathbf{j}$

$15\cos60° = 30 + (v_{B/A})_x$

$15\sin60° = 0 + (v_{B/A})_y$

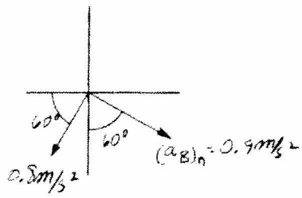
$(v_{B/A})_x = -22.5 = 22.5$ m/s $\leftarrow$

$(v_{B/A})_y = 12.99$ m/s $\uparrow$



$v_{B/A} = \sqrt{(22.5)^2 + (12.99)^2} = 26.0$ m/s **Ans**

$\theta = \tan^{-1}\left(\frac{12.99}{22.5}\right) = 30°$ **Ans**



$(a_B)_n = \frac{v^2}{\rho} = \frac{15^2}{250} = 0.9$ m/s²

$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

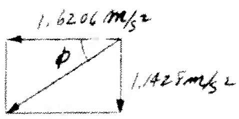
$-0.8\cos60°\mathbf{i} - 0.8\sin60°\mathbf{j} + 0.9\sin60°\mathbf{i} - 0.9\cos60°\mathbf{j} = 2\mathbf{i} + (a_{B/A})_x\mathbf{i} + (a_{B/A})_y\mathbf{j}$

$-0.8\cos60° + 0.9\sin60° = 2 + (a_{B/A})_x$

$-0.8\sin60° - 0.9\cos60° = (a_{B/A})_y$

$(a_{B/A})_x = -1.6206$ ft/s² $= 1.6206$ m/s² $\leftarrow$

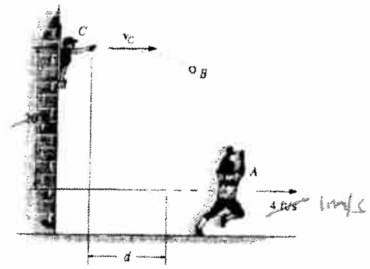
$(a_{B/A})_y = -1.1428$ ft/s² $= 1.1428$ m/s² $\downarrow$



$a_{B/A} = \sqrt{(1.6206)^2 + (1.1428)^2} = 1.98$ m/s² **Ans**

$\phi = \tan^{-1}\left(\frac{1.1428}{1.6206}\right) = 35.2°$ **Ans**

12-206. The boy A is moving in a straight line away from the building at a constant speed of 4 ft/s. The boy C throws the ball B horizontally when A is at d = 10 ft. At what speed must C throw the ball so that A can catch it? Also determine the relative speed of the ball with respect to boy A at the instant the catch is made.



For B:

$(\xrightarrow{+})\ s = s_0 + v_0 t$

$s_B = 0 + v_C t$

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$-20 = 0 + 0 + \frac{1}{2}(-32.2)t^2$

$t = 1.1146$ s

For A:

$(\xrightarrow{+})\ s = s_0 + v_0 t$

$s_A = 10 + 4t$

Require $s_B = s_A$ at $t = 1.1146$ s

$v_C(1.1146) = 10 + 4(1.1146)$

$v_C = 12.97 = 13.0$ ft/s **Ans**

When ball is caught

$(v_{Bx})_2 = 12.97$ ft/s $\rightarrow$

$(+\uparrow)\quad v = v_0 + a_c t$

$(v_{By})_2 = 0 - 32.2(1.114) = -35.89$ ft/s $= 35.89$ ft/s $\downarrow$

Thus,

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

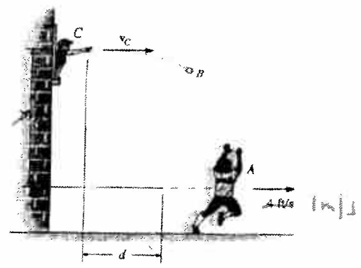
$12.97\mathbf{i} - 35.89\mathbf{j} = 4\mathbf{i} + (v_{B/A})_x\mathbf{i} - (v_{B/A})_y\mathbf{j}$

$(v_{B/A})_x = 8.97$ ft/s $\rightarrow$

$(v_{B/A})_y = 35.89$ ft/s $\downarrow$

$v_{B/A} = \sqrt{(8.97)^2 + (35.89)^2} = 37.0$ ft/s **Ans**

12-207. The boy A is moving in a straight line away from the building at a constant speed of 4 ft/s. At what horizontal distance d must he be from C in order to make the catch if the ball is thrown with a horizontal velocity of v_C = 10 ft/s? Also determine the relative speed of the ball with respect to the boy A at the instant the catch is made.



$(\xrightarrow{+})\quad s = s_0 + v_0 t$

$x = 0 + 10t$

$(+\uparrow)\quad s = s_0 + v_0 + \frac{1}{2}a_c t^2$

$-20 = 0 + 0 + \frac{1}{2}(-32.2)t^2$

$t = 1.1146$ s

$d = 11.146$ ft $= 11.1$ ft **Ans**

$(\xrightarrow{+})\quad (v_{Bx})_2 = 10$ ft/s $\rightarrow$

$(+\uparrow)\quad v = v_0 + a_c t$

$(v_{By})_2 = 0 - 32.2(1.1146) = -35.89$ ft/s $= 35.89$ ft/s $\downarrow$

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

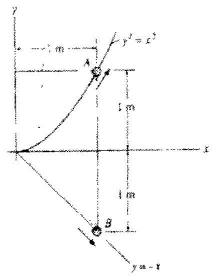
$10\mathbf{i} - 35.89\mathbf{j} = 4\mathbf{i} + (v_{B/A})_x\mathbf{i} - (v_{B/A})_y\mathbf{j}$

$(v_{B/A})_x = 6$ ft/s $\rightarrow$

$(v_{B/A})_y = 35.89$ ft/s $\downarrow$

$v_{B/A} = \sqrt{6^2 + (35.89)^2} = 36.4$ ft/s **Ans**

***12-208.** At a given instant, two particles A and B are moving with a speed of 8 m/s along the paths shown. If B is decelerating at 6 m/s^2 and the speed of A is increasing at 5 m/s^2, determine the acceleration of A with respect to B at this instant.



$y^2 = x^3$

$2y\frac{dy}{dx} = 3x^2 \qquad \frac{dy}{dx} = \frac{3x^2}{2y}\bigg|_{x=1,\,y=1} = 1.5$

$\theta = \tan^{-1}(1.5) = 56.31°$

$2\left(\frac{dy}{dx}\right)^2 + 2y\left(\frac{d^2y}{dx^2}\right) = 6x$

At $x = 1,\ y = 1$

$2(1.5)^2 + 2(1)\left(\frac{d^2y}{dx^2}\right) = 6(1) \qquad \frac{d^2y}{dx^2} = 0.75$

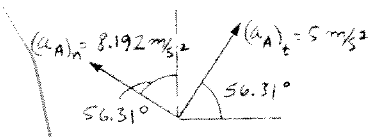
$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (1.5)^2\right]^{\frac{3}{2}}}{0.75} = 7.812$

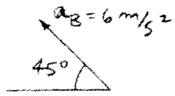
$(a_A)_n = \frac{v_A^2}{\rho} = \frac{8^2}{7.812} = 8.192$

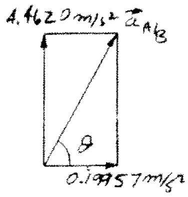
$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$

$-8.192 \sin 56.31°\mathbf{i} + 8.192 \cos 56.31°\mathbf{j} + 5 \cos 56.31°\mathbf{i} + 5 \sin 56.31°\mathbf{j}$

$= -6 \cos 45°\mathbf{i} + 6 \sin 45°\mathbf{j} + (a_{A/B})_x\mathbf{i} + (a_{A/B})_y\mathbf{j}$







$-8.192 \sin 56.31° + 5 \cos 56.31° = -6 \cos 45° + (a_{A/B})_x$

$8.192 \cos 56.31° + 5 \sin 56.31° = 6 \sin 45° + (a_{A/B})_y$

$(a_{A/B})_x = 0.199\,57 \qquad (a_{A/B})_y = 4.4620$

$a_{A/B} = \sqrt{(0.199\,57)^2 + (4.4620)^2} = 4.47$ m/s^2 **Ans**

$\theta = \tan^{-1}\left(\frac{4.4620}{0.1995}\right) = 87.4°$ ∠ **Ans**