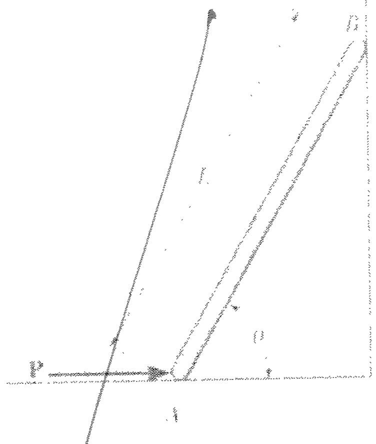
Problem 11-1

The thin rod of weight W rests against the smooth wall and floor. Determine the magnitude of force **P** needed to hold it in equilibrium.



Solution:

$$x_p = L\cos(\theta) \qquad \delta x_p = -L\sin(\theta)\delta\theta$$

$$y_W = \left(\frac{L}{2}\right)\sin(\theta) \qquad \delta y_W = \left(\frac{L}{2}\right)\cos(\theta)\delta\theta$$

$$\delta U = 0; \qquad -P\,\delta x_p - W\delta y_W = 0$$

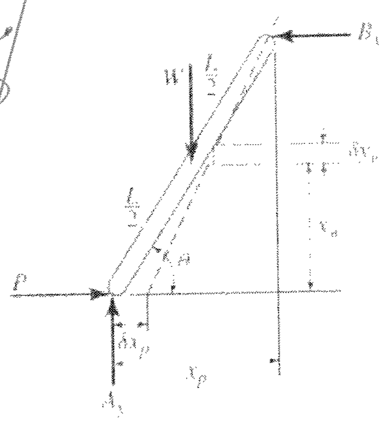
$$-P(-L\sin(\theta)\delta\theta) - W\left(\frac{L}{2}\cos(\theta)\delta\theta\right) = 0$$

$$\delta\theta\left[P\,L\sin(\theta) - \left(\frac{W\,L}{2}\right)\cos(\theta)\right] = 0$$

Since $\delta\theta \neq 0$

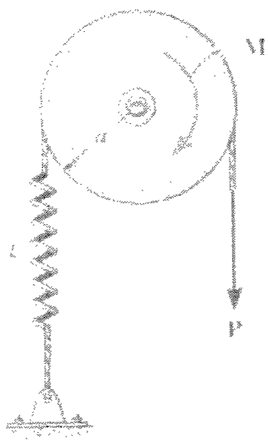
$$P\;L\sin(\theta) - \left(\frac{W\,L}{2}\right)\cos(\theta) = 0$$

$$P = \frac{W}{2}\cot(\theta)$$



Problem 11-2

The disk has a weight W and is subjected to a vertical force **P** and a couple moment M. Determine the disk's rotation θ if the end of the spring wraps around the periphery of the disk as the disk turns. The spring is originally unstretched.



Given:

$W = 10$ ~~lb~~ N

$P = 8$ ~~lb~~ N

$M = 8$ ~~lb-ft~~ N m

$a = 1.5$ ~~ft~~ m

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$k = 12\ \frac{\text{lb}}{\text{ft}}$

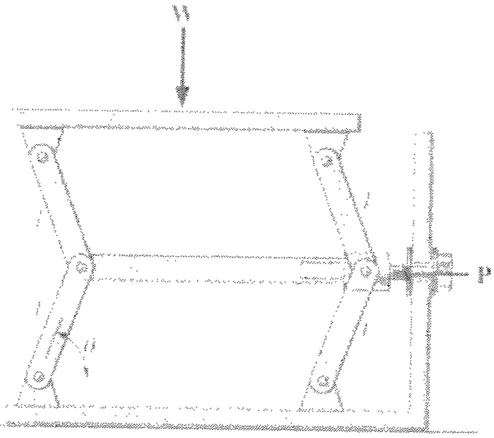
Solution:

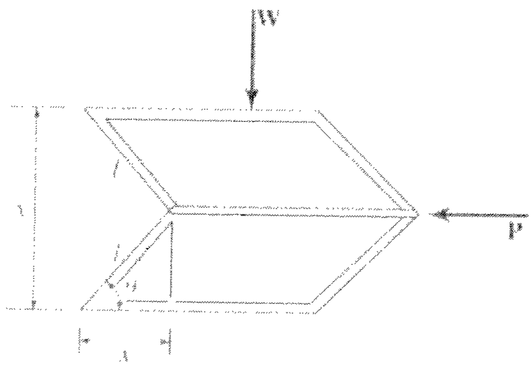
$$\delta U = P\ a\delta\ \theta + M\ \delta\theta - ka\theta a\delta\theta = \left(P\ a + M - ka^2\theta\right)\ \delta\ \theta = 0$$

$$P\ a + M - ka^2\theta = 0 \qquad \theta = \frac{P\ a + M}{ka^2} \qquad \theta = 42.4\ \text{deg}$$

Problem 11-3

The platform supports a load W. Determine the horizontal force **P** that must be supplied by the screw in order to support the platform when the links are at the arbitrary angle θ.





Solution:

$$x = l\cos(\theta) \qquad \delta x = -l\sin(\theta)\delta\theta$$

$$y = 2\ l\sin(\theta) \qquad \delta y = 2\ l\cos(\theta)\delta\theta$$

$$\delta\ U = -W\ \delta\ y - P\ \delta\ x = 0$$

$$-W(2\ l\cos(\theta)\delta\ \theta) - P(-l\sin(\theta)\delta\ \theta) = 0$$

$$-2\ W\cos(\theta) + P\sin(\theta) = 0$$

$$P = 2\ W\cot(\theta)$$

Problem 11-4

Each member of the pin-connected mechanism has mass m_l. If the spring is unstretched when $\theta = 0°$, determine the angle θ for equilibrium.

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Given:

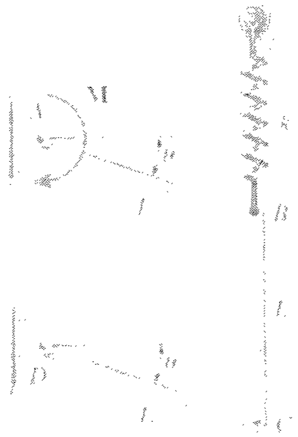
$m_1 = 8\ \text{kg}$

$k = 2500\ \dfrac{\text{N}}{\text{m}}$

$L = 300\ \text{mm}$

$M = 50\ \text{N m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution:

$y_1 = \left(\dfrac{L}{2}\right)\sin(\theta) \qquad \delta y_1 = \left(\dfrac{L}{2}\right)\cos(\theta)\delta\theta \qquad y_2 = L\sin(\theta) \qquad \delta y_2 = L\cos(\theta)\delta\theta$

$\delta U = 2\ m_1 g\delta\ y_1 + m_1 g\delta\ y_2 - k y_2 \delta y_2 + M\delta\ \theta = 0$

$\delta U = \left[m_1 g L\left[2\left(\dfrac{1}{2}\right)\cos(\theta) + \cos(\theta)\right] - kL\sin(\theta)L\cos(\theta) + M\right]\delta\theta = 0$

There are 2 solutions found by starting with different guesses

Guess $\theta = 10\ \text{deg}$ Given

$m_1\ gL\ 2\ \cos(\theta) - kL^2\sin(\theta)\cos(\theta) + M = 0 \qquad \theta = \text{Find}(\theta) \qquad \theta = 27.4\ \text{deg}$

Guess $\theta = 60\text{deg}$ Given

$m_1\ g\ L\ 2\cos(\theta) - kL^2\sin(\theta)\cos(\theta) + M = 0 \qquad \theta = \text{Find}(\theta) \qquad \theta = 72.7\ \text{deg}$

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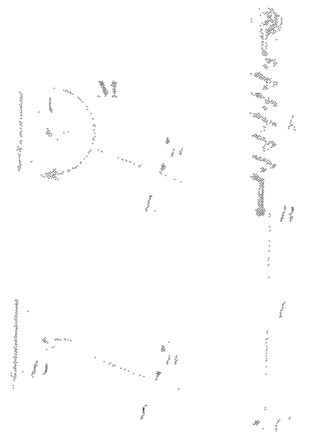
Problem 11-~~5~~ 4



Each member of the pin-connected mechanism has mass m_1. If the spring is unstretched when $\theta = 0°$, determine the required stiffness k so that the mechanism is in equilibrium when $\theta = \theta_0$.

Units Used:

$$kN = 10^3\ N$$

Given:

$$m_1 = 8\ kg$$

$$\theta = 30\ deg$$

$$L = 300\ mm$$

$$M = 0\ N \cdot m$$

$$g = 9.81\ \frac{m}{s^2}$$

Solution:

$$y_1 = \left(\frac{L}{2}\right)\sin(\theta) \qquad \delta y_1 = \left(\frac{L}{2}\right)\cos(\theta)\delta\,\theta \qquad y_2 = L\sin(\theta) \qquad \delta y_2 = L\cos(\theta)\delta\,\theta$$

$$\delta U = 2m_1 g\delta y_1 + m_1 g\delta y_2 - k\,y_2\delta y_2 + M\,\delta\,\theta = 0$$

$$\delta U = \left[m_1 g L\left[2\left(\frac{1}{2}\right)\cos(\theta) + \cos(\theta)\right] - kL\sin(\theta)L\cos(\theta) + M\right]\delta\,\theta = 0$$

Guess $k = 1\ \frac{kN}{m}$ Given

$$m_1 g L 2\cos(\theta) - kL^2\sin(\theta)\cos(\theta) + M = 0 \qquad k = \text{Find}(k) \qquad k = 1.046\ \frac{kN}{m}$$

Problem 11-~~6~~ 5

The crankshaft is subjected to torque M. Determine the horizontal compressive force **F** applied to the piston for equilibrium when $\theta = \theta_0$

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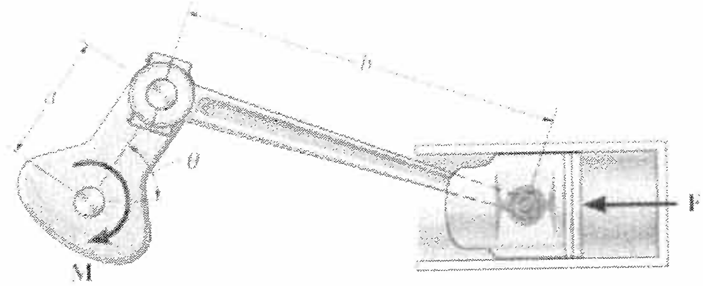
Given:

$M = 50\ \text{N m}$

$\theta_0 = 60\ \text{deg}$

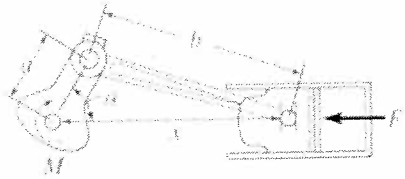
$a = 100\ \text{mm}$

$b = 400\ \text{mm}$



Solution: $\theta = \theta_0$

$b^2 = a^2 + x^2 - 2\,a\,x\cos(\theta)$



$0 = 2\,x\,\delta x - 2\,a\cos(\theta)\delta x + 2\,a x\sin(\theta)\,\delta\theta$

$$\delta x = \left(\frac{a x\sin(\theta)}{x - a\cos(\theta)}\right)\delta\theta$$

$$\delta U = -F\,\delta x + M\,\delta\theta = \left[-F\left(\frac{a x\sin(\theta)}{x - a\cos(\theta)}\right) + M\right]\delta\theta = 0$$

Guesses $x = 1\ \text{m}$ $F = 1\ \text{N}$

Given $b^2 = a^2 + x^2 - 2ax\cos(\theta)$ $-F\left(\frac{a x\sin(\theta)}{x - a\cos(\theta)}\right) + M = 0$

$\begin{pmatrix} F \\ x \end{pmatrix} = \text{Find}(F, x)$ $x = 440.512\ \text{mm}$ $F = 512\ \text{N}$

Problem 11-7

The crankshaft is subjected to torque **M**. Determine the horizontal compressive force **F** and plot the result of **F** (ordinate) versus θ(abscissa) for $0° <= \theta <= 90°$.

Units Used:

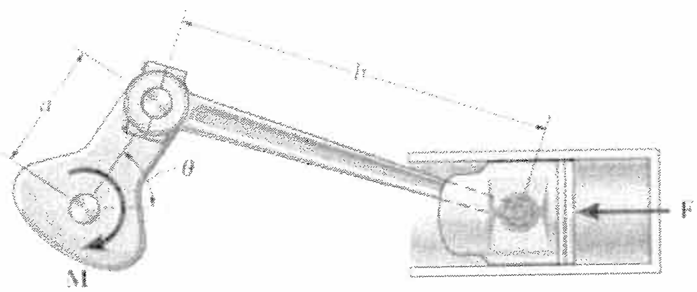
$\text{kN} = 10^3\ \text{N}$

Given:

$M = 0.05\ \text{kN}\cdot\text{m}$

$a = 0.1\ \text{m}$

$b = 0.4\ \text{m}$



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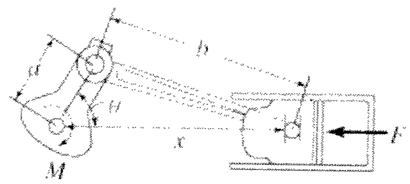
Solution:



$$b^2 = a^2 + x^2 - 2ax\cos(\theta)$$

Solving

$$x = a\cos(\theta) + \sqrt{a^2\cos(\theta)^2 + b^2 + a^2}$$

Virtual displacements

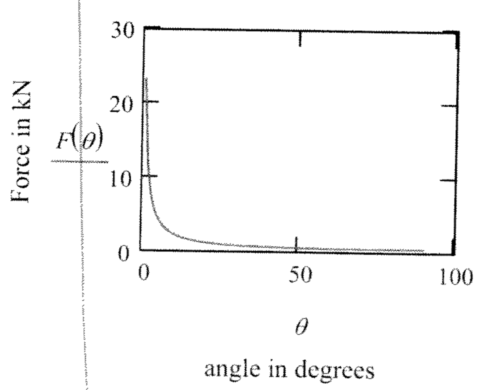
$$b^2 = a^2 + x^2 - 2ax\cos(\theta)$$

$$0 = 2x\delta x - 2a\cos(\theta)\delta x + 2ax\sin(\theta)\delta\theta \qquad \delta x = \left(\frac{ax\sin(\theta)}{x - a\cos(\theta)}\right)\delta\theta$$

$$\delta U = -F\,\delta x + M\delta\theta = \left[-F\left(\frac{ax\sin(\theta)}{x - a\cos(\theta)}\right) + M\right]\delta\theta = 0 \qquad F = M\left(\frac{x - a\cos(\theta)}{ax\sin(\theta)}\right)$$

$$\theta = 0..90 \qquad x(\theta) = a\cos(\theta\text{deg}) + \sqrt{a^2\cos(\theta\,\text{deg})^2 + b^2 + a^2}$$

$$F(\theta) = M\left(\frac{x(\theta) - a\cos(\theta\text{deg})}{ax(\theta)\sin(\theta\text{deg})}\right)$$



$$F(60) = 0.515$$

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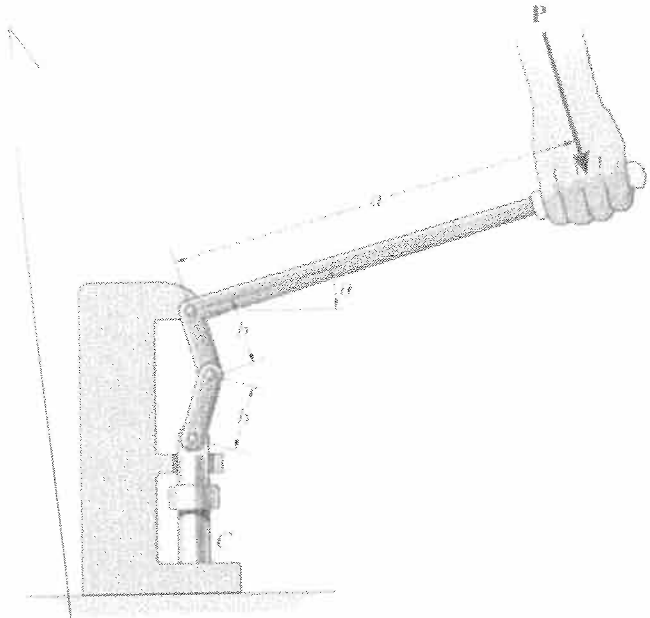
Problem 11-8

If a force **P** is applied perpendicular to the handle of the toggle press, determine the compressive force developed at *C*.



Given:

$P = 30 \text{ lb}$

$\theta = 30 \text{ deg}$

$a = 12 \text{ in}$

$b = 2 \text{ in}$

Solution:

$\delta s = a\ \delta\ \theta$

$y = 2\ b\cos(\theta)$

$\delta y = -2b\sin(\theta)$

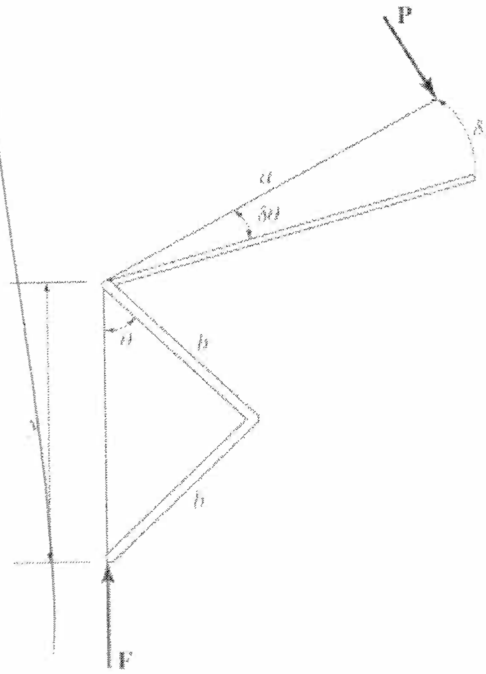
$\delta U = -P\ \delta_s + -F\ \delta_y = 0$

$-P\ a\delta\theta + F2b\sin(\theta)\delta\theta = 0$

$F2b\sin(\theta) = P\ a$

$$F = \frac{1}{2}\ P\left(\frac{a}{b\ \sin(\theta)}\right)$$

$F = 180\text{ lb}$



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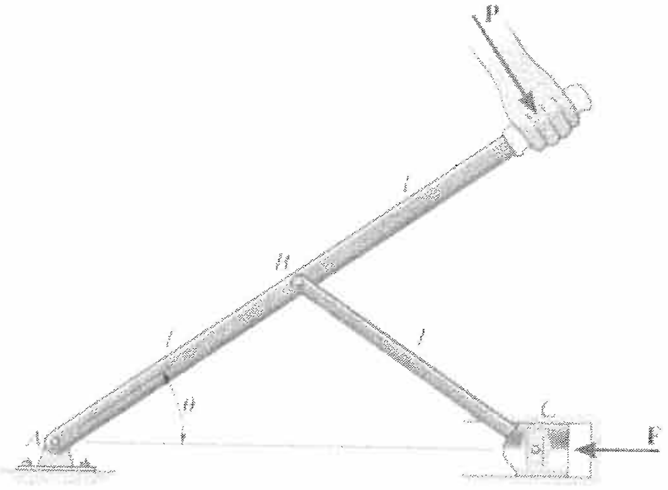
Problem 11-9 6

A force **P** is applied to the end of the lever. Determine the horizontal force **F** on the piston for equilibrium.



Solution:

$$\delta s = 2\ l\ \delta\theta$$

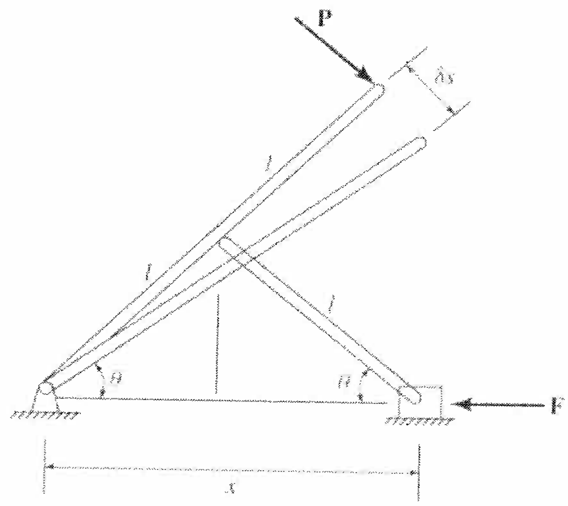
$$x = 2\ l\ \cos(\theta)$$

$$\delta x = -2\ l\ \sin(\theta)\ \delta\theta$$

$$\delta U = -P\ \delta s - F\ \delta x = 0$$

$$-P\,2l\delta\theta + F2l\sin(\theta)\delta\theta = 0$$

$$F = P\csc(\theta)$$



Problem 11-10 7

The mechanism consists of the four pin-connected bars and three springs, each having a stiffness k and an unstretched length l_0 Determine the horizontal forces **P** that must be applied to the pins in order to hold the mechanism in the horizontal position for equilibrium.

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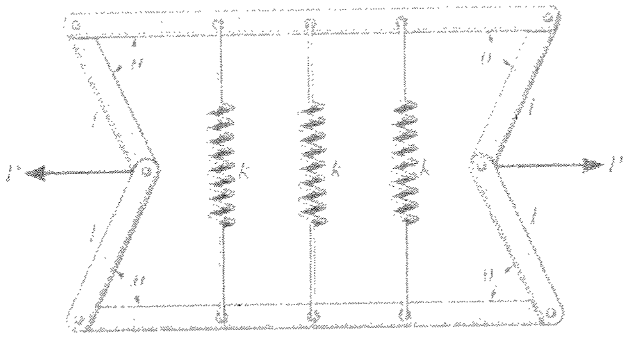

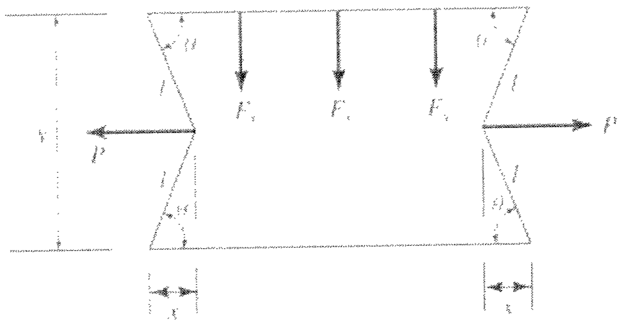

Solution:

$x = l\cos(\theta)$ $\qquad$ $\delta x = -l\sin(\theta)\,\delta\theta$

$y = 2\,l\sin(\theta)$ $\qquad$ $\delta y = 2\,l\cos(\theta)\,\delta\theta$

$\delta U = 0;\quad -2\,P\delta x - 3\,F_s = 0$

$$2\,P\,l\sin(\theta)\delta\theta - 3\,F_s\,2l\cos(\theta)\delta\theta = 0$$

$$P\sin(\theta) = 3\,F_s\cos(\theta)$$

Since $F_s = k\left(2\,l\sin(\theta) - l_0\right)$,then

$$P = 3\,k\cot(\theta)\left(2\,l\sin(\theta) - l_0\right)$$

Problem 11-11

When $\theta = \theta_0$, the uniform block of weight W_b compresses the two vertical springs a distance δ. If the uniform links AB and CD each weigh W_L, determine the magnitude of the applied couple moments **M** needed to maintain equilibrium.

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Given:

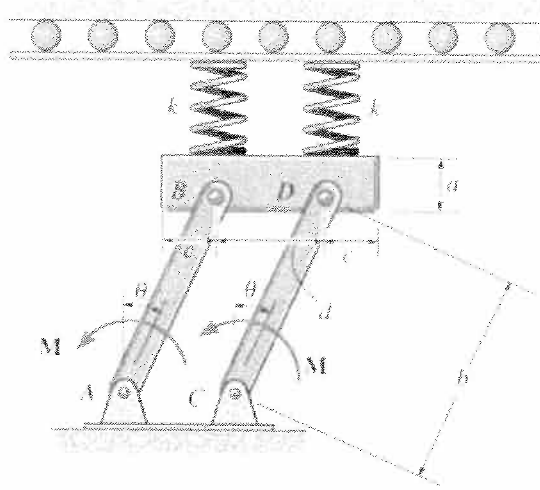
$\theta_0 = 20\ \text{deg}$ $a = 1\ \text{ft}$

$W_b = 50\ \text{lb}$ $b = 4\ \text{ft}$

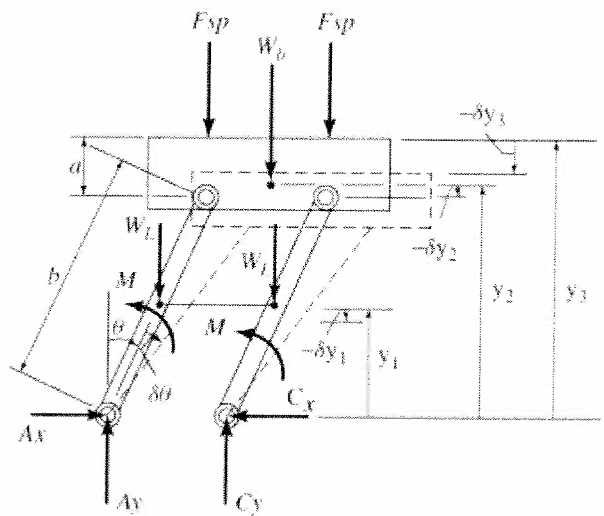
$\delta = 4\ \text{in}$ $c = 1\ \text{ft}$

$W_L = 10\ \text{lb}$ $d = 2\ \text{ft}$

$k = 2\ \frac{\text{lb}}{\text{in}}$



Solution: $\theta = \theta_0$



$$y_1 = \frac{b}{2}\cos(\theta)$$

$$\delta y_1 = \frac{-b}{2}\sin(\theta)\delta\theta$$

$$y_2 = \frac{a}{2} + b\cos(\theta)$$

$$\delta y_2 = -b\sin(\theta)\delta\theta$$

$$y_3 = y_2 + \frac{a}{2}$$

$$\delta y_3 = \delta y_2$$

$$\delta U = -2W_L\delta y_1 - W_b\delta y_2 - 2k\delta\,\delta y_3 - 2M\delta\theta = 0$$

$$\delta U = \left[2W_L\left(\frac{b}{2}\right)\sin(\theta) + W_b b\sin(\theta) + 2k\delta b\sin(\theta) - 2M\right]\delta\theta = 0$$

$$M = \left[\left(\frac{W_L + W_b}{2}\right)b + k\delta b\right]\sin(\theta) \qquad M = 52.0\ \text{lb}\cdot\text{ft}$$

Problem 11-12

The spring is unstretched when $\theta = 0$. Determine the angle θ for equilibrium. Due to the roller guide, the spring always remains vertical. Neglect the weight of the links.

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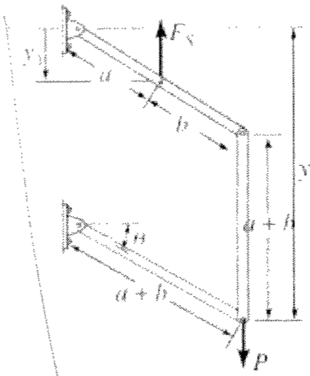
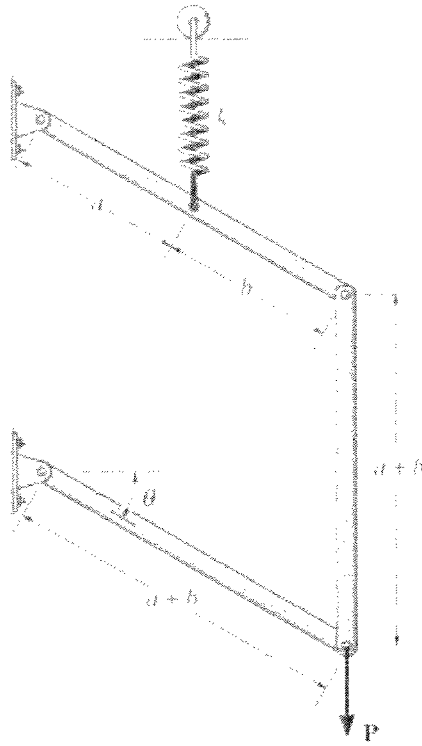
Given:





$$P = 8\ \text{lb}$$

$$k = 50\ \frac{\text{lb}}{\text{ft}}$$

$$a = 2\ \text{ft}$$

$$b = 2\ \text{ft}$$

Solution:

$$y_1 = a\sin(\theta)$$

$$\delta y_1 = a\cos(\theta)\delta\theta$$

$$y_2 = (a+b)\sin(\theta) + a + b$$

$$\delta y_2 = (a+b)\cos(\theta)\delta\theta$$

$$\delta U = -k\,y_1\delta y_1 + P\delta y_2 = \left[-k\,a\sin(\theta)a\cos(\theta) + P(a+b)\cos(\theta)\right]\delta\theta = 0$$

$$\cos(\theta)\left[P(a+b) - k a^2\sin(\theta)\right] = 0$$

There are 2 answers $\theta_1 = \text{acos}(0)$ $\theta_1 = 90\,\text{deg}$

$$\theta_2 = \text{asin}\left[\frac{P(a+b)}{k a^2}\right] \qquad \theta_2 = 9.207\,\text{deg}$$

Problem 11-13

Determine the force **P** required to lift the block of mass *M* using the differential hoist. The lever arm is fixed to the upper pulley and turns with it.

Given:

$$a = 800\ \text{mm}$$

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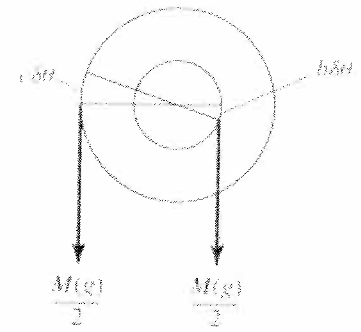
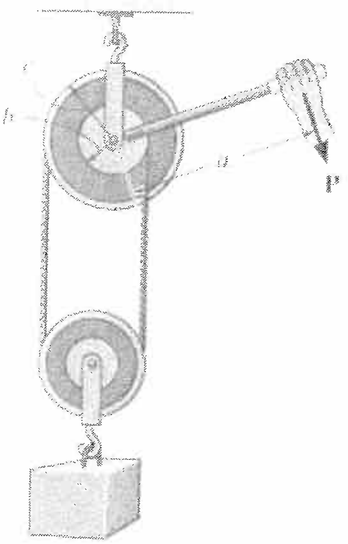
$M = 15\ \text{kg}$

$b = 150\ \text{mm}$

$c = 300\ \text{mm}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$





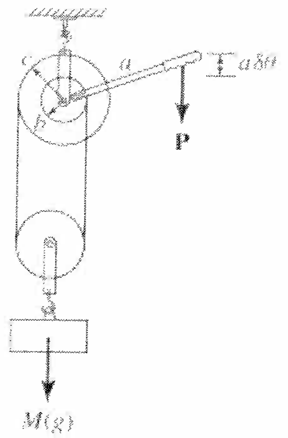
Solution:

$\delta U = 0; \quad P\ a\ \delta\theta + \dfrac{Mg}{2}(-c\ \delta\theta + b\delta\theta) = 0$

$$P\ a + \frac{Mg}{2}(b - c) = 0$$

$$P = \frac{M\ g}{2}\left(\frac{c-b}{a}\right)$$

$$P = 13.8\ \text{N}$$



Problem 11-14

Determine the magnitude of the applied couple moments **M** needed to maintain equilibrium at θ. The plate E has a weight W. Neglect the weight of the links AB and CD.

Given:

$a = 0.5\ \text{ft}$

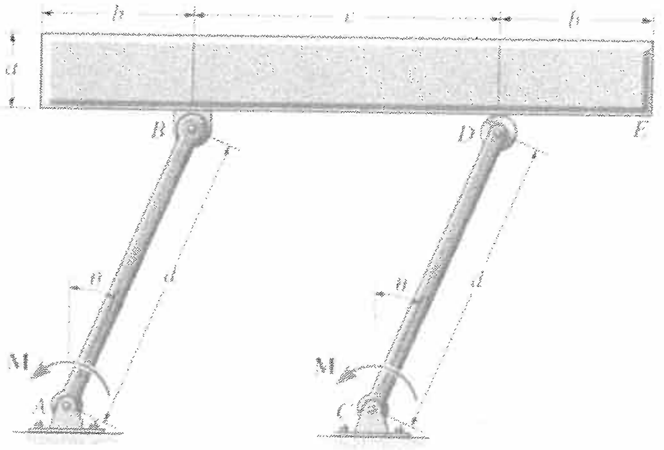
$d = 2\ \text{ft}$

$b = 1\ \text{ft}$

$c = 2\ \text{ft}$

$W = 50\ \text{lb}$

$\theta = 20\ \text{deg}$



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Solution:

$$y_m = d\cos(\theta) + \frac{a}{2} \qquad \delta y_m = -d\sin(\theta)\delta\theta$$

$$\delta U = -2\ M\ \delta\theta - W[-d\sin(\theta)(\delta\theta)] = (W\ d\sin(\theta) - 2\ M)\delta\theta = 0$$

$$M = \frac{1}{2}\ W\ d\sin(\theta)$$

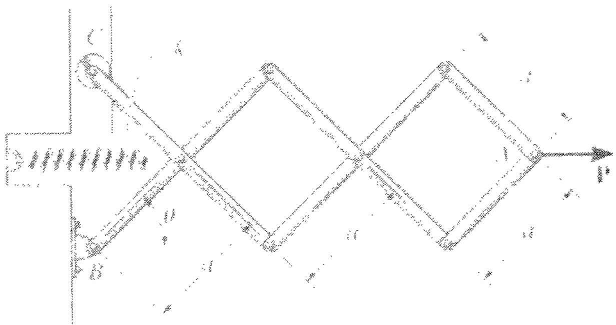
$$M = 17.1\ \text{~~lb·ft~~}\ \text{N·m}$$

Problem 11-~~15~~ 10

The members of the mechanism are pin connected. If a horizontal force **P** acts at *A*, determine the angle θ for equilibrium. The spring is unstretched when $\theta = 90°$.

Units Used:

$$kN = 10^3\ N$$



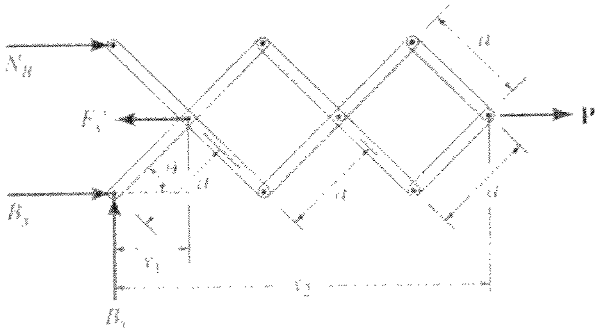
Given:

$$a = 0.5\ m$$

$$k = 20\ \frac{kN}{m}$$

$$P = 400\ N$$

Solution:



$$x_1 = a\cos(\theta)$$

$$\delta x_1 = -a\sin(\theta)\delta\theta$$

$$x_2 = 5a\cos(\theta)$$

$$\delta x_2 = -5\ a\sin(\theta)\ \delta\theta$$

$$\delta U = [P(-5\ a\sin(\theta)) - ka\cos(\theta)(-a\sin(\theta))]\delta\theta = 0$$

$$\sin(\theta)(-5\ P + ka\cos(\theta)) = 0$$

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There are 2 equilibrium angles.

$\theta_1 = \operatorname{asin}(0)$ $\quad\quad\quad\quad$ $\theta_1 = 0\,\text{deg}$

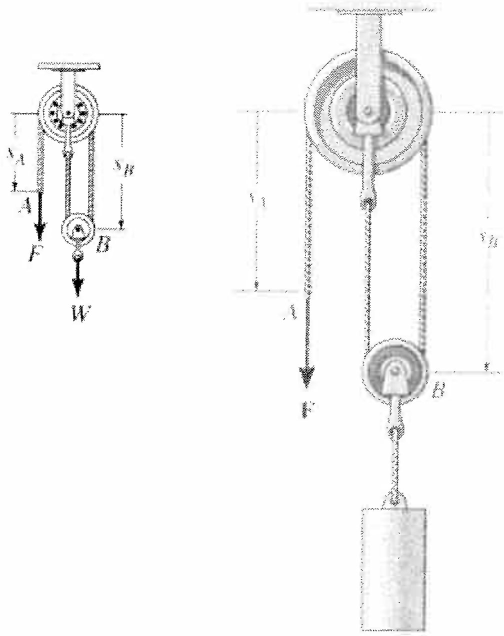
$\theta_2 = \operatorname{acos}\left(\frac{5P}{k\,a}\right)$ $\quad\quad\quad\quad$ $\theta_2 = 78.5\,\text{deg}$

Problem 11-16

Determine the force F needed to lift the block having weight W. *Hint:* Note that the coordinates S_A and S_B can be related to the constant vertical length l of the cord.

Given:

$W = 100\,\text{lb}$



Solution:

$l = s_A + 2\,s_B$

$0 = \delta s_A + 2\,\delta s_B$

$\delta s_A = -2\,\delta s_B$

$\delta U = F\delta s_A + W\delta s_B = (-2\,F + W)\,\delta s_B = 0$

$F = \frac{W}{2}$ $\quad\quad$ $F = 50\,\text{lb}$

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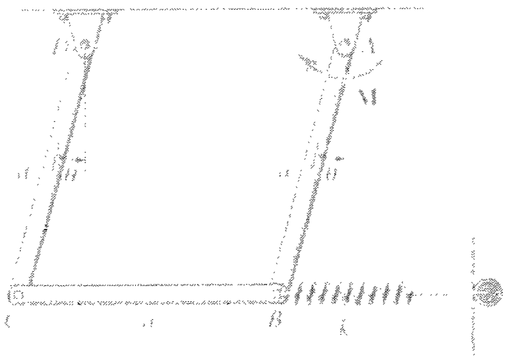
Problem 11-17



Each member of the pin-connected mechanism has a mass m_1. If the spring is unstretched when $\theta = 0°$ determine the angle θ for equilibrium.

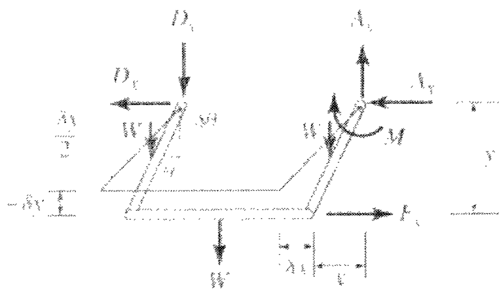
Given:

$a = 300\text{ mm}$

$k = 2500\ \dfrac{\text{N}}{\text{m}}$

$m_1 = 8\text{ kg}$

$M = 50\text{ N}\cdot\text{m}$



Solution:

$x = a\sin(\theta) \qquad \delta x = a\cos(\theta)\delta\theta$

$y = a\cos(\theta) \qquad \delta y = -a\sin(\theta)\delta\theta$

$F_s = kx \qquad F_s = ka\sin(\theta)$

$$\delta U = m_1 g\left[2\left(\frac{\delta y}{2}\right) + \delta y\right] - F_s\delta x + M\delta\theta = 0$$

$$\delta U = \left(-2m_1\, g a\sin(\theta) - ka\sin(\theta)a\cos(\theta) + M\right)\delta U = 0$$

Initial Guesses: $\theta = 10\text{ deg}$

Given

$-2\, m_1\, g a\sin(\theta) - ka\sin(\theta)a\cos(\theta) + M = 0 \qquad \theta = \text{Find}(\theta) \qquad \theta = 10.7\text{ deg}$

Now starting with a different guess we find anothe answer. $\theta = 90\text{ deg}$

Given

$-2\, m_1\, g a\sin(\theta) - ka\sin(\theta)a\cos(\theta) + M = 0 \qquad \theta = \text{Find}(\theta) \qquad \theta = 89.3\text{ deg}$

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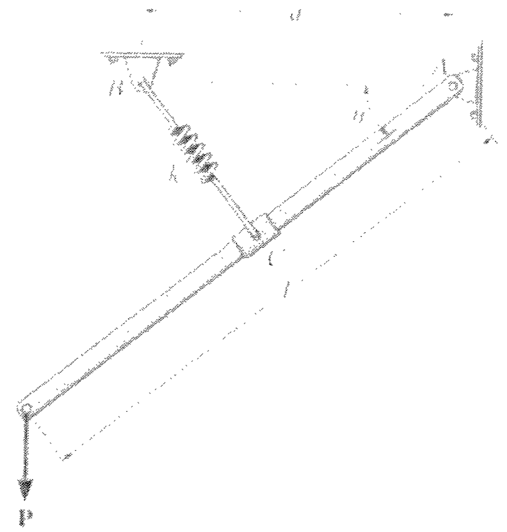
Problem 11-18

The bar is supported by the spring and smooth collar that allows the spring to be always perpendicular to the bar for any angle θ. If the unstretched length of the spring is l_0, determine the force **P** needed to hold the bar in the equilibrium position θ. Neglect the weight of the bar.



Solution:

$s = a\sin(\theta)$ $\qquad$ $\delta_s = a\cos(\theta)\delta\theta$

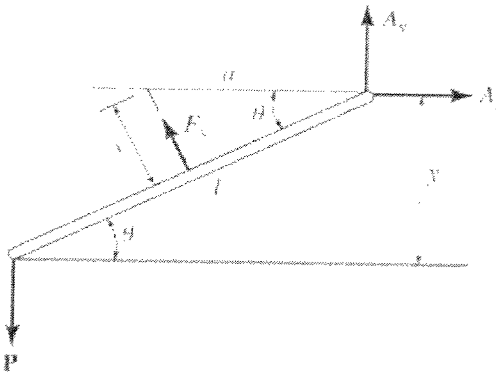
$y = l\sin(\theta)$ $\qquad$ $\delta_y = l\cos(\theta)\delta\theta$

$F_s = k(a\sin(\theta) - l_0)$

$\delta U = P\delta y - F_s\delta s = 0$

$\delta U = P\,l\cos(\theta)\delta\theta - k(a\sin(\theta) - l_0)a\cos(\theta)\delta\theta = 0$

$$P = \frac{k\,a}{l}(a\sin(\theta) - l_0)$$



Problem 11-19

The scissors jack supports a load **P**. Determine the axial force in the screw necessary for equilibrium when the jack is in the position θ. Each of the four links has a length L and is pin-connected at its center. Points B and D can move horizontally.

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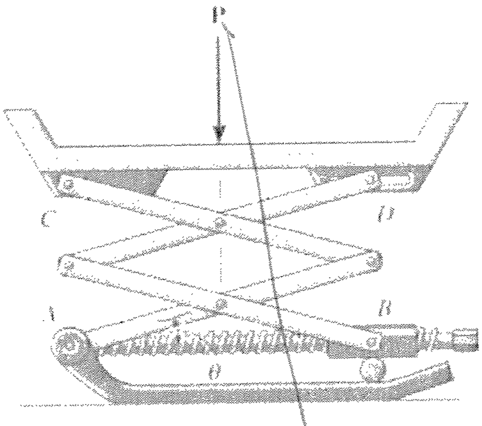
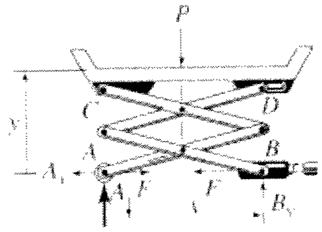




Solution:

$x = L\cos(\theta)$ $\delta x = -L\sin(\theta)\delta\theta$

$y = 2L\sin(\theta)$ $\delta y = 2L\cos(\theta)\delta\theta$

$\delta U = -P\,\delta y - F\,\delta x = (-P\,2L\cos(\theta) + F\,L\sin(\theta))\delta\theta = 0$ $F = 2P\cot(\theta)$

Problem 11-20 14

Determine the masses m_A and m_B of A and B required to hold the desk lamp of mass M in balance for any angles θ and ϕ. Neglect the weight of the mechanism and the size of the lamp.

Given:

$M = 400$ gm

$a = 75$ mm

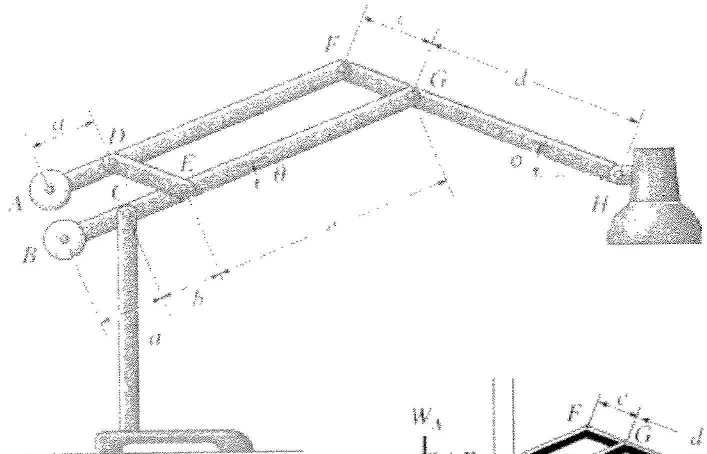
$b = 75$ mm

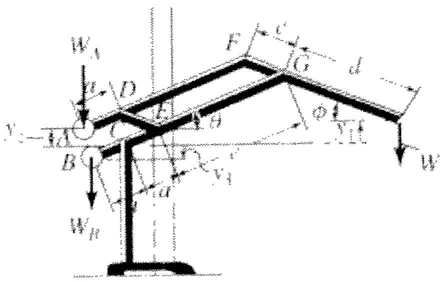
$c = 75$ mm

$d = 300$ mm

$e = 300$ mm

$g = 9.81\ \dfrac{m}{s^2}$





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Solution:

$y_1 = (b+e)\sin(\theta) - d\sin(\phi)$ $\qquad \delta y_1 = (b+e)\cos(\theta)\,\delta\theta - d\cos(\phi)\,\delta\phi$

$y_2 = b\sin(\theta) + c\sin(\phi) - a\sin(\theta)$ $\qquad \delta y_2 = (b-a)\cos(\theta)\delta\theta + c\cos(\phi)\delta\phi$

$y_3 = -a\sin(\theta)$ $\qquad \delta y_3 = -a\cos(\theta)\delta\theta$

$$\delta U = -Mg\delta y_1 - m_A g\delta y_2 - m_B g\delta y_2 = 0$$

$$\delta U = g\left[\left[-M(b+e) - m_A(b-a) + m_B a\right]\cos(\theta)\delta\theta + \left(Md - m_A c\right)\cos(\phi)\delta\phi\right] = 0$$

We now solve by setting both coefficients to zero.

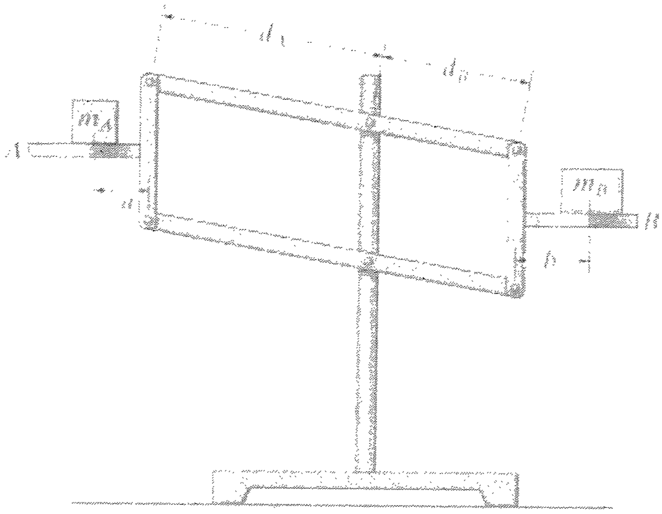
Guesses $\quad m_A = 1\ \text{kg} \qquad m_B = 1\ \text{kg}$

Given $\quad -M(b+e) - m_A(b-a) + m_B a = 0 \qquad Md - m_A c = 0$

$$\begin{pmatrix} m_A \\ m_B \end{pmatrix} = \text{Find}(m_A, m_B) \qquad \begin{pmatrix} m_A \\ m_B \end{pmatrix} = \begin{pmatrix} 1.6 \\ 2 \end{pmatrix}\ \text{kg}$$

Problem 11-21

The *Roberval balance* is in equilibrium when no weights are placed on the pans *A* and *B*. If two masses m_A and m_B are placed at *any* location *a* and *b* on the pans, show that equilibrium is maintained if $m_A d_A = m_B d_B$.



Solution:

$y_A = d_A \sin(\theta)$

$\delta y_A = d_A \cos(\theta)\delta\theta$

$y_B = -d_B \sin(\theta)$

$\delta y_B = -d_B \cos(\theta)\delta\theta$

$$\delta U = -m_A\, g d_A \cos(\theta)\delta\theta - m_B\, g\left(-d_B\cos(\theta)\right)\delta\theta = 0$$

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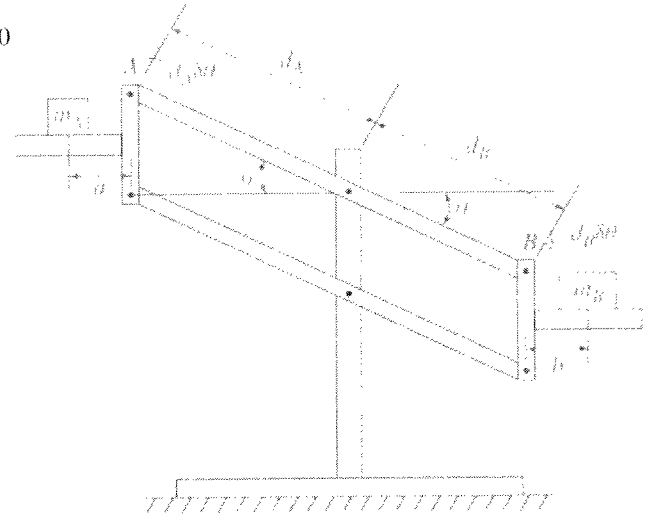
$\delta U = (m_B d_B - m_A d_A)\ g\cos(\theta)\delta\theta = 0$

$m_A d_A = m_B d_B$ Q.E.D



Problem 11-22

The chain puller is used to draw two ends of a chain together in order to attach the "master link." The device is operated by turning the screw S, which pushes the bar AB downward, thereby drawing the tips C and D towards one another. If the sliding contacts at A and B are smooth, determine the force **F** maintained by the screw at E which is required to develop a drawing tension **T** in the chains.

Given:

$T = 120$ lb

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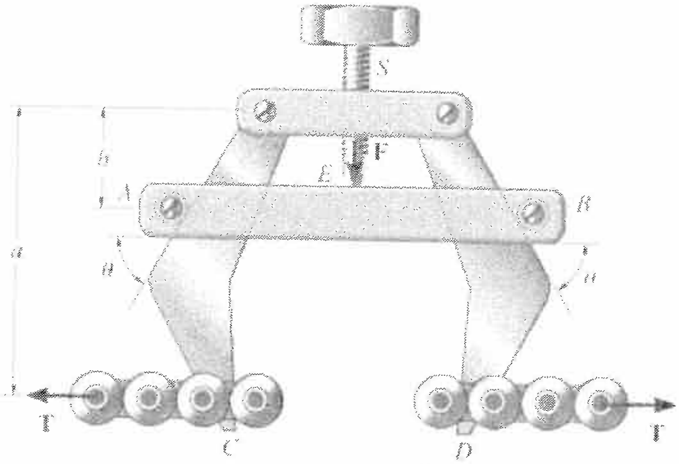
$\theta = 60 \text{ deg}$

$a = 3 \text{ in}$

$b = 1 \text{ in}$



Solution:

$$\delta s = \frac{b\ \delta\theta}{\cos(\theta)}$$

$$\delta y = \left(\frac{b}{\cos(\theta)}\right)\left(\frac{1}{\sin(\theta)}\right)\delta\theta$$

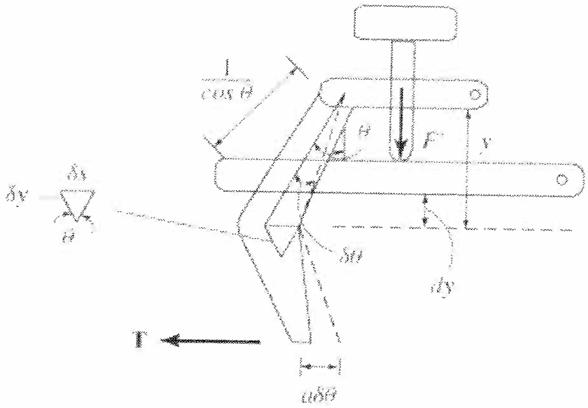
$$\delta U = 0$$

$$F\delta y - 2T(a\delta\theta) = 0$$

$$\frac{F\ b}{\cos(\theta)\sin(\theta)} = 2\ T\ a$$

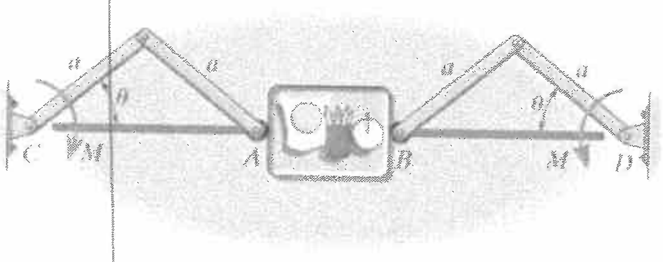
$$F = 2\ T\ a\cos(\theta)\left(\frac{\sin(\theta)}{b}\right)$$

$$F = 312\,\text{lb}$$



Problem 11-23

The service window at a fast-food restaurant consists of glass doors that open and close automatically using a motor which supplies a torque **M** to each door. The far ends, A and B, move along the horizontal guides. If a food tray becomes stuck between the doors as shown, determine the horizontal force the doors exert on the tray at the position θ.



Solution:

$$x = 2a\cos(\theta)$$

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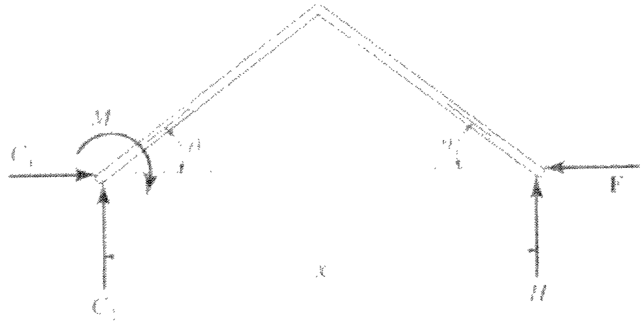
$$\delta x = -2a\sin(\theta)\delta\theta$$

$$\delta U = 0;\; -M\,\delta\theta - F\,\delta x = 0$$

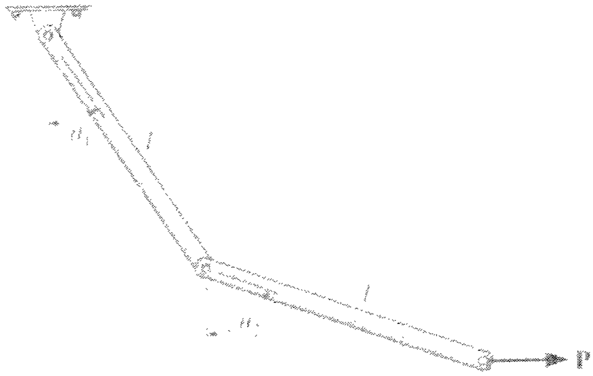
$$-M\,\delta\theta + F\;2\;a\;\sin(\theta)\delta\theta = 0$$

$$F = \frac{M}{2\;a\;\sin(\theta)}$$



Problem 11-24

A horizontal force acts on the end of the link as shown. Determine the angles θ_1 and θ_2 for equilibrium of the two links. Each link is uniform and has a mass m.

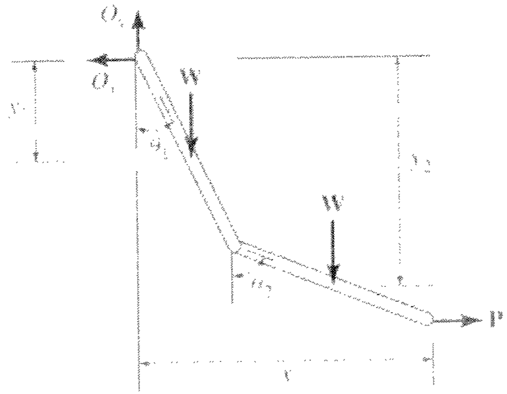


Solution:

$$x = l\sin(\theta_1) + l\sin(\theta_2)$$

$$y_1 = \frac{l}{2}\cos(\theta_1)$$

$$y_2 = l\cos(\theta_1) + \frac{l}{2}\cos(\theta_2)$$



$$\delta x = l\cos(\theta_1)\delta\theta_1 + l\cos(\theta_2)\delta\theta_2$$

$$\delta y_1 = -\frac{l}{2}\sin(\theta_1)\delta\theta_1$$

$$\delta y_2 = -l\sin(\theta_1)\delta\theta_1 - \frac{l}{2}\sin(\theta_2)\delta\theta_2$$

$$\delta U = P\delta_x + mg\delta y_1 + mg\delta y_2 = 0$$

$$\delta U = P\,l\left(\cos(\theta_1)\,\delta\theta_1 + \cos(\theta_2)\,\delta\theta_2\right) - mg\left(\frac{l}{2}\right)\left(3\sin(\theta_1)\delta\theta_1 + \sin(\theta_2)\,\delta\theta_2\right) = 0$$

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$$\delta U = \left(P\,l\cos(\theta_1) - \frac{3}{2} m g l \sin(\theta_1)\right)\delta\theta_1 + \left(P\,l\cos(\theta_2) - \frac{1}{2} m g l \sin(\theta_2)\right)\delta\theta_2 = 0$$

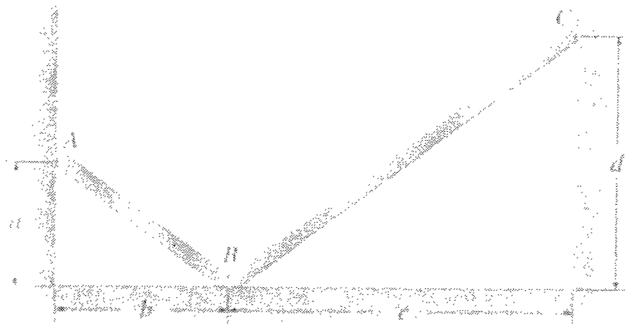
Thus we have 2 equations:

$$P\,l\cos(\theta_1) - \frac{3}{2}\,m\,g\,l\,\sin(\theta_1) = 0 \qquad \theta_1 = \operatorname{atan}\left(\frac{2\,P}{3\,m\,g}\right)$$

$$P\,l\cos(\theta_2) - \frac{1}{2}\,m\,g\,l\,\sin(\theta_2) = 0 \qquad \theta_2 = \operatorname{atan}\left(\frac{2\,P}{m\,g}\right)$$

Problem 11-25

Rods AB and BC have centers of mass located at their midpoints. If all contacting surfaces are smooth and BC has mass m_{BC} determine the appropriate mass m_{AB} of AB required for equilibrium.



Given:

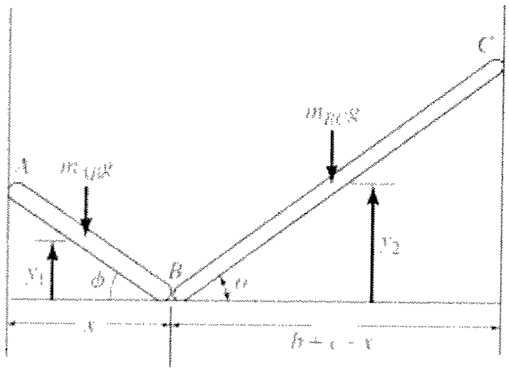
$m_{BC} = 100$ kg

$a = 0.75$ m

$b = 1$ m

$c = 2$ m

$d = 1.5$ m



Solution:

Use θ as the independent variable

Define $L_1 = \sqrt{a^2 + b^2}$ $\quad L_2 = \sqrt{c^2 + d^2}$ $\quad \theta = \operatorname{atan}\left(\frac{d}{c}\right)$ $\quad \phi = \operatorname{atan}\left(\frac{a}{b}\right)$

Then $L_1\cos(\phi) + L_2\cos(\theta) = b + c$ $\qquad -L_1\sin(\phi)\delta\phi - L_2\sin(\theta)\delta\theta = 0$

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Thus $\delta\phi = \left(\frac{L_2 \sin(\theta)}{L_1 \sin(\phi)}\right)\delta\theta$

Also $y_1 = \frac{L_1}{2}\sin(\phi)$ $\delta y_1 = \frac{L_1}{2}\cos(\phi)\delta\phi = \left(\frac{-L_2 \sin(\theta)\cot(\phi)}{2}\right)\delta\theta$

$y_2 = \frac{L_2}{2}\sin(\theta)$ $\delta y_2 = \frac{L_2}{2}\cos(\theta)\delta\theta$

$$\delta U = -m_{AB}g\delta y_1 - m_{BC}g\delta y_2 = g\left[m_{AB}\left(\frac{L_2 \sin(\theta)\cot(\phi)}{2}\right) - m_{BC}\left(\frac{L_2}{2}\cos(\theta)\right)\right]\delta\theta = 0$$

$m_{AB} = m_{BC}\tan(\phi)\cot(\theta)$ $m_{AB} = 100\text{ kg}$

Problem 11-26

If the potential energy for a conservative two-degree-of-freedom system is expressed by the relation $V = ay^2 + bx^2$, where y and x, determine the equilibrium positions and investigate the stability at each position.

Given:

$a = 3\ \frac{\text{N}}{\text{m}}$ $b = 2\ \frac{\text{N}}{\text{m}}$

Solution:

$$V = a y^2 + b x^2$$

Equilibrium position:

$\frac{\partial}{\partial x}V = 2bx = 0$ $x = 0$

$\frac{\partial}{\partial y}V = 2ay = 0$ $y = 0$

Stability:

At $(0, 0)$ $\frac{\partial^2}{\partial x^2}V = 2b$ $2b = 4\frac{\text{N}}{\text{m}}$ > 0

At $(0, 0)$ $\frac{\partial^2}{\partial y^2}V = 2a$ $2a = 6\frac{\text{N}}{\text{m}}$ > 0

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At $(0,0)$ $\quad \frac{\partial}{\partial x}\frac{\partial}{\partial y}V = 0$

At $(0,0)$ $\quad \left[\left(\frac{\partial}{\partial x}\frac{\partial}{\partial y}V\right)^2 - \left(\frac{\partial^2}{\partial x^2}V\right)\left(\frac{\partial^2}{\partial y^2}V\right)\right] = -4ab \qquad -4ab = -24\frac{N^2}{m^2} \qquad < 0$

Stable at (0,0)

Problem 11-27

If the potential energy for a conservative one-degree-of-freedom system is expressed by the relation $V = (ax^3 + bx^2 + cx + d)$, determine the equilibrium positions and investigate the stability at each position.

Given: $\quad a = 4\frac{\text{lb}}{\text{ft}^2} \qquad b = -1\frac{\text{lb}}{\text{ft}} \qquad c = -3\ \text{lb} \qquad d = 10\ \text{ft}\cdot\text{lb}$

Solution:

$$V = ax^3 + bx^2 + cx + d$$

Required Position:

$$\frac{d}{dx}V = 3ax^2 + 2bx + c = 0$$

$$x_1 = \frac{-2b + \sqrt{4b^2 - 4(3a)c}}{2(3a)}$$

$$x_1 = 0.59\ \text{ft}$$

$$x_2 = \frac{-2b - \sqrt{4b^2 - 4(3ac)}}{2\ (3a)}$$

$$x_2 = -0.424\ \text{ft}$$

Stability:

$$\frac{d^2}{dx^2}V = V'' = 6ax + 2b$$

At $x = x_1 \qquad V''_1 = 6ax_1 + 2b \qquad V''_1 = 12.2\frac{\text{lb}}{\text{ft}} \qquad V''_1 > 0 \qquad$ Stable

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At $x = x_2$ $\quad V''_2 = 6ax_2 + 2b \quad V''_2 = -12.2\,\dfrac{\text{lb}}{\text{ft}} \quad V''_2 < 0 \quad$ Unstable

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Problem 11-28

If the potential energy for a conservative one-degree-of-freedom system is expressed by the relation $V = a\sin(\theta) + b\cos(2\theta)$, $0\text{ deg} \le \theta \le 180\text{ deg}$, determine the equilibrium positions and investigate the stability at each position.

Given:

$a = 24\ \text{ft}\cdot\text{lb} \qquad b = 10\ \text{ft}\cdot\text{lb}$

Solution:

$$V = a\sin(\theta) + b\cos(2\theta)$$

$$\frac{d}{d\theta}V = V' = a\cos(\theta) - 2b\sin(2\theta) = a\cos(\theta) - 4b\sin(\theta)\cos(\theta)$$

$$V' = \cos(\theta)(a - 4b\sin(\theta)) = 0$$

$$\frac{d^2}{dx^2}V = V'' = -a\sin(\theta) - 4b\cos(2\theta)$$

Equilibrium Positions:

$\theta_1 = \text{acos}(0) \qquad \theta_1 = 90\text{ deg}$

$\theta_2 = \text{asin}\left(\dfrac{a}{4b}\right) \qquad \theta_2 = 36.87\text{ deg}$

$\theta_3 = \pi - \theta_2 \qquad \theta_3 = 143.13\text{ deg}$

Check Stability

$V''_1 = -a\sin(\theta_1) - 4b\cos(2\theta_1) \qquad V''_1 = 16\text{ lb ft} \qquad$ Stable

$V''_2 = -a\sin(\theta_2) - 4b\cos(2\theta_2) \qquad V''_2 = -25.6\text{ lb ft} \qquad$ Unstable

$V''_3 = -a\sin(\theta_3) - 4b\cos(2\theta_3) \qquad V''_3 = -25.6\text{ lb ft} \qquad$ Unstable

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Problem 11-29 18

If the potential energy for a conservative two-degree-of-freedom system is expressed by the relation $V = ay^2 + bx^2$, where y and x, determine the equilibrium positions and investigate the

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stability at each position.

Given:

$$a = 6\,\frac{\text{N}}{\text{m}} \qquad b = 2\,\frac{\text{N}}{\text{m}}$$

Solution:

$$V = a y^2 + b x^2$$

Equilibrium position:

$$\frac{\partial}{\partial x} V = 2 b x = 0 \qquad x = 0$$

$$\frac{\partial}{\partial y} V = 2 a y = 0 \qquad y = 0$$

Stability:

$$\text{At } (0,0) \quad \frac{\partial^2}{\partial x^2} V = 2b \qquad 2b = 4\,\frac{\text{N}}{\text{m}} \qquad > 0$$

$$\text{At } (0,0) \quad \frac{\partial^2}{\partial y^2} V = 2a \qquad 2a = 12\,\frac{\text{N}}{\text{m}} \qquad > 0$$

$$\text{At } (0,0) \quad \frac{\partial}{\partial x}\frac{\partial}{\partial y} V = 0$$

$$\text{At } (0,0) \quad \left[\left(\frac{\partial}{\partial x}\frac{\partial}{\partial y} V\right)^2 - \left(\frac{\partial^2}{\partial x^2} V\right)\left(\frac{\partial^2}{\partial y^2} V\right)\right] = -4 a b \qquad -4 a b = -48\,\frac{\text{N}^2}{\text{m}^2} \qquad < 0$$

Stable at (0,0)

Problem 11-30 19

The spring of the scale has an unstretched length a. Determine the angle θ for equilibrium when a weight W is supported on the platform. Neglect the weight of the members. What value W would be required to keep the scale in neutral equilibrium when $\theta = 0°$?

Solution:

Potential Function: The datum is established at point A. Since the weight W is above the datum, its potential energy is positive.

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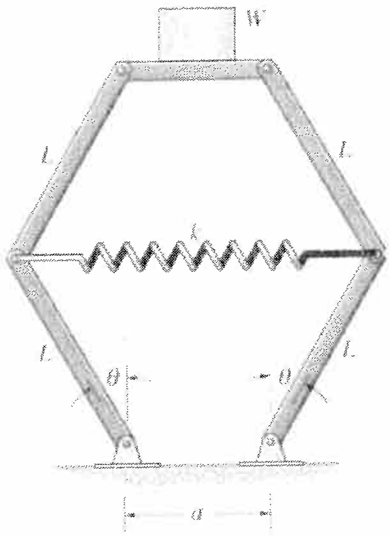
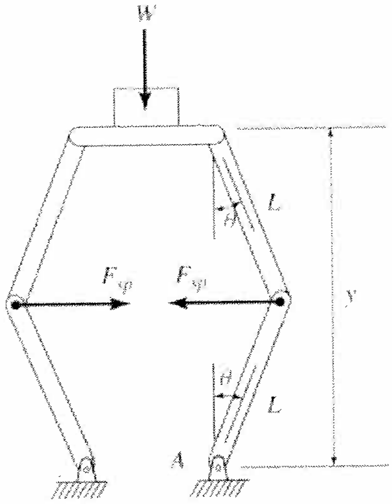
$$V = \frac{1}{2}k(2L\sin(\theta))^2 + W2L\cos(\theta) = 2kL^2\sin(\theta)^2 + 2WL\cos(\theta)$$

Equilibrium Position: The system is in equilibrium if

$$\frac{d}{d\theta}V = 4kL^2\sin(\theta)\cos(\theta) - 2WL\sin(\theta) = 2L\sin(\theta)(2kL\cos(\theta) - W) = 0$$





Solving, $\theta = 0°$ or $\theta = \mathrm{acos}\left(\frac{W}{2kL}\right)$

To have neutral stability at $\theta = 0$, we require that

$$\frac{d^2}{d\theta^2}V = 4kL^2\cos(2\times 0) - 2WL\cos(0) = 4kL^2 - 2WL = 0 \qquad W = 2kL$$

Problem 11-31

The two bars each have weight W. Determine the required stiffness k of the spring so that the two bars are in equilibrium at $\theta = \theta_0$. The spring has an unstretched length δ.

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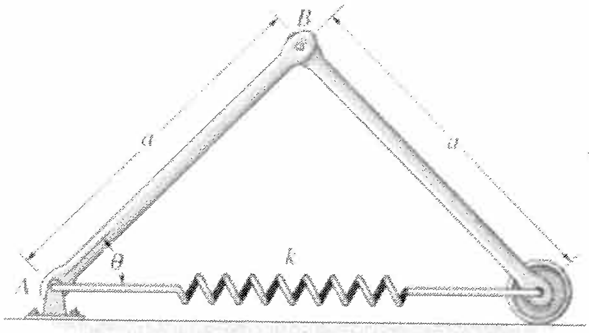
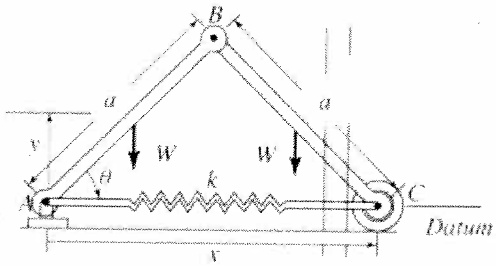




Given: $W = 8\ \text{lb}$ $\theta_0 = 30\ \text{deg}$ $\delta = 1\ \text{ft}$ $a = 2\ \text{ft}$

Solution: $\theta = \theta_0$

$$V = 2W\left(\frac{a}{2}\right)\sin(\theta) + \frac{1}{2}k\left(2a\cos(\theta) - \delta\right)^2$$

$$\frac{dV}{d\theta} = Wa\cos(\theta) - k\left(2a\cos(\theta) - \delta\right)2a\sin(\theta) = 0$$

$$k = \frac{Wa\cos(\theta)}{\left(2a\cos(\theta) - \delta\right)2a\sin(\theta)} \qquad k = 2.812\ \frac{\text{lb}}{\text{ft}}$$

Problem 11-32

Each of the two springs has an unstretched length δ. Determine the mass M of the cylinder when it is held in the equilibrium position shown, i.e., $y = a$.

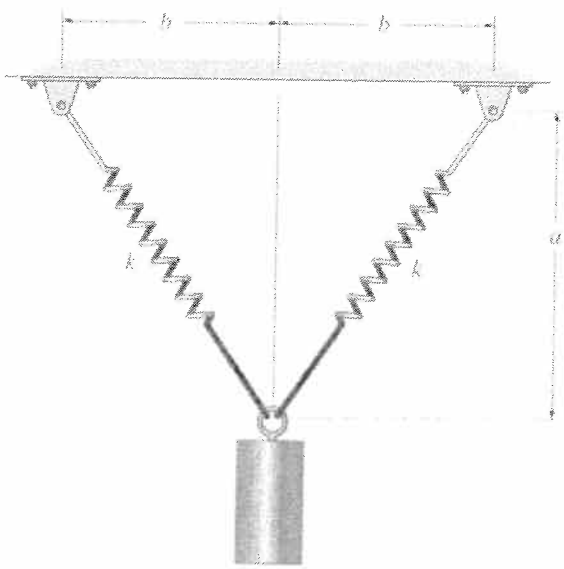
Given:

$a = 1\ \text{m}$

$b = 500\ \text{mm}$

$\delta = 500\ \text{mm}$

$k = 200\ \frac{\text{N}}{\text{m}}$



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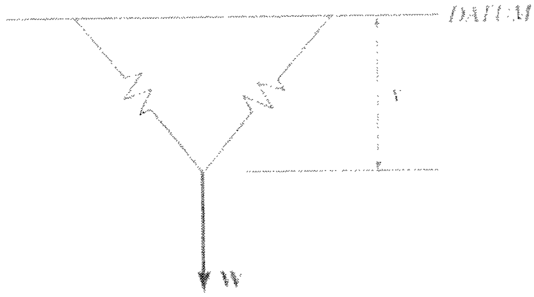
Solution:

$$V = 2\frac{k}{2}\left(\sqrt{y^2 + b^2} - \delta\right)^2 - Mgy$$



$$\frac{dV}{dy} = 2k\left(\sqrt{y^2 + b^2} - \delta\right)\left(\frac{y}{\sqrt{y^2 + b^2}}\right) - Mg$$

Set $y = a$

Guess $M = 1\ \text{kg}$

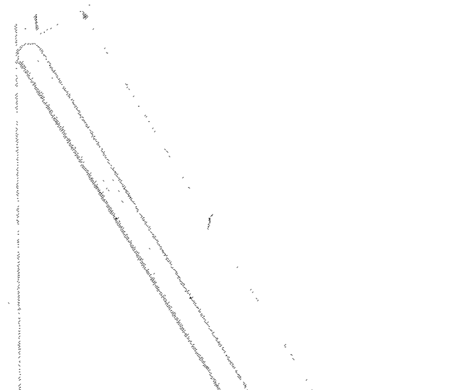
Given

$$2k\left(\sqrt{y^2 + b^2} - \delta\right)\left(\frac{y}{\sqrt{y^2 + b^2}}\right) - Mg = 0$$

$M = \text{Find}(M)$ $\quad M = 22.5\ \text{kg}$

Problem 11-33

The uniform beam has mass M. If the contacting surfaces are smooth, determine the angle θ for equilibrium and investigate the stability of the beam when it is in this position. The spring has an unstretched length of δ.



Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given:

$M = 200\ \text{kg}$ $\quad k = 1.2\ \frac{\text{kN}}{\text{m}}$

$\delta = 0.5\ \text{m}$ $\quad l = 2\ \text{m}$

Solution:

$$V = Mg\left(\frac{l}{2}\right)\sin(\theta) + \frac{1}{2}k\left(l\cos(\theta) - \delta\right)^2$$

$$\frac{d}{d\theta}V = V' = Mg\left(\frac{l}{2}\right)\cos(\theta) - k\left(l\cos(\theta) - \delta\right)l\sin(\theta) = 0$$

$$\frac{d^2}{d\theta^2}V = V'' = -Mg\left(\frac{l}{2}\right)\sin(\theta) + kl^2\sin(\theta)^2 - k\left(l\cos(\theta) - \delta\right)l\cos(\theta)$$

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There are 2 equilibrium points

Guess $\theta = 30\ \text{deg}$ Given $Mg\left(\frac{l}{2}\right)\cos(\theta) - k(l\cos(\theta) - \delta)l\sin(\theta) = 0$

$\theta_1 = \text{Find}(\theta)$ $\theta_1 = 36.4\,\text{deg}$

Guess $\theta = 60\ \text{deg}$ Given $Mg\left(\frac{l}{2}\right)\cos(\theta) - k(l\cos(\theta) - \delta)l\sin(\theta) = 0$

$\theta_2 = \text{Find}(\theta)$ $\theta_2 = 62.3\,\text{deg}$

Check Stability

$$V''_1 = -Mg\left(\frac{l}{2}\right)\sin(\theta_1) + kl^2\sin(\theta_1)^2 - k(l\cos(\theta_1) - \delta)l\cos(\theta_1)$$

$$V''_2 = -Mg\left(\frac{l}{2}\right)\sin(\theta_2) + kl^2\sin(\theta_2)^2 - k(l\cos(\theta_2) - \delta)l\cos(\theta_2)$$

$V''_1 = -1.624\,\text{kN}\cdot\text{m}$ Unstable $V''_2 = 1.55\,\text{kN}\cdot\text{m}$ Stable

---

Problem 11-34

The bar supports a weight W at its end. If the springs are originally unstretched when the bar is vertical, determine the required stiffness $k_1 = k_2 = k$ of the springs so that the bar is in neutral equilibrium when it is vertical.

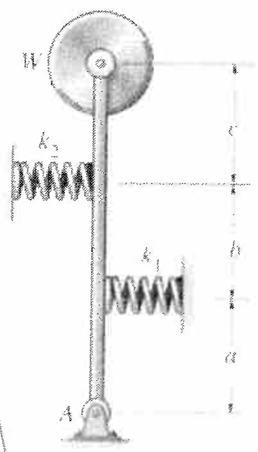
Given:

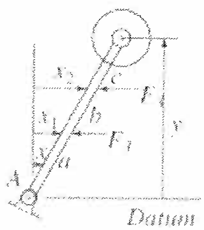
$W = 500\ \text{lb}$

$a = 3\ \text{ft}$

$b = 3\ \text{ft}$

$c = 3\ \text{ft}$





Solution:

$$V = W(a + b + c)\cos(\theta) + \frac{1}{2}k(a\sin(\theta))^2 + \frac{1}{2}k[(a + b)\sin(\theta)]^2$$

$$\frac{d}{d\theta}V = -W(a + b + c)\sin(\theta) + \frac{k}{2}\left[a^2 + (a + b)^2\right]\sin(2\theta)$$

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$$\frac{d^2}{d\theta^2}V = -W(a+b+c)\cos(\theta) + k\left[a^2 + (a+b)^2\right]\cos(2\theta)$$

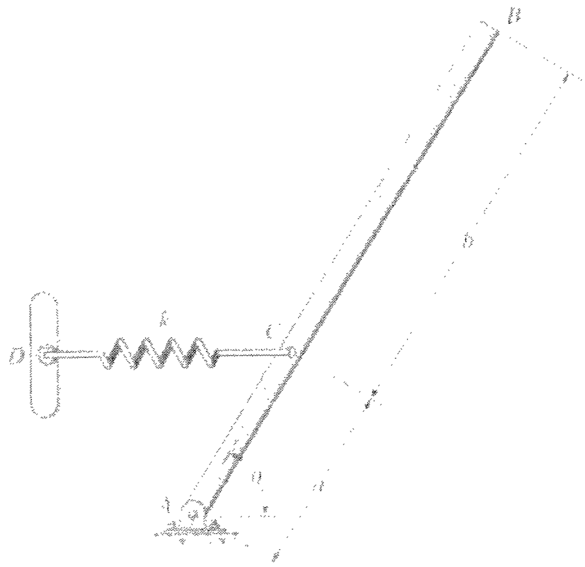
at $\theta = 0$

$$\frac{d^2}{d\theta^2}V = -W(a+b+c) + k\left[a^2 + (a+b)^2\right] = 0 \qquad \text{for neutral stability}$$

$$k = \frac{W(a+b+c)}{a^2 + (a+b)^2} \qquad k = 100\,\frac{\text{lb}}{\text{ft}}$$

Problem 11-35

The uniform rod AB has a mass M. If spring DC is unstretched at $\theta = 90$ deg, determine the angle θ for equilibrium and investigate the stability at the equilibrium position. The spring always acts in the horizontal position due to the roller guide at D.



Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given:

$M = 80$ kg $\qquad a = 1$ m

$k = 2\,\frac{\text{kN}}{\text{m}} \qquad b = 2$ m

Solution:

$$V = Mg\left(\frac{a+b}{2}\right)\sin(\theta) + \frac{1}{2}k\left(a\cos(\theta)\right)^2$$

$$V' = \frac{d}{d\theta}V = Mg\left(\frac{a+b}{2}\right)\cos(\theta) - \frac{k}{2}a^2\sin(2\theta)$$

$$V'' = \frac{d^2}{d\theta^2}V = -Mg\left(\frac{a+b}{2}\right)\sin(\theta) - ka^2\cos(2\theta)$$

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Equilibrium

Guess $\theta = 30\ \text{deg}$ Given $Mg\left(\frac{a+b}{2}\right)\cos(\theta) - \frac{k}{2}a^2\sin(2\theta) = 0$ $\theta_1 = \text{Find}(\theta)$

Guess $\theta = 70\ \text{deg}$ Given $Mg\left(\frac{a+b}{2}\right)\cos(\theta) - \frac{k}{2}a^2\sin(2\theta) = 0$ $\theta_2 = \text{Find}(\theta)$

Check Staibility

$$V''_1 = -Mg\left(\frac{a+b}{2}\right)\sin(\theta_1) - ka^2\cos(2\theta_1)$$

$$V''_2 = -Mg\left(\frac{a+b}{2}\right)\sin(\theta_2) - ka^2\cos(2\theta_2)$$

$\theta_1 = 36.1\ \text{deg}$ $V''_1 = -1.3\ \text{kN}\cdot\text{m}$ Unstable

$\theta_2 = 90.0\ \text{deg}$ $V''_2 = 0.82\ \text{kN}\cdot\text{m}$ Stable

---

Problem 11-36

Determine the angle θ for equilibrium and investigate the stability at this position. The bars each have mass m_b and the suspended block D has mass m_D Cord DC has a total length of L.

Given:

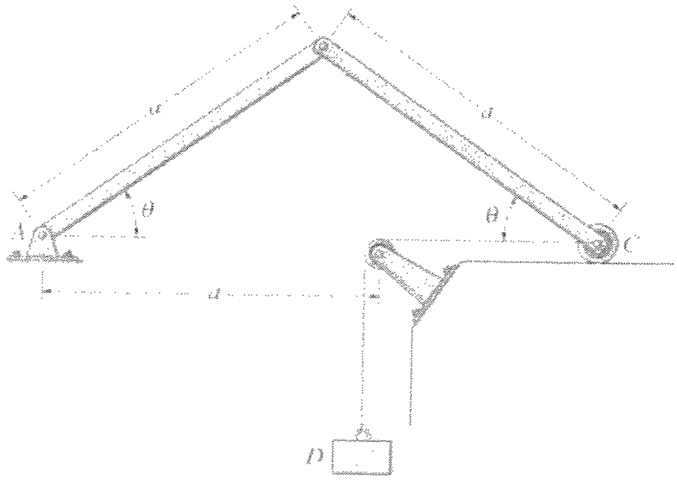
$m_b = 3\ \text{kg}$

$m_D = 7\ \text{kg}$

$a = 500\ \text{mm}$

$L = 1\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution: Equilibrium

$$V = 2m_b g\frac{a}{2}\sin(\theta) - m_D g(L + a - 2a\cos(\theta)) = 0$$

$$V = m_b g a\sin(\theta) + 2m_D g a\cos(\theta) - m_D g(L + a)$$

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$$\frac{d}{d\theta}V = m_b g a\cos(\theta) - 2m_D g a\sin(\theta) = 0$$

$$\tan(\theta) = \frac{m_b}{2m_D} \qquad \theta = \mathrm{atan}\left(\frac{m_b}{2m_D}\right) \qquad \theta = 12.095\ \mathrm{deg}$$

Stability

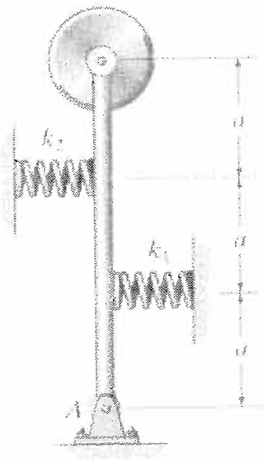
$$V'' = \frac{d^2}{d\theta^2}V = -m_b g a\sin(\theta) - 2m_D g a\cos(\theta)$$

$$V'' = -m_b g a\sin(\theta) - 2m_D g a\cos(\theta) \qquad V'' = -70.229\ \mathrm{N\cdot m}$$

Since $V'' < 0$ the equilibrium point is unstable.

Problem 11-37

The bar supports a weight of W at its end. If the springs are originally unstretched when the bar is vertical, investigate the stability of the bar when it is in the vertical position.



Given:

$$k_1 = 300\ \frac{\mathrm{lb}}{\mathrm{ft}}$$

$$k_2 = 500\ \frac{\mathrm{lb}}{\mathrm{ft}}$$

$$W = 500\ \mathrm{lb}$$

$$a = 3\ \mathrm{ft}$$

Solution:

$$V = W3a\cos(\theta) + \frac{1}{2}k_1(a\sin(\theta))^2 + \frac{1}{2}k_2(2a\sin(\theta))^2$$

$$V = 3Wa\cos(\theta) + \frac{a^2}{2}(k_1 + 4k_2)\sin(\theta)^2$$

$$V' = \frac{d}{d\theta}V = -3Wa\sin(\theta) + \frac{a^2}{2}(k_1 + 4k_2)\sin(2\theta)$$

$$V'' = \frac{d^2}{d\theta^2}V = -3Wa\cos(\theta) + a^2(k_1 + 4k_2)\cos(2\theta)$$

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At $\theta = 0$ deg

$$V'' = -3Wa\cos(\theta) + a^2(k_1 + 4k_2)\cos(2\theta)$$

Since $V'' = 1.62 \times 10^4$ lb ft > 0, then the vertical position is stable.

Problem 11-38

If each of the three links of the mechanism has a weight W, determine the angle θ for equilibrium. The spring, which always remains vertical, is unstretched when $\theta = 0°$.

Solution:

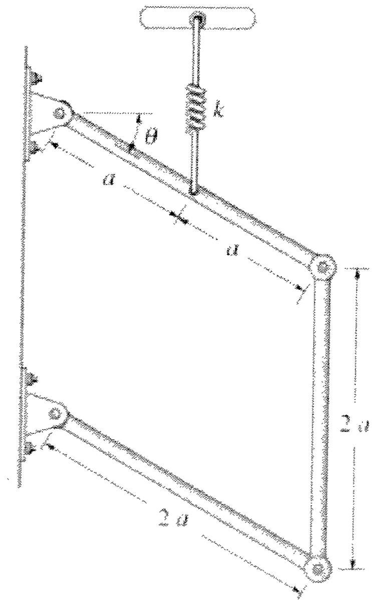
$$V = \frac{1}{2}k(a\sin(\theta))^2 - 2Wa\sin(\theta) - W(2a)\sin(\theta)$$

$$V = \frac{ka^2}{2}\sin(\theta)^2 - 4Wa\sin(\theta)$$

$$\frac{d}{d\theta}V = ka^2\sin(\theta)\cos(\theta) - 4Wa\cos(\theta) = 0$$

$\cos(\theta) = 0$ $\qquad \theta = 90$ deg

$\sin(\theta) = \frac{4W}{ka}$ $\qquad \theta = \operatorname{asin}\left(\frac{4W}{ka}\right)$



Problem 11-39

The small postal scale consists of a counterweight W_1 connected to the members having negligible weight. Determine the weight W_2 that is on the pan in terms of the angles θ and ϕ and the dimensions shown. All members are pin connected.

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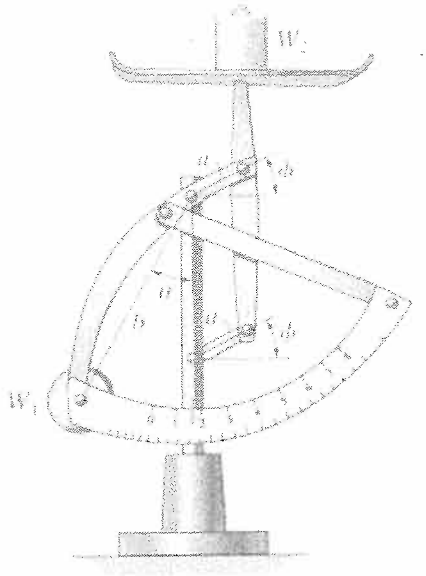
Solution:

$$\phi = -\theta + \text{constant}$$

$$V = W_2 a \sin(\phi) - W_1 b \cos(\theta)$$

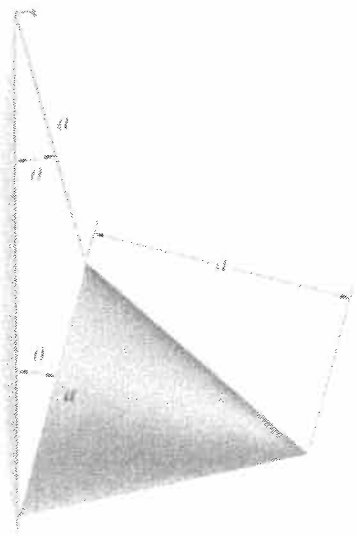
$$V' = \frac{d}{d\theta} V = -W_2 a \cos(\phi) + W_1 b \sin(\theta) = 0$$

$$W_2 = W_1 \left(\frac{b \sin(\theta)}{a \cos(\phi)} \right)$$



Problem 11-40

The uniform right circular cone having a mass m is suspended from the cord as shown. Determine the angle θ at which it hangs from the wall for equilibrium. Is the cone in stable equilibrium?



Solution:

$$V = -\left(\frac{3a}{2} \cos(\theta) + \frac{a}{4} \sin(\theta) \right) m g$$

$$V' = \frac{dV}{d\theta} = -\left(\frac{-3a}{2} \sin(\theta) + \frac{a}{4} \cos(\theta) \right) m g$$

$$V'' = \frac{d^2 V}{d\theta^2} = -\left(\frac{-3a}{2} \cos(\theta) - \frac{a}{4} \sin(\theta) \right) m g$$

Equilibrium

$V' = 0$ $\qquad \frac{3}{2} \sin(\theta) = \frac{1}{4} \cos(\theta)$ $\qquad \tan(\theta) = \frac{1}{6}$ $\qquad \theta = \operatorname{atan}\left(\frac{1}{6}\right)$ $\qquad \theta = 9.462 \text{ deg}$

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$$V'' = -\left(\frac{-3}{2}\cos(\theta) - \frac{1}{4}\sin(\theta)\right)\ amg \qquad V'' = 1.5\ amg \qquad \text{Stable}$$

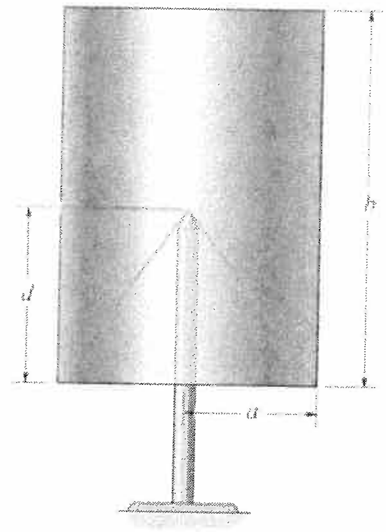
Problem 11-41

The homogeneous cylinder has a conical cavity cut into its base as shown. Determine the depth d of the cavity so that the cylinder balances on the pivot and remains in neutral equilibrium.

Given:

$a = 50\ \text{mm}$

$b = 150\ \text{mm}$

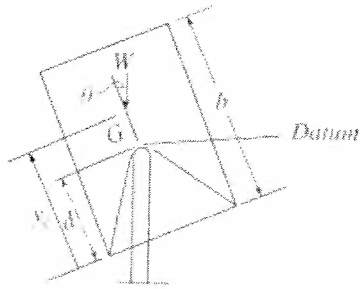


Solution:

$$y_C = \frac{\frac{b}{2}a^2\pi b - \frac{d}{4}\left(\frac{1}{3}\pi a^2 d\right)}{\pi a^2 b - \frac{1}{3}\pi a^2 d}$$

$$V = (y_C - d)\cos(\theta)W$$

$$\frac{d}{d\theta}V = -W\sin(\theta)(y_C - d)$$



$\theta = 0\ \text{deg}$ (equilibrium position)

$$\frac{d^2}{d\theta^2}V = -W\cos(\theta)(y_C - d) = 0$$

$$d = y_C$$

Guess $d = 10\ \text{mm}$

Given $$d = \frac{\frac{b}{2}a^2\pi b - \frac{d}{4}\left(\frac{1}{3}\pi a^2 d\right)}{\pi a^2 b - \frac{1}{3}\pi a^2 d} \qquad d = \text{Find}(d) \qquad d = 87.868\ \text{mm}$$

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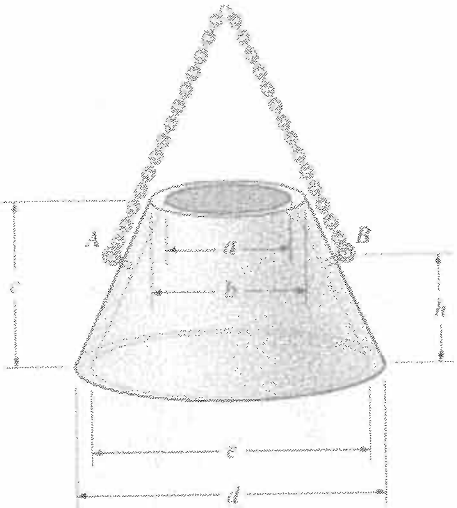
Problem 11-42

The conical manhole cap is made of concrete and has the dimensions shown. Determine the critical location $h = h_{cr}$ of the pick-up connectors at A and B so that when hoisted with constant velocity the cap is in neutral equilibrium. Explain what would happen if the connectors were placed at a point $h > h_{cr}$.



Given:

$a = 2$ ft

$b = 2.5$ ft

$c = 3$ ft

$d = 5$ ft

$e = d - (b - a)$

Solution:

$$V = W(y_c - h)\cos(\theta)$$

$$\frac{d}{d\theta}V = W(h - y_c)\sin(\theta) = 0$$

Equilibrium at

$\sin(\theta) = 0$ $\theta = 0\text{deg}$

For neutral equilibrium require

$$\frac{d^2}{d\theta^2}V = W(h - y_c)\cos(\theta) = 0$$ Thus $y_c = h$

Thus, A and B must be at the elevation of the center of gravity of the cap. $c_1 = \dfrac{cd}{d-b}$ $c_2 = \dfrac{ce}{e-a}$

$$y_c = \frac{\left(\frac{d}{2}\right)^2\left(\frac{c_1}{3}\right)\left(\frac{c_1}{4}\right) - \left(\frac{b}{2}\right)^2\left(\frac{c_1-c}{3}\right)\left(\frac{c_1+3c}{4}\right) - \left(\frac{e}{2}\right)^2\left(\frac{c_2}{3}\right)\left(\frac{c_2}{4}\right) + \left(\frac{a}{2}\right)^2\left(\frac{c_2-c}{3}\right)\left(\frac{c_2+3c}{4}\right)}{\left(\frac{d}{2}\right)^2\left(\frac{c_1}{3}\right) - \left(\frac{b}{2}\right)^2\left(\frac{c_1-c}{3}\right) - \left(\frac{e}{2}\right)^2\left(\frac{c_2}{3}\right) + \left(\frac{a}{2}\right)^2\left(\frac{c_2-c}{3}\right)}$$

$h_{cr} = y_c$ $h_{cr} = 1.32$ ft If $h > h_{cr}$ then stable.

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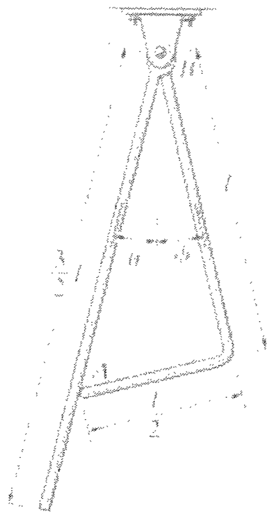
Problem 11-43

Each bar has a mass per length of m_0. Determine the angles θ and ϕ at which they are suspended in equilibrium. The contact at A is smooth, and both are pin con-nected at B.



Solution:

$$\theta + \phi = \operatorname{atan}\left(\frac{1}{2}\right)$$

$$V = -\frac{3l}{2} m_0 \left(\frac{3l}{4}\right)\cos(\theta) - l m_0 \left(\frac{l}{2}\right)\cos(\phi) - \frac{l}{2} m_0 \left(l\cos(\phi) + \frac{l}{4}\sin(\phi)\right)$$

$$\frac{d}{d\theta}V = \frac{9 m_0 l^2}{8}\sin(\theta) - m_0 l^2 \sin(\phi) + \frac{m_0 l^2}{8}\cos(\phi) = 0$$

Guess $\theta = 10 \text{ deg}$ $\phi = 10 \text{ deg}$

Given $\theta + \phi = \operatorname{atan}\left(\frac{1}{2}\right)$ $\frac{9}{8}\sin(\theta) - \sin(\phi) + \frac{1}{8}\cos(\phi) = 0$

$$\begin{pmatrix}\theta \\ \phi\end{pmatrix} = \operatorname{Find}(\theta, \phi) \qquad \begin{pmatrix}\theta \\ \phi\end{pmatrix} = \begin{pmatrix}9.18 \\ 17.38\end{pmatrix} \text{deg}$$

Problem 11-44

The triangular block of weight W rests on the smooth corners which are a distance a apart. If the block has three equal sides of length d, determine the angle θ for equilibrium.

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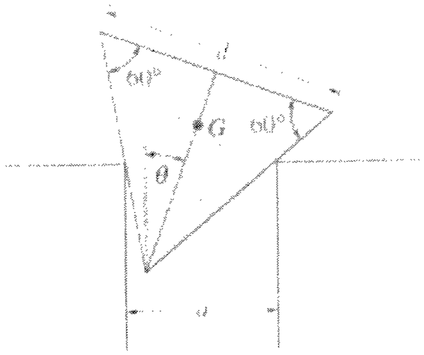

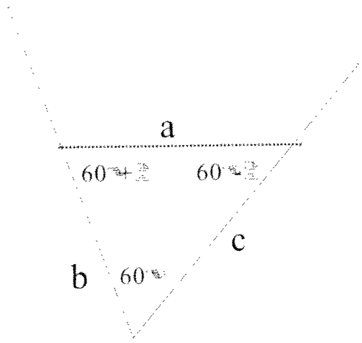

Solution:

$$\frac{a}{\sin(60\text{ deg})} = \frac{b}{\sin(60\text{ deg} - \theta)} \qquad b = a\frac{\sin(60\text{ deg} - \theta)}{\sin(60\text{ deg})}$$

$$V = W\left(\frac{2}{3}d\sin(60\text{ deg})\cos(\theta) - b\cos(30\text{ deg} - \theta)\right)$$

$$V = \frac{W}{2\sqrt{3}}(2d\cos(\theta) - 2a\cos(2\theta) - a)$$

$$\frac{d}{d\theta}V = \frac{W}{2\sqrt{3}}(-2d\sin(\theta) + 8a\sin(\theta)\cos(\theta)) = 0$$

$$\theta_1 = \text{asin}(0) \qquad \theta_1 = 0\text{ deg}$$

$$\theta_2 = \text{acos}\left(\frac{d}{4a}\right)$$

Problem 11-45

A homogeneous cone rests on top of the cylindrical surface. Derive a relationship between the radius r of the cylinder and the height h of the cone for neutral equilibrium. *Hint:* Establish the potential function for a *small* angle θ of tilt of the cone, i.e., approximate sin $\theta \approx 0$ and cos $\theta \approx 1-\theta^2/2$.

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Solution:

$$V = \left[\left(r + \frac{h}{4}\right)\cos(\theta) + r\theta\sin(\theta)\right]W$$

$$V_{app} = \left[\left(r + \frac{h}{4}\right)\left(1 - \frac{\theta^2}{2}\right) + r\theta^2\right]W$$

$$\frac{d}{d\theta}V_{app} = \left[-\left(r + \frac{h}{4}\right)\theta + 2r\theta\right]W = 0$$

$$\frac{dV_{app}}{d\theta} = \left(r - \frac{h}{4}\right)\theta W = 0$$

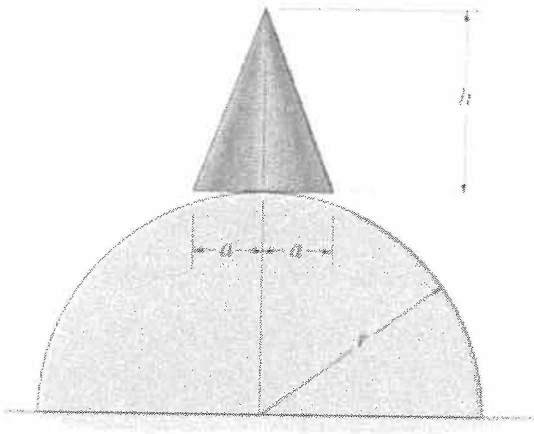
$$\frac{d^2}{d\theta^2}V_{app} = r - \frac{h}{4} = 0$$

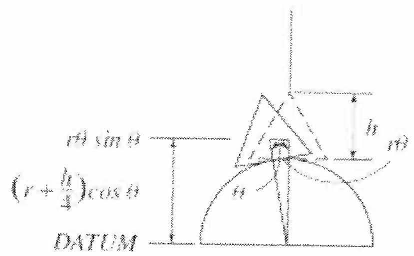
Equilibrium

$$\theta = 0 \text{ deg}$$

For neutral equilibrium:

$$r = \frac{h}{4}$$





Problem 11-46

The door has a uniform weight W_1. It is hinged at A and is held open by the weight W_2 and the pulley. Determine the angle θ for equilibrium.

Given:

$W_1 = 50 \text{ lb}$

$W_2 = 30 \text{ lb}$

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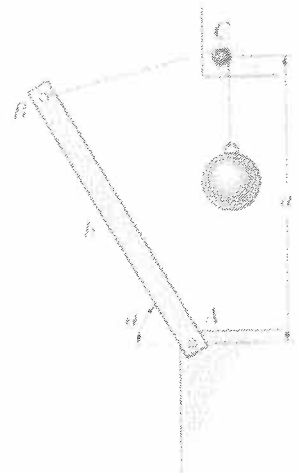
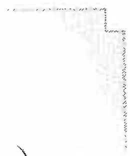
$a = \sout{6\ \text{ft}}$ 2 m

$b = \sout{6\ \text{ft}}$ 2 m



Solution:



$$V = W_1\left(\frac{b}{2}\right)\sin(\theta) + W_2\sqrt{a^2 + b^2 - 2ab\sin(\theta)}$$

$$\frac{d}{d\theta}V = W_1\left(\frac{b}{2}\right)\cos(\theta) - W_2\left(\frac{ab\cos(\theta)}{\sqrt{a^2 + b^2 - 2ab\sin(\theta)}}\right) = 0$$

Guess $\theta = 10\ \text{deg}$

Given $W_1\left(\frac{b}{2}\right)\cos(\theta) - W_2\left(\frac{ab\cos(\theta)}{\sqrt{a^2 + b^2 - 2ab\sin(\theta)}}\right) = 0$ $\theta = \text{Find}(\theta)$

$\theta = 16.26\ \text{deg}$

Problem 11-47

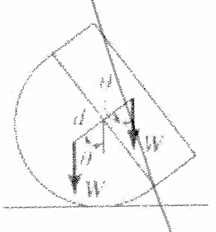
The hemisphere of weight W supports a cylinder having a specific weight γ. If the radii of the cylinder and hemisphere are both a., determine the height h of the cylinder which will produce neutral equilibrium in the position shown.

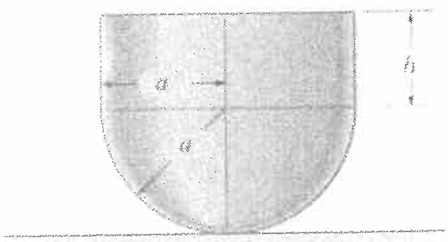
Given:

$W = 60\ \text{lb}$

$a = 5\ \text{in}$

$\gamma = 311\ \frac{\text{lb}}{\text{ft}^3}$





Solution:

$$V = -W\left(\frac{3a}{8}\right)\cos(\theta) + \gamma\pi a^2 h\left(\frac{h}{2}\right)\cos(\theta)$$

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$$V = \left(\frac{\gamma \pi a^2 h^2}{2} - \frac{W3a}{8}\right)\cos(\theta)$$

$$\frac{d}{d\theta}V = -\left(\frac{\gamma \pi a^2 h^2}{2} - \frac{W3a}{8}\right)\sin(\theta)$$

$$\frac{d^2}{d\theta^2}V = -\left(\frac{\gamma \pi a^2 h^2}{2} - \frac{W3a}{8}\right)\cos(\theta)$$

For neutral equilibrium we must have

$$\frac{\gamma \pi a^2 h^2}{2} - \frac{W3a}{8} = 0 \qquad h = \sqrt{\frac{W3}{4\pi \gamma a}} \qquad h = 3.99 \text{ in}$$

Problem 11-48 28

Compute the force developed in the spring required to keep the rod of mass M_{rod} in equilibrium at θ. The spring remains horizontal due to the roller guide.

Given:

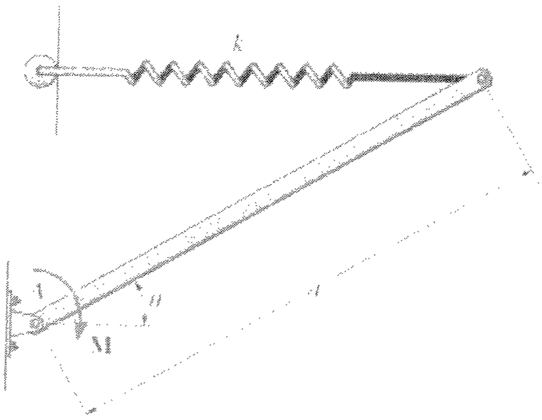
$k = 200 \ \frac{\text{N}}{\text{m}}$

$M = 40 \ \text{N}\cdot\text{m}$

$a = 0.5 \ \text{m}$

$\theta = 30 \ \text{deg}$

$M_{rod} = 6 \ \text{kg}$



Solution:

$$V = M\theta + M_{rod}g\left(\frac{a}{2}\right)\sin(\theta) + \frac{1}{2}k(a\cos(\theta) - \delta)^2$$

$$\frac{d}{d\theta}V = M + M_{rod}g\left(\frac{a}{2}\right)\cos(\theta) - k(a\cos(\theta) - \delta)a\sin(\theta) = 0$$

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Guess $\delta = 100 \text{ mm}$

Given $M + M_{rod} g \left(\frac{a}{2}\right) \cos(\theta) - k(a\cos(\theta) - \delta) a \sin(\theta) = 0$

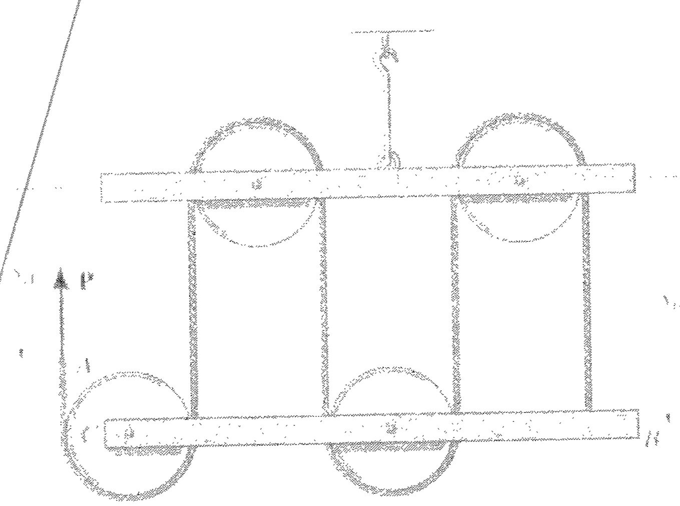
$\delta = \text{Find}(\delta)$ $\delta = -0.622 \text{ m}$ $F = k(a\cos(\theta) - \delta)$ $F = 211.0 \text{ N}$

Problem 11-49

Determine the force **P** acting on the cord which is required to maintain equilibrium of the horizontal bar *CB* of mass *M*. *Hint:* First show that the coordinates s_A and s_B are related to the *constant* vertical length *l* of the cord by the equation $5s_B - s_A = L$.

Given:

$M = 20 \text{ kg}$



Solution:

$L = 4s_B + (s_B - s_A)$

$L = 5s_B - s_A$

$\Delta L = 5\Delta s_B - \Delta s_A = 0$

$\Delta s_A = 5\Delta s_B$

$V = -Mg s_B + P s_A$

$\Delta V = -Mg\Delta s_B + P\Delta s_A = (-Mg + 5P)\Delta s_B = 0$

$P = \frac{Mg}{5}$ $P = 39.2 \text{ N}$

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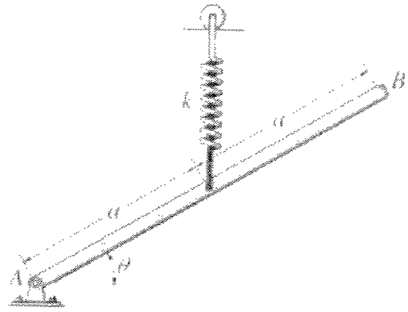
Problem 11-50

The uniform bar AB has weight W. If the attached spring is unstretched when $\theta = 90$ deg, use the method of virtual work and determine the angle θ for equilibrium. Note that the spring always remains in the vertical position due to the roller guide.



Given:

$W = 10\ \text{lb}$

$k = 5\ \dfrac{\text{lb}}{\text{ft}}$

$a = 4\ \text{ft}$

Solution:

$y = a\sin(\theta) \qquad \delta y = a\cos(\theta)\delta\theta$

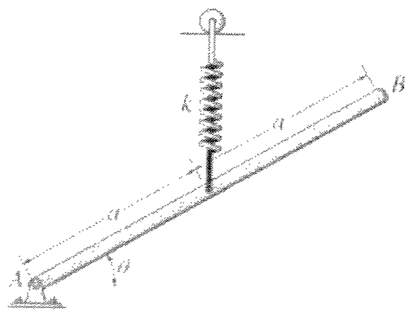
$\delta U = (-W + F_s)\delta_y = [k(a - a\sin(\theta)) - W]a\cos(\theta)\delta\theta = 0$

$\cos(\theta_1) = 0 \qquad \theta_1 = \text{acos}(0) \qquad \theta_1 = 90\,\text{deg}$

$\sin(\theta_2) = 1 - \dfrac{W}{ka} \qquad \theta_2 = \text{asin}\left(1 - \dfrac{W}{ka}\right) \qquad \theta_2 = 30\,\text{deg}$

Problem 11-51

The uniform bar AB has weight W. If the attached spring is unstretched when $\theta = 90$ deg, use the principle of potential energy and determine the angle θ for equilibrium. Investigate the stability of the equilibrium positions. Note that the spring always remains in the vertical position due to the roller guide.



Given:

$W = 10\ \text{lb}$

$k = 5\ \dfrac{\text{lb}}{\text{ft}}$

$a = 4\ \text{ft}$

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Solution:

$$V = Wa\sin(\theta) + \frac{1}{2}k(a - a\sin(\theta))^2 = Wa\sin(\theta) + \frac{1}{2}ka^2(1 - \sin(\theta))^2$$

Equilibrium

$$\frac{d}{d\theta}V = Wa\cos(\theta) - ka^2(1 - \sin(\theta))\cos(\theta) = 0$$

$\cos(\theta_1) = 0$ $\qquad$ $\theta_1 = \operatorname{acos}(0)$ $\qquad$ $\theta_1 = 90\,\text{deg}$

$\sin(\theta_2) = 1 - \dfrac{W}{ka}$ $\qquad$ $\theta_2 = \operatorname{asin}\left(1 - \dfrac{W}{ka}\right)$ $\qquad$ $\theta_2 = 30\,\text{deg}$

Check Stability If $V'' > 0$ the equilibrium point is stable. If $V'' < 0$, then unstable

$$V'' = \frac{d^2V}{d\theta^2} = -Wa\sin(\theta) + ka^2\sin(\theta) + ka^2\cos(2\theta)$$

$V''_1 = -Wa\sin(\theta_1) + ka^2\sin(\theta_1) + ka^2\cos(2\theta_1)$ $\qquad$ $V''_1 = -40\,\text{lb}\cdot\text{ft}$

$V''_2 = -Wa\sin(\theta_2) + ka^2\sin(\theta_2) + ka^2\cos(2\theta_2)$ $\qquad$ $V''_2 = 60\,\text{lb}\cdot\text{ft}$

Problem 11-52

The punch press consists of the ram R, connecting rod AB, and a flywheel. If a torque M is applied to the flywheel, determine the force **F** applied at the ram to hold the rod in the position $\theta = \theta_0$.

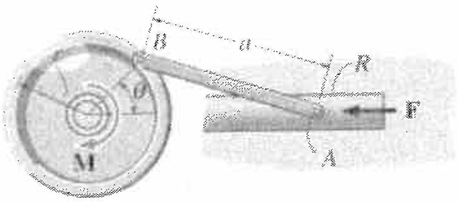
Given:

$M = 50\ \text{N}\cdot\text{m}$

$\theta_0 = 60\ \text{deg}$

$r = 0.1\ \text{m}$

$a = 0.4\ \text{m}$



Solution: $\quad \theta = \theta_0$

Free Body Diagram: The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only force **F** and Moment M do work.

$$a^2 = x^2 + r^2 - 2xr\cos(\theta)$$

$0 = 2x\delta x - 2r\cos(\theta)\delta x + 2xr\sin(\theta)\delta\theta$ $\qquad$ $\delta x = \left(\dfrac{xr\sin(\theta)}{r\cos(\theta) - x}\right)\delta\theta$

$$\delta U = -F\,\delta x - M\delta\theta = \left[-F\left(\frac{xr\sin(\theta)}{r\cos(\theta) - x}\right) - M\right]\delta\theta = 0$$

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Guesses $F = 1\ \text{N}$ $x = 0.1\ \text{m}$

Given $a^2 = x^2 + r^2 - 2xr\cos(\theta)$ $-F\left(\dfrac{xr\sin(\theta)}{r\cos(\theta) - x}\right) - M = 0$

$\begin{pmatrix} x \\ F \end{pmatrix} = \text{Find}(x, F)$ $x = 0.441\ \text{m}$ $F = 512\ \text{N}$

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